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Buckling Analysis of Single-Walled Carbon Nanotubes Embedded in an Elastic Medium under Axial Compression Using Non-Local Timoshenko Beam Theory

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Abstract – *Based on non-local elasticity theory, Timoshenko beam model is developed to study the elastic buckling of single-walled carbon nanotubes (SWCNTs) embedded in an elastic medium under axial compression. The non-local effects in the normal and transverse shear stress components are considered. The effects of the surrounding elastic medium based on a Winkler model are taken into account. Considering the small-scale effects, the governing equilibrium equations are derived and the critical buckling loads under axial compression are obtained. The numerical results are reported using the non-local Timoshenko beam theory and compared with those obtained using the non-local Euler–Bernoulli beam theory. The results show that the critical buckling load can be overestimated by the local beam model if the small-scale effect is overlooked for long nanotubes. Furthermore, in order to estimate the non-local critical buckling load of SWCNTs under axial compression, a simplified analysis is carried out and the results are compared with those obtained using molecular mechanics.*

Keywords: Single-Walled Carbon Nanotubes, Buckling, Axial Compression, Timoshenko Beam Theory, Elastic Medium, Non-Local

1.0 INTRODUCTION

Carbon nanotubes (CNTs) were first introduced by Kroto et al. [1] in 1985 and later on discovered by Iijima [2] in 1991. They exhibit superior mechanical properties such as high elastic modulus and high strength. There are several studies on the axial and torsional buckling analyses of CNTs in the literature. Ru [3] presented an elastic double-shell model for infinitesimal buckling of a single-walled carbon nanotube (SWCNT) embedded in an elastic matrix under axial compression. His analysis was based on the Winkler model for the surrounding elastic medium and a simplified model for the van der Waals interaction between the inner and the outer nanotubes. His results show that the critical axial strain of the embedded SWCNT is lower than that of an embedded single-walled carbon nanotube (SWCNT). Ranjbartoreh et al. [4] investigated the buckling behaviour and critical axial pressure of the SWCNT with the surrounding elastic medium under axial pressure. They found the critical axial forces and the pressures of SWCNTs for different axial half sine and circumferential sine wave numbers.

Moreover, they realized that the minimum critical axial force of a SWCNT surrounded with an elastic medium is higher than that of an SWCNT under similar conditions. Using the energy method, Zhang et al. [5] analysed the elastic buckling of a long SWCNT embedded in an elastic medium subjected to a far-field hydrostatic pressure. Their study was based on the elastic shell model at the nano-scale taking into account the effects of vdW forces. They derived an expression relating the external pressure to the buckling mode number. They also showed that the inner-radius-to-thickness ratio, the material parameters of the elastic medium, and the vdW force all have significant effects on the critical pressure. Ranjbartoreh et al. [6] studied the variations of critical axial forces for the inner and the outer tubes of a SWCNT in different buckling modes. They considered the effects of changes in the radius, length, and aspect ratio on the stability of a SWCNT. Wang et al. [7] used the Timoshenko beam model for free vibration analysis of a single-walled carbon nanotube. By comparing the results of Timoshenko and the Euler–Bernoulli beam theories, they showed that the frequencies are significantly overpredicted by the Euler–Bernoulli beam theory (EBBT) for small aspect ratios.

Wang et al. [8] investigated the elastic buckling of an embedded elastic medium of an SWCNT under combined torsional and axial loadings for various radius-to-thickness ratios. They realized that the radial constraint increases the critical shear stress and enhanced the ability of resisting combined torsional and axial loadings. Yao and Han [9] studied the thermal effects for the axially compressed buckling of an SWCNT embedded in an elastic medium. They considered the effects of temperature change, surrounding elastic medium, a dvdW forces between the inner and outer nanotubes. They found that the axial buckling load of an SWCNT under thermal loads is dependent on the wave numbers of axially buckling loads. Their results showed that at low temperatures, the critical axial load for infinitesimal buckling of an SWCNT increases by increasing the temperature.

