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Determination of Vibration Characteristics of Beams Application for Functional Gradient Materials (FGM)

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Abstract – *This paper relates to the dynamic behavior of the composite material beams gradually evaluated through the thickness. Our work is to analyze the natural frequencies of composite beams FGM used in building structures in Civil Engineering often subjected to vibration loads due to earthquake. The vibration characteristics specific beams such as free fixed beams are studied and orthotropic without including distortion due to shear and rotational inertia. On the one hand we introduce the effects of the shear deformation due to shear and rotational inertia for the accurate prediction of natural frequencies.*

Keywords: Beam, Vibration, Frequencies, Composite, FGM

1.0 INTRODUCTION

In the last decade, there has been is interested in beams composites evaluated gradients (FGM) as robotic arms, helicopter blades and many other applications with the main objective the improvement of vibration characteristics [1].

During these years, several authors have tried to predict the natural frequencies of composite beams (FGM) [2], but there were some forecasts for these beams [3]. Some researchers studied the free vibration characteristics embedded orthotropic beams without including the shear deformation and rotary inertia [4]. An exact solution for free vibration of composite beams (FGM) simply supported without shear deformation and rotational inertia has also been reported in this work [5]. The effects of transverse shear strain is imposed for composites, because of the high ratio of longitudinal to transverse shear modulus of the shear modulus [6], and the classical theory of stratification is unsatisfactory for accurate prediction of natural frequencies [7].

In the current work, exact solutions were presented for shear beams symmetrical composite materials was evaluated gradients [8]. The presented method is applicable to solution conditions to arbitrary levels and the results can be used as a reference to the approximate solutions [9]. In the composite beam (FGM), shown in table 1, the displacement field

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assumed for this beam based on the theory of the first order shear deformation and takes the following form [10].

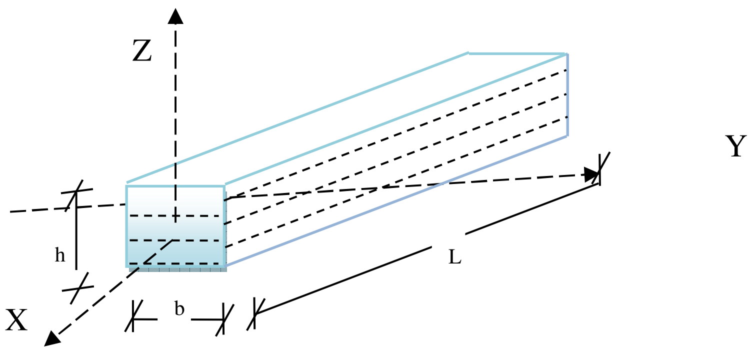

Figure 1: Geometry of a beam of composite materials functionally graded

2.0 THE DISPLACEMENT FIELD

$$\bar{u}(x,z,t) = u(x,t) + z\psi(x,t) \tag{1}$$
$$\bar{w}(x,z,t) = w(x,t)$$

u and w are such that the two components of displacement of a point in the neutral axis and ψ Means rotation around the normal of the neutral axis [11].
The equation of deformations is written

$$\varepsilon_x = \varepsilon_x^0 + zk_x - \alpha\Delta T - \beta\Delta C \tag{2}$$

$$\gamma_{xz} = \psi + \frac{\partial w}{\partial x} \tag{3}$$

with

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} \quad \text{et} \quad k_x = \frac{\partial \psi}{\partial x} \tag{4}$$

The constraint is given by

$$\sigma_x = E_{11}\varepsilon_x \tag{5}$$

3.0 DETERMINATION OF BENDING MOMENT AND NORMAL FORCE

The normal force and bending moment are given by the following expression

$$(N_x, M_x) = \int_A \sigma_x (1, z) dA \tag{6}$$

Replacing the expression of the stress we will

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$$(N_x, M_x) = \int_A E_{11}(\varepsilon_x^0 + zk_x - \alpha\Delta T - \beta\Delta C)(1, z)bdz \tag{7}$$

So:

$$N_x = \int_A E_{11}(\varepsilon_x^0 + zk_x - \alpha\Delta T - \beta\Delta C)bdz \tag{8}$$

$$M_x = \int_A E_{11}(\varepsilon_x^0 + zk_x - \alpha\Delta T - \beta\Delta C)bzdz \tag{9}$$

