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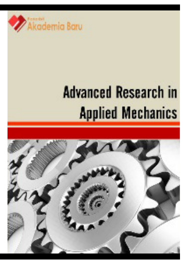

Q-parameterization control method for a class of two wheel mobile robot

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ABSTRACT

This project presents an application of a robust controller based on the "Q-Parameterization" theory to control the motion of the common and well known problem in control engineering the " Two-Wheeled Inverted Pendulum" (TWIP). This controller is applied to attain stability and good dynamic performance. The Q-parameter is designed such that it characterizes the set of all stabilizing controllers for the " Two-Wheeled Inverted Pendulum" (TWIP) system. The selection and design of the Q-Parameter is based along the system behaviour constraining. The work is carried out in three stages. First, the derivation of the mathematical model for the " Two-Wheeled Inverted Pendulum" (TWIP) system. Second the design methodology of the proposed Q-Parameterization controller is presented. Finally a simulation process is carried out to evaluate the resulting controller and its performance. For comparison purposes an LQR controller are designed and the results were carried out.

Keywords:
Q-parameterization, Robust control, Two-wheeled balancing robot, Linear Model, LQR

1. INTRODUCTION

Controller choice and design are one of the most important steps in any control system, where it represents the heart of the system. By choosing an appropriate controller we can achieve the predetermined goals of the plant and meet the required robust stability and performance. Among the years, many controllers have been discovered and found to serve this purpose, whether they were nonlinear controllers such as sliding mode control (SMC) [1] or linear such as H2, LQG [2] and Q-Parameterization [3,4] controllers. In this project the Q-Parameterization controller is applied to control the motion of a Two-Wheeled Inverted Pendulum.

The inverted pendulum [5] has been in the control engineering field since 1908 where Stephenson [6] examined an inverted pendulum and demonstrated that it could be stabilized by applying rapid, vertical, harmonic oscillations to its base. The inverted pendulum on a cart, then became a very common problem in the field of control engineering [5, 7] for its unstable nature where it can provide a good experiment to evaluate the performance of the controllers.

The inverted pendulum is mounted on a car that moves freely in a line (one axis). The objective is to maintain the angle θ as small as possible. In this setup the cart is free to move along one axis while the pendulum should maintain its balance. This experiment has evolved to become free to move on a plane (two axis) which is now known as the "Two-Wheeled Inverted Pendulum" (TWIP).

The TWIP problem is more complicated and considered a highly coupled nonlinear unstable control problem. Since the TWIP is mounted on two separately driven wheels it can move on a plane and the chosen controller should maintain the plant balance while driving it to its destination. The whole system needs to be modelled first to design a controller based on the Q-Parameterization theory to achieve the control objectives. The two-wheeled inverted pendulum found to be nonlinear, thus a linearized model is acquired. After the linearization process is done the plant is controlled and stabilized to follow a reference signal.

2. Literature review

An inverted pendulum problem existed for a long time in the field of control engineering. Unlike non-inverted pendulums, inverted pendulums are unstable and they fall over rather than swinging back and forth around their equilibrium. In [6] analysed an inverted pendulum and concluded that it can be controlled to stabilization by applying rapid, vertical, harmonic oscillations to its base. The work kept developing until in [9] developed and derived a general motion equation for an inverted pendulum and his conditions of stability are similar to what Stephenson's has stated.

Yun-Su Ha and Shin'ichi Yuta [10], developed and built an autonomous robot to navigate in a plane while keeping its own balance. The control algorithm is a collection of three different parts: balance control, steering control and straight line tracking control.

Felix Grasser et al [8] developed a TWIP called "JOE" and derived its model. A state feedback controller was built based on the pole placement technique by decoupling the plant to separate the steering torque from the balancing and positioning torque. The work was intended as a mobile transportation table to help the waiters in cafeterias and restaurants to carry the orders.