Yakobsonetal [10] studied the buckling analysis of an SWCNT with the Tersoff-Brenner molecular dynamics and supported the applicability of a continuum shell theory. On the other hand, it has been shown that the small-scale effect has a substantial role in the analysis of CNTs. Zhang et al. [11] used a non-local multiple-shell model for the elastic buckling of an SWCNT under uniform external radial pressure. They derived an explicit expression for the critical buckling pressure of a SWCNT and also considered the influence of the small length-scale on the buckling pressure.

Wang et al. [12] developed the non-local elastic beam and shell models, investigating the small-scale effect on buckling analysis of CNTs under axial compression. They showed that the buckling solutions for CNTs via local continuum mechanics are overestimated and hence the scale effect is indispensable to provide more accurate results for mechanical behaviors of CNTs via continuum mechanics.

Ghorbanpour Arani et al. [13] presented the torsional and axial buckling analyses of the individual embedded SWCNTs subjected to internal and external pressures. They considered both the small length-scale effect and the surrounding elastic medium. Using the continuum cylindrical shell model, they derived explicit formulas for vdW interaction between the layers of an SWCNT. Their results showed that the internal pressure increases the critical load, whereas the external pressure tends to decrease it. Some researchers [14–16] have used different continuum models, including the non-local elasticity models, to study the mechanical properties of CNTs. Using consistent equations of motion for the non-local Euler–Bernoulli

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and the Timoshenko beam models, Lu et al. [14] investigated the vibration properties of an SWCNT and an MWCNT. They obtained some interesting results based on the non-local beam model which are useful for understanding the wave properties of the structures in nanoscale. Wang and Liew [15] considered the scale effects on static deformation of micro- and nanorods or tubes through the nonlocal Euler–Bernoulli and Timoshenko beam theories. They derived explicit solutions for static deformation of such structures with standard boundary conditions.

Recently, Reddy and Pang [16] obtained the equations of motion based on the Euler–Bernoulli and the Timoshenko beam theories using the non-local theory for SWCNTs. They used the equations of motion to evaluate the bending, vibration, and buckling responses of beams with various boundary conditions. They also presented numerical results to bring out the effect of the non-local behaviour on deflections, natural frequencies, and buckling loads of CNTs using the non-local theory.

Ghorbanpour Arani et al. [17] investigated the transverse vibrations of SWCNTs and DWCNTs under axial load by applying the Euler–Bernoulli and Timoshenko beam models and the Donnell shell model. They concluded that the Euler–Bernoulli beam model and the Donnell shell model predictions have the lowest and highest accuracies, respectively. In order to predict the vibration behaviour of the CNT more accurately, the classical models were modified using the non-local theory. Moreover, they obtained the natural frequencies and amplitude coefficient for the simply supported boundary conditions. To the best of authors' know ledge, the previous studies on buckling of CNTs were restricted to SWCNTs and/or classical (local) beam theories, and the nonlocal buckling analysis of SWCNTs using Timoshenko beam theory (TBT) has not been studied yet.

In this study, the elastic buckling of a SWCNT embedded in an elastic medium under axial compression is studied using the non-local Timoshenko beam model. The effects of the surrounding elastic medium based on the Winkler model and the vdW force between the inner and outer nanotubes are considered. The small-scale effect is clearly considered in the formulation. Using the governing equilibrium equations, the critical buckling load under axial compression is obtained. The results show that the critical buckling load is overestimated by the local beam model when the small-scale effect is ignored. The Timoshenko beam results are obtained and compared with those obtained by Euler–Bernoulli beam. In addition, the shear effect clearly indicates the importance of applying shear deformation beam models for CNTs. The results obtained using the TBT and the EBBT for SWCNTs in the absence of the surrounding elastic medium under axial compression are compared with the available results obtained using molecular mechanics (MM). It is shown that the results of non-local and local theories are close to those of the MM for long SWCNTs. A simplified analysis is also carried out to estimate the non-local critical buckling load of a SWCNT under axial compression using TBT.