In another way:

$$N_x = \int E_{11}\varepsilon_x^0 bdz + \int E_{11}zk_x bdz - \int E_{11}\alpha\Delta T bdz - \int E_{11}\beta\Delta C bdz \tag{10}$$

$$M_x = \int E_{11}\varepsilon_x^0 bzdz + \int E_{11}k_x bz^2 dz - \int E_{11}\alpha\Delta T bzdz - \int E_{11}\beta\Delta C bzdz \tag{11}$$

Eventually the normal force N_x and the bending moment M_x can be written in this form

$$N_x = A_{11}\varepsilon_x^0 + B_{11}k_x + U \tag{12}$$

$$M_x = B_{11}\varepsilon_x^0 + D_{11}k_x + R \tag{13}$$

with:

$$\begin{aligned}
A_{11} &= \int E_{11} bdz \\
B_{11} &= \int E_{11} bzdz \\
D_{11} &= \int E_{11} bz^2 dz \\
U &= -\int E_{11}\alpha\Delta T bdz - \int E_{11}\beta\Delta C bdz \\
R &= -\int E_{11}\alpha\Delta T bzdz - \int E_{11}\beta\Delta C bzdz
\end{aligned} \tag{14}$$

The equations of N_x and M_x are written in matrix form

$$\begin{Bmatrix} N_x \\ M_x \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x \end{Bmatrix} + \begin{Bmatrix} U \\ R \end{Bmatrix} \tag{15}$$

and

$$Q_{xz} = A_{55}\gamma_{xz} \tag{16}$$

A_{11}, B_{11}, and D_{11} designate terms of rigidity of the membrane matrix, coupling and flexing

$$(A_{11}, B_{11}, D_{11}) = b \int_{-h/2}^{h/2} E_{11}\left(1, z, z^2\right) dz \tag{17}$$

A_{55} is the shear factor, defined by

$$A_{55} = bk \int_{-h/2}^{h/2} G_{13} dz \tag{18}$$

Or G_{13} is the term of expression

$$G_{13} = \frac{G_f G_m}{(1-V_f)G_f + V_f G_m} \tag{19}$$

And k is the shear correction factor.

4.0 THE EQUILIBRIUM EQUATIONS

We can establish the equilibrium equations from the principle of virtual work [12]. Static balance can be described by the sum of the internal work (δW_{int}) and the external work (δW_{ext}) developed by the displacement field of a point located on a part of the virtual work. The border every effort (interior and external) acting the system is zero [13].

5.0 VIRTUAL W.ORK INTERNAL AND EXTERNAL EFFORTS

$$\delta W_{ext}(\delta U) + \delta W_{\text{int}}(\delta U) = 0, \forall\, \delta U \tag{20}$$

The work of external forces is written

$$\delta W_{ext}(\delta U) = \int_D f^V \delta U \, dV + \int_{\partial D_f} f^S \delta U \, dS \tag{21}$$

The work of the internal forces is written

$$\delta W_{\text{int}}(\delta U) = -\int_D \sigma : \delta \in dV, \tag{22}$$

In this case the principle of virtual work is expressed as follows

$$\delta\pi = 0 = \int_0^t \int_0^l \left\{ \begin{aligned} & N_x \delta\varepsilon_x^0 + M_x \delta k_x + Q_{xz} \delta\gamma_{xz} + (I_1 \dot{u} + I_2 \dot{\psi}) \delta \dot{u} \\ & + I_1 \dot{w} \delta \dot{w} + (I_3 \dot{\psi} + I_2 \dot{u}) \delta \dot{\psi} \end{aligned} \right\} dxdt \tag{23}$$

with

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$$(I_1, I_2, I_3) = b\int_{-h/2}^{h/2} \rho(1, z, z^2)\,dz \tag{24}$$

The expression of ρ is given by

$$\rho = V_f \rho_f + V_m \rho_m \tag{25}$$

The values of V_f and V_m are given as follows

$$V_f + V_m = 1 \tag{26}$$

with

$$V_f = V_2 + (V_1 - V_2)\frac{|z|^n}{h} \tag{27}$$

The equation of motion for a beam of composite material evaluated gradient can be obtained by substituting equation (1), (13) and (14) in equation (21), in by incorporating part of the displacement gradients and the coefficients are fixed u, w and ψ separately. For a symmetrical composite laminates B_{11} is equal to zero and in the plane of movement can be considered as negligible compared to the displacement due to bending [14]. The equations of motion are to be deducted [15]