S. W. Nawawi et al [1] designed a controller for a TWIP. The controller was developed based on the PISMC strategy by decoupling the system too. In [11] the TWIP was built and controlled in real time by applying the pole placement technique.

Morrell and Field [12] have published a paper on the design of the control algorithm used to control the Segway (commercial TWIP for human transportation 2002). Fig shows a timeline of the main events in the development of the inverted pendulum problem.

The need for better stability and better robustness led the researchers to develop and be always in the search for better control algorithms. With the appearance of the state-space theory in 1950s, control algorithms evolved and made big step where state feedback represents the basis for most of the control strategies. For instance, LQG-control are constructed with a Kalman filter and states feedback [2], where the states should be measurable. Safonov [13] showed that the LQG controller can achieve better robustness with 60 phase margin and infinity amplitude margin when all states are available. However, these results do not hold for output feedback when one or more of the states are missing, in such case a state estimator is required.

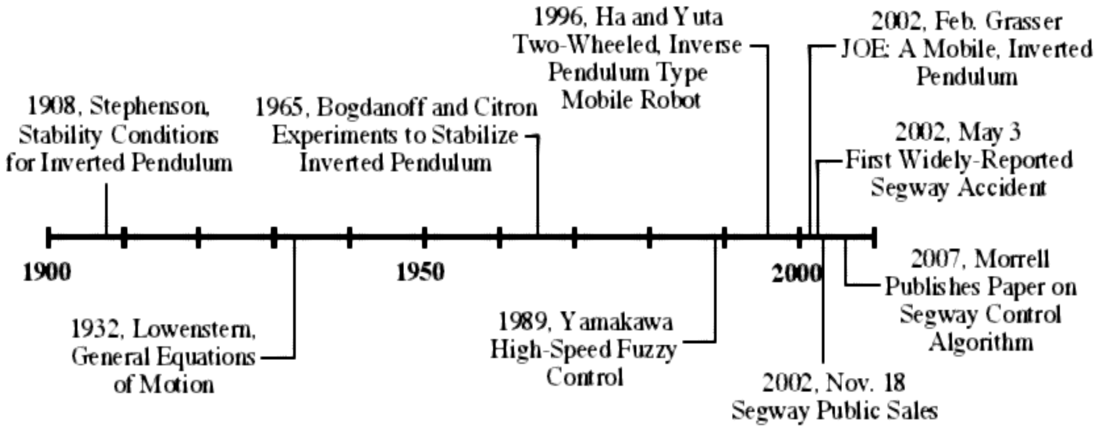


Fig. 1. Timeline of inverted pendulum literature [1]

Robust control attracted and still attracting wide attention for its preferable performance. However, this performance comes with higher complexity. Coprime parameterization is one of the techniques to maintain the robustness of a controller it was established and defined by [14] and [15]. This approach is centred around the coprime factorization of the transfer function of any given plant over a real rational, stable and proper function space R. In this case the transfer functions:

$$P(s) = \frac{N(s)}{M(s)}\ ; \text{where } N(s), M(s) \in \Re \qquad (1)$$

are coprime if and only if there exist functions X(s) and Y(s) such that:

$$N(s)X(s) + M(s)Y(s) = 1$$

The functions N(s), M(s), X(s) and Y(s) form a ring with the identity (equation (**Error! Reference source not found.**)). By adding a free parameter Q(s) all stabilizing compensators can be found in this function ring and are defined by the parameterized set:

$$C \in \left\{\frac{X(s)+M(s)Q(s)}{Y(s)-N(s)Q(s)}, Q(s) \in \Re \ \&\ Y(s) - N(s)Q(s) \neq 0\right\} \qquad (1)$$

Antsaklis, P.J. [16], has defined the relations between the left and right coprime factorization and state-space controller/observer pair for a transfer function of real number, $\mathcal{R}$. Deriving any right coprime factorization in real number is equivalent to solving a state feedback stabilization problem.