2.0 NON-LOCAL TIMOSHENKO BEAM THEORY

The TBT considered the shear deformation of the beam. Based on this theory, the equilibrium equations are

$$\psi-\frac{\partial w}{\partial x}=\frac{V}{k\mathrm{A}G}\quad \frac{\partial \psi}{\partial x}=\frac{M}{EI} \tag{1}$$

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where $\psi - \frac{\partial w}{\partial x}$ is the shear angle, $\frac{\partial w}{\partial x}$ denotes the slope of the center line of the beam, ψ the rotation angle of cross-section of the beam, and E and G are Young's and shear modulus, respectively. Also, A is the cross-section area of the beam, I the moment of inertia, w the transverse displacement, M and V are the resultant bending moment and the resultant shear force, respectively, and k a correction factor depending on the shape of the cross-section of the considered beam. In classical or local theory of continuum mechanics, the stress at a point is only proportional to the strain at that point. This theory is valid for large scale. In small scale, the stress at a point is proportional to the strain at all points of the body [18].This phenomenon is known as small-scale effect which is cleared in constitutive equations by the parameter e_0a and its theory is identified as small-scale or non-local theory. For a beam in the nanoscale, it is not reasonable to ignore the small-scale effect (e_0a). By ignoring this term ($e_0a = 0$), the non-local theory reduces to local or classical theory which have no desired accuracy for the analysis of CNTs. The constitutive equations of non-local theory can be given by [18]

$$\left[1-(e_0a)^2\nabla^2\right]\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

where C_{ijkl}'s are the elasticity tensor components of classical (local) isotropic elasticity and σ_{ij} and ε_{ij} are the components of stress and strain tensors, respectively. Also e_0 is a material constant and a is an internal characteristic length of the material (e.g. length of C– C bond, lattice spacing granular distance). For a beam structure, the non-local constitutive relation (2) can be approximated to one-dimensional form as

$$\begin{aligned}\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\sigma_x &= E\varepsilon_x \\ \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\sigma_{xy} &= G\gamma_{xy}\end{aligned} \tag{3}$$

where ε_x is the axial strain and γ_{xy} denotes the shear strain which is equal to $\psi - \frac{\partial w}{\partial x}$. Using the free body diagram of an infinitesimal element of the beam structure subjected to an axial loading p, the force equilibrium equations in vertical direction and the moment on the one-dimensional structure can be derived as follows

$$\begin{aligned}&\frac{\partial V}{\partial x} + q(x) = 0 \\ &V = \frac{\partial M}{\partial x} + p\frac{\partial w}{\partial x}\end{aligned} \tag{4}$$

Where p is the axial compression and $q(x)$ is vdW force between the inner and outer tubes. Substituting the kinematics relationships, bending moment, and shear force into equations (3), the bending moment M and the shear force V for the non-local model can be expressed as

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$$\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]M = EI\frac{\partial\psi}{\partial x}$$

$$\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]V = kAG\left(\psi-\frac{\partial w}{\partial x}\right) \tag{5}$$

Substituting equations (5) into equations (4) and eliminating ψ yield the following differential equation

$$EI\frac{d^4w}{dx^4}+\left[1-e0a^2\frac{d^2}{dx^2}\right]\left[1-\frac{EI}{kAG}\frac{d^2}{dx^2}\right]q(x)+\left[1-e0a^2\frac{d^2}{dx^2}\right]p\frac{d^2w}{dx^2}=0 \tag{6}$$

The above equation is the equilibrium equation of a Timoshenko beam considering the non-local effects.