$$\int_0^t\int_0^L [A_{11}\varepsilon_x^0 + B_{11}k_x]\frac{\partial(\delta u)}{\partial x} + \left[B_{11}\frac{\partial u}{\partial x} + D_{11}\frac{\partial \psi}{\partial x}\right]\frac{\partial(\delta\psi)}{\partial x} + \left[A_{55}\left(\psi + \frac{\partial w}{\partial x}\right)\right]\delta\left(\psi + \frac{\partial w}{\partial x}\right)$$
$$+\left(I_1\frac{\partial u}{\partial t} + I_2\frac{\partial \psi}{\partial t}\right)\frac{\partial(\delta u)}{\partial t} + I_1\frac{\partial w}{\partial t}\frac{\partial(\delta w)}{\partial t} + I_3\frac{\partial \psi}{\partial t}\frac{\partial(\delta\psi)}{\partial t} + \left(I_3\frac{\partial \psi}{\partial t} + I_2\frac{\partial u}{\partial t}\right)\frac{\partial \psi}{\partial t} = 0 \tag{28}$$

We calculate each term separately

$$\int_0^L D_{11}\frac{\partial \psi}{\partial x}\frac{\partial(\delta\psi)}{\partial x} = \left[D_{11}\frac{\partial \psi}{\partial x}\delta\psi\right]_0^L - \int_0^L D_{11}\frac{\partial^2 \psi}{\partial x^2}\delta\psi \tag{30}$$

$$\int_0^L\left[A_{55}\left(\psi + \frac{\partial w}{\partial x}\right)\right]\partial\left(\psi + \frac{\partial w}{\partial x}\right) = \int_0^L\left[A_{55}\left(\psi + \frac{\partial w}{\partial x}\right)\right]\delta\psi + \left[A_{55}\left(\psi + \frac{\partial w}{\partial x}\right)\right]_0^L\delta w - \int_0^L [A_{55}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right)]\delta w \tag{31}$$

$$\int_0^t I_1\frac{\partial w}{\partial t}\frac{\partial(\delta w)}{\partial t} = \left[I_1\frac{\partial w}{\partial t}\delta w\right]_0^t - \int_0^t I_1\frac{\partial^2 w}{\partial t^2}\delta w \tag{32}$$

$$\int_0^t I_3\frac{\partial \psi}{\partial t}\frac{\partial(\delta\psi)}{\partial t} = \left[I_3\frac{\partial \psi}{\partial t}\delta\psi\right]_0^t - \int_0^t I_3\frac{\partial^2 \psi}{\partial t^2}\delta\psi \tag{33}$$

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6.0 THE EQUATIONS OF MOTION

The equations can be written as follows:

$$A_{55}\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-I_1\frac{\partial^2 w}{\partial t^2}=0 \tag{34}$$

$$D_{11}\frac{\partial^2 \psi}{\partial x^2}-A_{55}\left(\psi+\frac{\partial w}{\partial x}\right)-I_3\frac{\partial^2 \psi}{\partial t^2}=0 \tag{35}$$

Equations (28) and (29) are used to determine the free vibration of the composite beam (FGM) [16].

7.0 HARMONICS SOLUTIONS

For the solution of these equations we will follow the procedure

$$w=We^{i\omega t} \tag{36}$$

$$\psi=\Psi e^{i\omega t} \tag{37}$$

With ω is the circular frequency. Using equations (30) and (31), equations (28) and (29) can be expressed as

$$\frac{d^2 W}{d\xi^2}+a^2c^2W+L\frac{d\Psi}{d\xi}=0 \tag{38}$$

$$c^2\frac{d^2\Psi}{d\xi^2}-(1-a^2b^2c^2)\Psi-\frac{1}{L}\frac{dW}{d\xi}=0 \tag{39}$$

with

$$a^2=\frac{I_1L^4\omega^2}{D_{11}}$$
$$b^2=\frac{I_3}{I_1L^2} \tag{40}$$
$$c^2=\frac{D_{11}}{A_{55}L^2} \quad \text{and} \quad \xi=\frac{x}{L}$$

Equations (32) and (33) are transformed into two differential equations through the following steps. From equation (32) was

$$\frac{d\Psi}{d\xi}=-\frac{d^2W}{Ld\xi^2}-\frac{a^2c^2W}{L} \tag{41}$$

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Equation 33 is derived once a report ξ is obtained

$$c^2 \frac{d^3\Psi}{d\xi^3} - (1-a^2b^2c^2)\frac{d\Psi}{d\xi} - \frac{1}{L}\frac{d^2W}{d\xi^2} = 0 \tag{42}$$