Similarly, deriving any left coprime factorization in $\mathcal{R}$ is equivalent to design a full-order full-state observer. Normalized Coprime factorization was introduced by Vidyasagar. M [17]. This concept was used in developing H∞ optimal control theory. Applications of Q-parameterization are seen in the processes with dead-time, and flexible arms control. For a multi-objectives system, the control objectives are not met simultaneously by just tuning the parameter Q. This limits the utilization of the Q-Parameterization method.

Mohamed, A.[3], has introduced a robust controller for the benchmark problem (two moving masses connected by springs) using the Q-parameterization theory to reject two disturbance classes. The first one was a class of impulse disturbance and the second was the class of sinusoidal disturbance. In the work of Mohamed, A. The more generalized form of feedback was used, the two-parameter-control, and two free parameters were found q1 and q2 to describe the domain of all stabilizing controllers.

Mohamed, A.[18] used the Q-Parameterization theory to design a controller for a magnetic bearing system to compensate imbalance and automate the balance in this system caused by the gap in the flux of the bearing. Abdelfatah M. Mohamed et al. [19], designed a Q-parameterization controller to control a variable speed magnetic bearing. Two controllers were found, each for different class of disturbance signals. Optimization methods were used to acquire the final controllers. S. W. Nawawi et al. [20], Q-Parameterization Control for a DC motor, where only the disturbance rejection was taken into consideration without any constraints on the system.

3. Mathematical modeling of two wheeled inverted pendulum mobile robot

The model used in this work is based on the work of Felix Grasser et al [8]. This model is based on the parameters characterizing the vehicle where its behaviour can be influenced by the motor torque and disturbance forces. Fig shows the free body diagram for the TWIP used to derive the model.

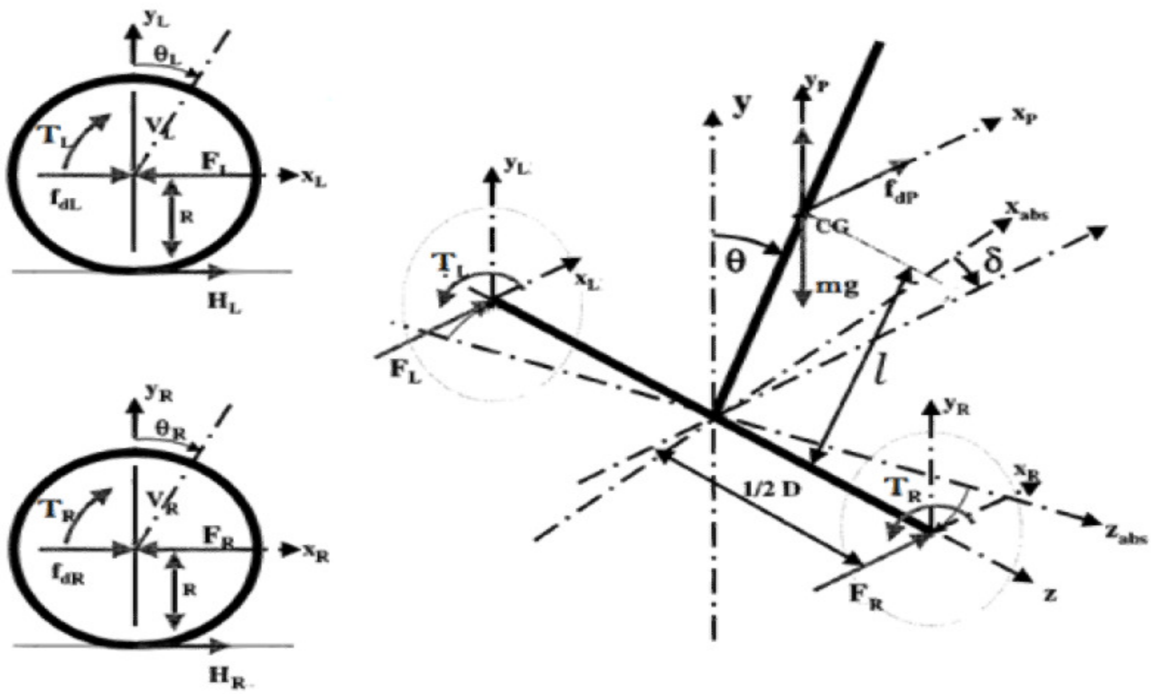


Fig. 2. Free body diagram of TWIP (Felix Grasser-2002)