2.1 SINGLE-WALLED CARBON NANOTUBES

Table 1 shows a DWCNT under axial compression embedded in an elastic medium. In this table, *R* is the radius of the tubes. Also p is the buckling pressure of the SWCNT and h the thickness of nanotubes. In this study, the buckling analysis of SWCNTs has been investigated using the TBT and EBBT based on the non-local continuum model. In this model, only the effect of vdW forces needs to be taken into account [3,4]. Whereas, in the MM model, in addition to the vdW forces, other forces due to the interaction

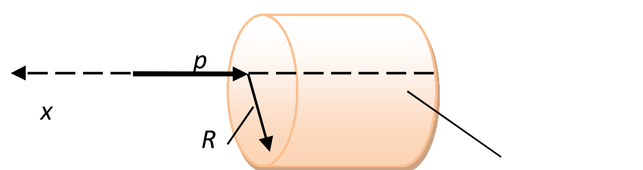

Figure1: SWCNT under axial compression embedded in an elastic medium

between the atoms such as bond stretch interaction, bond angle bending, dihedral angle torsion, and no bonded vdW interactions have also been taken into consideration. The continuum model does not need these forces. For the outer tube which specified with subscript 1, the normal pressure q1 can be defined

$$q_1 = q_1^w \tag{7}$$

where q_1^w is the interaction pressure due to the elastic medium. Based on the Winkler model, the elastic medium force can be written as

$$q_1^w = hdw_1 \tag{8}$$

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where d is the spring constant of the Winkler-type foundation. P is the buckling pressure for the tubes [19].That is

$$p = p_1 = p_2 = p_3 \ldots\ldots\ldots \tag{9}$$

Using equation and applying equations (8) and (9), the governing equilibrium equations can be written as

$$EI\frac{d^4w}{dx^4} + \left[1 - e0a^2\frac{d^2}{dx^2}\right]\left[1 - \frac{EI}{kAG}\frac{d^2}{dx^2}\right]hdw_1 + \left[1 - (e_0a)^2\frac{d^2}{dx^2}\right]p\frac{d^2w}{dx^2} = 0 \tag{10}$$

According to reference [20], for the longer SWCNTs (with large aspect ratio (e.g. L/R > 20)), the nanotube Buckles in rod buckling mode; however, for the shorter SWCNTs (with small aspect ratio (e.g. L/R < 20)). Similar conclusion was also obtained in some papers (for example, see references [21] and [22]). Let us assume the buckling modes as [20–22]

$$\begin{aligned} w_1 &= A\sin(\frac{m\pi}{L}x) \\ w_2 &= B\sin(\frac{m\pi}{L}x) \end{aligned} \tag{11}$$

The above equations satisfy the simply supported boundary conditions which are

$$w = \frac{d^2w}{\partial x^2} = 0 \text{ at } x = 0, L \tag{12}$$

Replacing equation (11) into equations (13), one can easily obtain

$$\begin{aligned} K_{11}w_1 - K_{12}w_2 &= 0 \\ -K_{21}w_1 + K_{22}w_2 &= 0 \end{aligned} \tag{13}$$

Where K_{11} and K_{12} in equations (13) are defined as

$$\begin{aligned} K_{11} &= EI(\frac{m\pi}{L})^4 + ((hd)\times\left[1 + \frac{EI}{kAG}(\frac{m\pi}{L})^2\right] - p(\frac{m\pi}{L})^2)\times\left[1 - (e_0a)^2(\frac{m\pi}{L})^2\right] \\ K_{12} &= (h)\left[1 + \frac{EI}{kAG}(\frac{m\pi}{L})^2\right]\times\left[1 - (e_0a)^2(\frac{m\pi}{L})^2\right] = K_{21} \\ K_{22} &= EI(\frac{m\pi}{L})^4 + (h\times\left[1 + \frac{EI}{kAG}(\frac{m\pi}{L})^2\right] - p(\frac{m\pi}{L})^2)\times\left[1 - (e_0a)^2(\frac{m\pi}{L})^2\right] \end{aligned} \tag{14}$$