It replaces the expression $\frac{d\Psi}{d\xi}$ in this equation we will, so the expression as

$$\frac{d^4W}{d\xi^4} + a^2(b^2+c^2)\frac{d^2W}{d\xi^2} - a^2(1-a^2b^2c^2)W = 0 \tag{43}$$

In an identical manner and from the equation (33) was

$$\frac{dW}{d\xi} = Lc^2\frac{d^2\Psi}{d\xi^2} - L(1-a^2b^2c^2)\Psi \tag{44}$$

Hence the equation (49) is written as follows

$$\frac{d^4\Psi}{d\xi^4} + a^2(b^2+c^2)\frac{d^2\Psi}{d\xi^2} - a^2(1-a^2b^2c^2)\Psi = 0 \tag{45}$$

So the equations (43) and (45) can be expressed as follows

$$\frac{d^4W}{d\xi^4} + a^2(b^2+c^2)\frac{d^2W}{d\xi^2} - a^2(1-a^2b^2c^2)W = 0 \tag{46}$$

$$\frac{d^4\Psi}{d\xi^4} + a^2(b^2+c^2)\frac{d^2\Psi}{d\xi^2} - a^2(1-a^2b^2c^2)\Psi = 0 \tag{47}$$

8.0 SOLUTIONS OF DIFFERENTIAL EQUATIONS

The solutions of the equations (42) and (50) can be written as the following steps [17]

$$\begin{aligned} W = {} & C_1 e^{(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2-\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4})}\xi} \\ & + C_2 e^{(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2+\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4})}\xi} \\ & + C_3 e^{-(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2+\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4})}\xi} \\ & + C_4 e^{-(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2-\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4})}\xi} \end{aligned} \tag{48}$$

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We will have the general form

$$W = C_1 e^{a\alpha\xi} + C_2 e^{ia\beta\xi} + C_3 e^{-ia\beta\xi} + C_4 e^{-a\alpha\xi} \tag{49}$$

$$cosh = \frac{e^{a\alpha\xi} + e^{-a\alpha\xi}}{2} \qquad sinh = \frac{e^{a\alpha\xi} - e^{-a\alpha\xi}}{2}$$

$$cos = \frac{e^{ia\beta\xi} + e^{-ia\beta\xi}}{2} \qquad sin = \frac{e^{ia\beta\xi} - e^{-ia\beta\xi}}{2i} \tag{50}$$

By performing addition and subtraction we have

$$\begin{aligned} &cosh\, a\alpha\xi + sinh\, a\alpha\xi = e^{a\alpha\xi} \\ &cosh\, a\alpha\xi - sinh\, a\alpha\xi = e^{-a\alpha\xi} \\ &cos\, a\beta\xi + i\, sin\, a\beta\xi = e^{ia\beta\xi} \\ &\cos a\beta\xi - i \sin a\beta\xi = e^{-ia\beta\xi} \end{aligned} \tag{51}$$

With α and β given by the following expressions

$$\begin{matrix}\alpha \\ \beta\end{matrix} = \frac{1}{\sqrt{2}} \left\{ \mp (b^2 + c^2) + \left[(b^2 - c^2)^2 + \frac{4}{a^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \tag{52}$$

So, W is written as

$$W = A_1\, cosh\, a\alpha\xi + A_2\, sinh\, a\alpha\xi + A_3\, cos\, a\beta\xi + A_4\, sin\, a\beta\xi \tag{53}$$

In a similar way to the function Ψ it will

$$\begin{aligned} \Psi = {} & C_5 e^{\left(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2-\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4}}\right)\xi} \\ & + C_6 e^{\left(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2+\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4}}\right)\xi} \\ & + C_7 e^{-\left(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2+\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4}}\right)\xi} \\ & + C_8 e^{-\left(\frac{\sqrt{2}}{2}\sqrt{-a(ab^2+ac^2-\sqrt{a^2b^4-2a^2b^2c^2+a^2c^4+4}}\right)\xi} \end{aligned} \tag{54}$$

So,

$$\Psi = C_5 e^{a\alpha\xi} + C_6 e^{ia\beta\xi} + C_7 e^{-ia\beta\xi} + C_8 e^{-a\alpha\xi} \tag{55}$$

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We will have the general form

$$\Psi = B_1 \sinh a\alpha\xi + B_2 \cosh a\alpha\xi + B_3 \sin a\beta\xi + B_4 \cos a\beta\xi \tag{56}$$