The system has 3-DOF which are:

1. Movement along X axis (Position).
2. Rotation around Z axis (Tilt or inclination).
3. Rotation around Y axis (Orientation).

These DOFs are to be controlled via the motor setup on the wheels hence the system can be described as under-actuated which represent a complexity in the whole control scheme, as shown in figure 3.1, the system consists of two separated wheels mounted on the chassis of the TWIP. Each wheel has its own driving motor which provides free movement on the XZ-plane. The wheel axis carries the inverted pendulum of certain length (l) and mass (m). We can see also the disturbance forces acting on the system f_{dp}, f_{dR} and f_{dL}.

The model Linearization is done using Taylors series around the operating point which is when θ is taken small (θ ≈ 0). Following this we can find that (Sin θ = θ ; Cos θ = 1). By applying this to the system differential equations yields:

$$\ddot{x} = \frac{T_L}{R\alpha} + \frac{T_R}{R\alpha} + \frac{f_{dL}}{\alpha} + \frac{f_{dR}}{\alpha} + \frac{J_{mo}+J_{po}-ml^2}{\alpha(J_{mo}+J_{po})} f_p - \frac{m^2 g l^2}{J_{mo}+J_{po}} \theta \tag{2}$$

$$\ddot{\theta} = \frac{(mgl\theta + f_p l)\left(M+m+4M_w+\frac{2J_W}{R^2}\right)+ml\left(\frac{T_L}{R}+\frac{T_R}{R}+f_{dL}+f_{dR}+f_p\right)}{\beta} \tag{3}$$

where:

$$\alpha = M + m + 4M_w + \frac{2J_w}{R^2} + \frac{m^2 l^2}{J_{mo} + J_{po}}$$

$$\beta = (J_{mo} + J_{po})(M + m + 4M_w + \frac{2J_w}{R^2})m^2 l^2$$

And the state-space representation is then given by

$$\begin{bmatrix} \dot{X} \\ \dot{v} \\ \dot{\theta} \\ \dot{\omega} \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ v \\ \theta \\ \omega \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} \\ 0 & 0 & 0 & 0 & 0 \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{45} \\ 0 & 0 & 0 & 0 & 0 \\ B_{61} & B_{62} & B_{63} & B_{64} & 0 \end{bmatrix} \begin{bmatrix} T_L \\ T_R \\ F_{dL} \\ F_{dR} \\ f_p \end{bmatrix} \tag{4}$$

Where,

$$A_{23} = -\frac{m^2 g l^2}{\alpha(J_{mo} + J_{po})}$$

$$B_{45} = \frac{\left(M + m + 4M_\omega + \frac{2J_\omega}{R^2}\right) l + ml}{\beta} \qquad A_{43} = \frac{\left(M + m + 4M_\omega + \frac{2J_\omega}{R^2}\right) mgl}{\beta}$$

$$B_{21} = B_{22} = \frac{1}{R\alpha}$$

$$B_{61} = \frac{D}{2R}\left[\frac{1}{J_\delta + \frac{D^2}{2}\left(\frac{J_\omega}{R^2} + M_\omega\right)}\right] \qquad B_{23} = B_{24} = \frac{1}{\alpha}$$

$$B_{62} = -B_{61} \qquad B_{25} = \frac{(J_{mo} + J_{po}) - ml^2}{\alpha(J_{mo} + J_{po})}$$