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The non-trivial solution for the homogeneous system (14) can be obtained by setting the determinant of coefficients equal to zero. Thus, the following second-order equation in terms of buckling pressure is obtained as

$$J_1 p^2 + J_2 p + J_3 = 0 \tag{15}$$

where J_1, J_2 and J_3 are

$$J_1 = (\frac{m\pi}{L})^4 + \left[1+(e_0 a)^2 (\frac{m\pi}{L})^2\right]^2$$

$$J_2 = -(\frac{m\pi}{L})^2 + (hd)\left[1+\frac{EI}{kAG}(\frac{m\pi}{L})^2\right] \times \left[1-(e_0 a)^2 (\frac{m\pi}{L})^2\right] - 2EI(\frac{m\pi}{L})^6 \times \left[1-(e_0 a)^2 (\frac{m\pi}{L})^2\right] \tag{16}$$

$$J_3 = (h^2 d)\left[1+\frac{EI}{kAG}(\frac{m\pi}{L})^2\right]^2 \times \left[1+(e_0 a)^2 (\frac{m\pi}{L})^2\right]^2 + hdEI(\frac{m\pi}{L})^4 \left[1+\frac{EI}{kAG}(\frac{m\pi}{L})^2\right] \times \left[1+(e_0 a)^2 (\frac{m\pi}{L})^2\right]$$
$$+ hEI(\frac{m\pi}{L})^4 \left[1+\frac{EI}{kAG}(\frac{m\pi}{L})^2\right] \times \left[1+(e_0 a)^2 (\frac{m\pi}{L})^2\right] - h^2 \left[1+\frac{EI}{kAG}(\frac{m\pi}{L})^2\right]^2 \times \left[1+(e_0 a)^2 (\frac{m\pi}{L})^2\right]^2 E^2 I^2 (\frac{m\pi}{L})^8$$

Solving equation (21) yields the buckling pressure of the SWCNT in which the effects of the small scale and elastic medium based on the Winkler model are shown.

3. NUMERICAL RESULTS

Based on Eringen's work [18], let e0= 0.39. Also, as the length of a C–C bond is a = 0.142nm, the parameter e_0a is equal to0.05538nm.The mechanical properties of the CNT are considered as [10]

$$\begin{aligned} &\nu = 0.19, \\ &h = 0.066\text{nm} \\ &E = 5500\text{nN/nm}^2, \\ &k = 0.5 \\ &d = 0.89995035\text{nN/nm}^3 \end{aligned} \tag{17}$$

Table 2 shows the non-local critical buckling load obtained using the TBT versus the length of the nanotube. It can be seen that for a nanotube with specified length, the non-local critical buckling load increases with increasing radius of the SWCNT because of the increasing flexural rigidity of cross-section of beam and the decreasing aspect ratio (L/R1); whereas, for specified inner and outer radii, the non-local critical buckling load decreases with increasing length of the SWCNT due to the increasing aspect ratio. Table 3 depicts the non-local critical buckling load obtained using the TBT and EBBT versus the length of the CNT. It can be concluded that the difference

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Table 1: Non-local critical buckling load under axial compression using TBT versus length of the nanotube for SWCNTs (e_0a = 0.05538nm)

e0a=0.05538			**d1=2.19**		
L	5	10	20	30	40
P	591	148.2	38.25	19.22	14.13
			d1=2.83		
L	5	10	20	30	40
P	1274	319.3	81.09	38.29	24.88
			d1=3.392		
L	5	10	20	30	40
P	219.4	549.6	138.7	63.93	39.36

Table 2: Comparison of TBT and EBBT (e_0a =0.05538nm)

e0a=0.05538			**d1=2.19**		
L	5	10	20	30	40
P	591.01	148.23	38.253	19.222	14.131
			d1=2.83		
L	5	10	20	30	40
P	1274.03	319.31	81.091	38.291	24.882
			d1=3.392		
L	5	10	20	30	40
P	219.5	549.61	138.72	63.931	39.361

between the critical buckling loads predicted by EBBT and TBT is negligible when L > 10nm. This is due to the fact that the shear effect is negligible for long nanotubes. Table 4 illustrates the non-local critical buckling load using TBT under axial compression versus the length of the nanotube (L) for two cases, with and without considering the Winkler elastic medium. It can be seen that the non-local critical buckling load under axial compression in the presence of the surrounding elastic medium (d = 0) is higher than that in the absence of the surrounding elastic medium (d = 0) for long nanotubes. Since considering the Winkler elastic medium causes to stiff the outer tube. Also, the difference between the two cases increases with increasing L; whereas, for short nanotubes, the difference between the two cases is negligible. This