For values of the constants B_1 B_2 B_3 B_4 used in equation (13) or is replaced expressions W and Ψ it comes

$$\begin{aligned}&(a^2\alpha^2 A_1 + a^2c^2 A_1 + a\alpha L B_1)\cosh a\alpha\xi + (a^2\alpha^2 A_2 + a^2c^2 A_2 + a\alpha L B_2)\sinh a\alpha\xi \\ &+(-a^2\beta^2 A_1 + a^2c^2 A_3 + a\beta L B_3)\cos a\beta\xi \\ &+(-a^2\beta^2 A_4 + a^2c^2 A_4 - a\beta L B_4)\sin a\beta\xi = 0\end{aligned} \tag{57}$$

Hence the values

$$B_1 = -\frac{a(\alpha^2 + c^2)A_1}{\alpha L} \tag{58}$$

$$B_2 = -\frac{a(\alpha^2 + c^2)A_2}{\alpha L} \tag{59}$$

$$B_3 = \frac{a(\beta^2 - c^2)A_3}{\beta L} \tag{60}$$

$$B_4 = -\frac{a(\beta^2 - c^2)A_4}{\beta L} \tag{61}$$

9.0 THE BOUNDARY CONDITIONS

The four boundary conditions for the beam are studied [17]

9.1 Simply Supported

$$\xi = 0,\ 1 \qquad W = 0 \quad \frac{d\Psi}{d\xi} = 0 \tag{62}$$

9.2 Clamped - Clamped

$$\xi = 0,\ 1 \qquad W = 0 \quad \Psi = 0 \tag{63}$$

9.3 Clamped - Free

$$\xi = 0,\ 1 \qquad W = 0 \quad \frac{1}{L}\frac{dW}{d\xi} + \Psi = 0 \tag{64}$$

9.4 Simply Supported - Clamped

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$\xi = 0, 1 \qquad W=0 \quad \Psi = 0$ (65)

The solutions for the four types of beam considered leads to the following matrix [18]

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ L_1 & L_2 & L_3 & L_4 \\ L_5 & L_6 & L_7 & L_8 \\ L_9 & L_{10} & L_{11} & L_{12} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{66}$$

The coefficients L_1 , L_2 up L_{12} depending on the boundary conditions of the beam.

10.0 THE DIFFERENT TYPES OF FIXATION

The equations are as follows

10.1 Simply Supported

$$\sin a\beta = 0 \tag{67}$$

10.2 Clampe – Clamped

$$2 - 2\cosh a\alpha \cos a\beta + \frac{a[a^2c^2(b^2-c^2)^2 + (3c^2-b^2)]\sinh a\alpha \sin a\beta}{(1-a^2b^2c^2)^{\frac{1}{2}}} = 0 \tag{68}$$

10.3 Clamped - Free

$$2 + [a^2(b^2-c^2)+2]\cosh a\alpha \cos a\beta - \frac{a(b^2+c^2)}{(1-a^2b^2c^2)^{\frac{1}{2}}}\sinh a\alpha \sin a\beta = 0 \tag{69}$$

10.4 Simply Supported – Clamped

$$\frac{\alpha(\alpha^2+b^2)}{\beta(\alpha^2+c^2)}\tanh a\alpha - \tan a\beta = 0 \tag{70}$$

Finally [19], that may have frequencies of composite beams symmetrical materials has gradients evaluated (FGM) were predicted using the first order theory which leads to a transverse shear deformation The numerical results presented for these beams are commonly met with various boundary conditions as they can be used as a reference for the approximate solutions [20].

11.0 THE INFLUENCE OF THE INDEX MATERIEL

In order to get the results regarding the influence of the physical evidence of the beam composite rated gradient [21], one must study the variation of displacement fields according

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to three parameters by varying (n, N, ξ), we analyzed the movements and non dimensional frequencies ϖ [22]. The studied beam will be considered a symmetrical beam gradient composites evaluated polymer matrix with carbon fibers [23]. The beam has a thickness h = 1 m, length L = 15m and the following material properties

$$E_m = 3.5\ Gpa \qquad E_f = 137.76\,Gpa$$
$$G_m = 1.6\,Gpa \qquad G_f = 12\,Gpa$$
$$\rho_m = 1200\,kg/m^3 \qquad \rho_f = 1450\,kg/m^3$$
$$V_1 = 0.6 \qquad V_2 = 0.4$$
$$k = 0.8333333$$

The results are shown in the following tables

Table 1: Variation of displacement W of ξ function for the first mode

MODE 1		*Simply – simply*	V_1=0.6 V_2=0.4	*n=0,1,2,3,4*	
$W=\sin a\beta\xi$	0	0.58	1	0.58	0
ξ	0	0.2	0.5	0.8	1