Table 3: Effect of the surrounding elastic medium on buckling load of DWCNTs (e_0a = 0.05538nm)

e0a=0.05538			**d1=0**		
L	5	10	20	30	40
P	590.918	147.863	36.974	16.433	9.244
			d1=0.89995035		
L	5	10	20	30	40
P	591.078	148.248	38.259	19.220	14.132

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Table 4: Non-local critical buckling load under axial compression for some values of e_0a

d1=1.095			e_0a=0		
L	5	10	20	30	40
P	591.793	148.293	38.262	19.220	14.132
Present study			**e_0a=0.05538**		
L	5	10	20	30	40
P	591.078	148.248	38.259	19.220	14.132
Wang et al 2006			**e_0a=1**		
L	5	10	20	30	40
P	424.335	135.006	37.372	19.042	14.075

means that for short nanotubes, the spring constant of the Winkler type can be ignored [23]. Table 4 shows the critical buckling load using TBT versus the length of the nanotube for some values of e_0a such as 0 (according to the local or classical model), 0.05538nm (according to the present study), 0.11664nm (according to Zhang et al. [11]), and 0.5,1 and 2nm (according to Wang et al. [12]). To study the effect of e_0a, all of these values for the small-scale effect (e_0a) have been used in the present TBT and EBBT for DWCNTs. It can be seen that the difference between the non-local theory [12] and the local theory is noticeable for short nanotubes.

Table 5: Effect of nanotube length on critical buckling load ratio using TBT

	L=5	**L=10**	**L=15**	**L=20**	**L=25**	**L=30**	**L=35**	**L=40**
e_0a=0	1	1	1	1	1	1	1	1
e_0a=0.05538	0.99897	0.99976	0.99987	0.99993	0.99996	0.99997	0.99998	0.99999
e_0a=0.11664	0.99545	0.99893	0.99944	0.99969	0.99981	0.99988	0.99993	0.99996

As the non-local elasticity theory is considered for the nanolength scales that the size effects are very important, then the non-local effect increases with decreasing length of the nanotube. Also, the results which are obtained using the present study (e_0a = 0.05538nm) are close to those obtained by Zhang et al. [11] (e_0a = 0.11664nm). Table 5 shows the effect of the nanotube length on critical buckling load using TBT. The obtained results show that the difference between the non-local the ories (present study (e_0a = 0.05538nm) and Zhang et al. (e_0a = 0.11664nm)) and the local theory is negligible for long nanotubes. This is because of the fact that the non-local effect decreases with increasing length of the nanotube.

4.0 COMPARISONWITH MOLECULAR MECHANICS

Sears and Batra [20] investigated the buckling an alysis of an SWCNT and MWCNT under axial compression using MM simulation. Their results were compared with Euler–Bernoulli theory and the finite element analysis (FEA). In this section, the obtained results for SWCNTs using both non-local and local theories have been compared with the MM results obtained by Sears and Batra [20]. To this end, consider the following material properties which are the same as those used by Sears and Batra [20]

$$\begin{aligned} &\nu = 0.19, \\ &h = 1.34Å\ (1Å = 0.1\text{nm}) \\ &E = 2530\text{Nn/nm2}, \\ &d = 0 \end{aligned} \tag{18}$$

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To study the effect of e_0a, Tables 5 and 6 show the critical buckling load of a SWCNT verses the length of nanotube for different values of the small-scale effect e0a based on the present study using the Timoshenko and Euler–Bernoulli beam theories, respectively. The results have also been compared with those of MM [20]. It can be seen that by using e_0a = 0.05538nm, the results are in good agreement with MM results in Comparison with the other values of e_0a.Moreover,the results obtained by Wang et al. [12] are different with MM results especially for short nanotubes. Also, for very short nanotubes (L < 100Å) all of the results are Different from MM results. This is reasonable, since for very short nanotubes the beam theory is not valid and the cylindrical shell theory must be used.

Table 5: Non-local critical axial strain versus length of the nanotube usingTBT (R= 5.95Å)

d1=5.95			***e₀a=0***	
Length	80	100	200	300
P	0.273	0.0174	0.004	0.0019
Present study			***e₀a=0.05538***	
Length	80	100	200	300
P	0.2731	0.01742	0.0041	0.001901
Wang et al 2006			***e₀a=1***	
Length	80	100	200	300
P	0.2721	0.01741	0.00408	0.01902

Table 6: Non-local critical axial strain versus length of nanotube using EBBT (R= 5.95Å)

d1=5.95			***e₀a=0***	
Length	80	100	200	300
P	0.273	0.0174	0.004	0.0019
Present study			***e₀a=0.05538***	
Length	80	100	200	300
P	0.2731	0.01742	0.0041	0.001901
Wang et al 2006			***e₀a=1***	
Length	80	100	200	300
P	0.2721	0.01741	0.00408	0.01902

5.0 IN THE ABSENCE OF THE VDW FORCES

In the absence of the vdW force, the tube is automatically a single-walled shell without the surrounding elastic medium. This is because of the fact that the surrounding elastic medium does not affect the internal tube when the vdW force is vanished. In this case, the critical buckling load of the tube under axial compression can be obtained using equation (18). In other words, buckling of the single walled nanotube occurs when the tube buckles. In this case, the non-local critical buckling load can be obtained using equation (18) as

$$p_n = \frac{EI(\frac{m\pi}{L})^2}{1+(e_0a)^2(\frac{m\pi}{L})^2} + \frac{h}{(\frac{m\pi}{L})^2}\left[1+\frac{EI}{kAG}(\frac{m\pi}{L})^2\right] \tag{19}$$

It can be concluded from equation (19) that the surrounding elastic medium causes an increase in the non-local critical buckling load of the outer tube.

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6.0 CONCLUSION

In this work, based on the non-local theory, the elastic buckling of SWCNTs embedded in an elastic medium under axial compression has been studied. The effects of the surrounding elastic medium based on a WinKler model are taken into account. The non-local terms have been clearly considered in the normal and transverse shear stress components. An explicit relation has been obtained for the non-local critical buckling load under axial compression. The results have been obtained using TBT and compared with those obtained using EBBT. The results show that the TBT yields larger buckling load than EBBT for short nanotubes. Also, the difference between the two theories is negligible when the length of the nanotube is large. The obtained results in the present study (Tables 4 to 6) show that the critical buckling load of SWCNTs is sensitive to the selected values for e_0a. Thus, the comparison of results with those obtained using the MM model indicates that e_0a = 0.05538nm should be considered in the future studies on the buckling of CNTs. This clarifies an important issue in the nanomechanics of CNTs that has not been considered before [25]. The non-local and local results using TBT and EBBT for a SWCNT in the absence of the surrounding elastic medium have been compared with the MM Results reported by Searsand Batra [20]. The difference between the non-local theory and MM is small for long Nanotubes [26]. However, for small aspect ratios, the results of beam theory are not in a good agreement with those obtained using MM. It is reasonable because the beam theory is not convenient for short nanotubes and the shell theory should be used. Also, it can be concluded that the results obtained using both the non-local and local theories are near to the MM results for long nanotubes. As a result, the non-local critical buckling load is overestimated by the local beam model because of omission of the small-scale effect for long nanotubes. It can be seen that the non-local critical buckling load in the presence of the surrounding elastic medium (d = 0) is higher than that in the absence of the surrounding elastic medium (d = 0) for long nanotubes. The difference between these two cases increases with increasing L. In addition, the shear effect clearly shows the importance of applying shear effect clearly shows the importance of applying shear deformation beam models for CNTs.

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