Table 2: Variation of displacement W of ξ function for the mode number 2

MODE 2		*Simply – simply*	V_1=0.6 V_2=0.4	*n=0,1,2,3,4*	
$W=\sin a\beta\xi$	0	1	0	-1	0
ξ	0	0.25	0.5	0.75	1

Table 3: Variation of displacement W of ξ function for the mode number 3

MODE 3		*Simply – simply*	V_1=0.6 V_2=0.4	*n=0,1,2,3,4*	
$W=\sin a\beta\xi$	0	1	-1	1	0
ξ	0	0.15	0.5	0.85	1

Table 4: Variation of displacement W of ξ function for the mode number 4

MODE 4			*Simply – simply*		V_1=0.6	V_2=0.4	*n=0,1,2,3,4*		
$W=\sin a\beta\xi$	0	1	0	-1	0	1	0	-1	0
ξ	0	0.1	0.25	0.35	0.5	0.6	0.75	0.85	1

Table 5: Variation of displacement W of ξ function for the mode number 5

MODE 5			*Simply – simply*			V_1=0.6	V_2=0.4	*n=0,1,2,3,4*			
$W=\sin a\beta\xi$	0	1	0	-1	0	1	0	-1	0	1	0
ξ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

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Table 6: Variation of displacement W of ξ function for the first mode

MODE 1	*Clamped – Clamped*		$V_1=0.6$	$V_2=0.4$ $n=0,1,2,3,4$	
$W=\sin a\beta\xi$	0	0.63	1.42	0.63	0
ξ	0	0.2	0.5	0.8	1

Table 7: Variation of displacement W of ξ function for the mode number 2

MODE 2	*Clamped – Clamped*		$V_1=0.6$	$V_2=0.4$ $n=0,1,2,3,4$	
$W=\sin a\beta\xi$	0	1.14	0	-1.14	0
ξ	0	0.25	0.5	0.75	1

Table 8: Variation of displacement W of ξ function for the mode number 3

MODE 3	*Clamped – Clamped*		$V_1=0.6$	$V_2=0.4$ $n=0,1,2,3,4$	
$W=\sin a\beta\xi$	0	1.08	-1.008	1.08	0
ξ	0	0.2	0.5	0.8	1

Table 9: Variation of displacement W of ξ function for the mode number 4

MODE 4		*Clamped – Clamped*			$V_1=0.6$	$V_2=0.4$ $n=0,1,2,3,4$			
$W=\sin a\beta\xi$	0	1	0	-1	0	1	0	-1	0
ξ	0	0.1	0.25	0.35	0.5	0.6	0.75	0.85	1

Table 10: Variation of displacement W of ξ function for the mode number 5

MODE 5		*Clamped – Clamped*			$V_1=0.6$	$V_2=0.4$ $n=0,1,2,3,4$					
$W=\sin a\beta\xi$	0	1	0	-1	0	1	0	-1	0	1	0
ξ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

To understand the influence of -n- physical evidence on the displacement field *W*, several types of fixation and vibration modes were made [24]. The importance of physical evidence exposed in our case on the beam behavior is guided by the perceived difference [25]. The displacement calculated from the expression (*W*) is compared to each mode between the six different values of (n) [26]. In the case of a simply supported beam Tables (1) to (5) clearly show that in the first five modes parameter (n) has no influence on the shape of patterns despite the change geometrical or physical characteristics of the beam[27]. By cons in the case of a doubly clamped beam the influence of the exponent (n) is illustrated in Tables (6) to (10) [28], this difference is mainly due to the variation of (n) which implies that this parameter plays a crucial role in the rigidity of beams for where fixed or moving is larger than that of a simply supported beam [29].

12.0 CONCLUSION

Determining the vibratory characteristics of the composite beams evaluated graded materials (FGM), to solve certain problems, we have developed the analytical method that uses the displacement fields of the beams [30].

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These fields based on the first order theory that requires the use of k shear factor that leads to a transverse shear deformation in the thickness direction. We can say that the frequencies of the symmetrical beams composites evaluated gradient (FGM) were predicted using first order theory which gives the possibility of certainty the shearing effect on the occurrence of vibration beams, while respecting the displacement field cancellation in the ends of the beam [31].

However, with this method we validated some applications to determine the different parameters that affect the movement. The numerical results obtained for these beams are commonly calculated for various boundary conditions and we can use them as references to approximate solutions.

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