$$B_{63} = \frac{D}{2}\left[\frac{1}{J_\delta + \frac{D^2}{2}\left(\frac{J_\omega}{R^2} + M_\omega\right)}\right] \qquad B_{41} = B_{42} = \frac{ml}{R\beta}$$

$$B_{64} = -B_{63} \qquad B_{43} = B_{44} = \frac{ml}{\beta}$$

4. Q-parametrization controller design

The controller K(s) a doubly coprime factorization must be constructed. This will ensure that closed loop poles are located in the prescribed area as shown in Fig. 1. This can be done by calculating the transfer functions $N, D, \tilde{N}, \tilde{D}, X, Y, \tilde{X}, \tilde{Y}$ where the following identities

$$YD + XN = I$$

$$\tilde{N}\tilde{X} + \tilde{D}\tilde{Y} = I$$

Must hold. then the transfer functions can be given by

$$N - C(sI - A_0)B$$

$$D - I - F_1(SI - A_0)^{-1}B$$

$$\tilde{N} - C(sI - \tilde{A}_0)^{-1}B$$

$$\tilde{D} - I - C(sI - \tilde{A}_0)^{-1}F_2$$

$$X - F_2(sI - \tilde{A}_0)^{-1}F_2$$

$$Y - I - F_1(sI - \tilde{A}_0)^{-1}B$$

$$\tilde{X} - F_1(sI - A_0)F_2$$

$$\tilde{Y} - I - C(sI - \tilde{A}_0)^{-1}B$$

(5)

and the plant *[P(s)]* right coprime factorization (rcf) and left coprime factorization (lcf) transfer function is then given by

$$P(s) = ND^{-1}(rcf) = \tilde{D}^{-1}\tilde{N}(lcf) \tag{6}$$

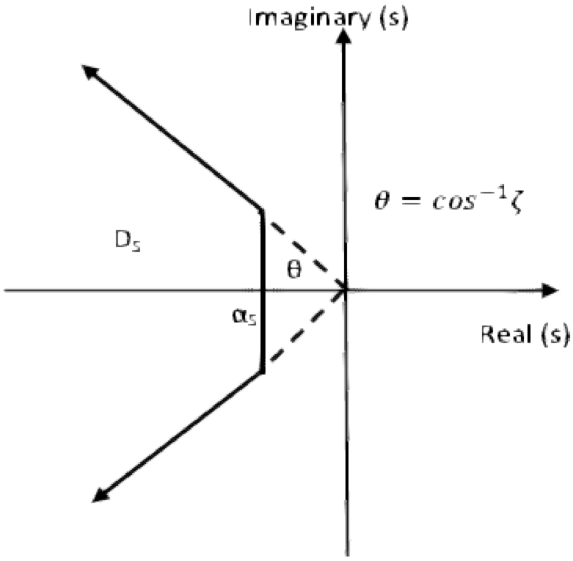


Fig. 1. Generalized stabilizing area

The vectors F_1 & F_2 are feedback gains (for the nominal internal controller) such that the matrices A_0 & $\tilde{A}_0$ are Hurwitz (have negative real part eigenvalues).

$$A_0 = A - BF_1$$

$$\tilde{A}_0 = A - F_2C$$

They can be chosen using any controller design technique. Here the pole placement is used to match 10% overshoot and 0.1 settling times.

$$\%O.S = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

$$10 = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} => \xi = 0.6$$

and from the settling time w_n can be found as follows

$$T_s = \frac{4}{\xi\omega_n}$$

$$0.1 = \frac{4}{\xi\omega_n} => \omega_n = 66.67\ rad/sec$$

using these values the desired poles are s_1 = [-40+53.33i, -40-53.33i, -240, -400] since the system is 4th order then the remaining two were chosen 6 times further than the desired dominant poles. These are used to find the gain F_1. For F_2 the poles were chosen further away s_2 = [-600, -800, -1000, -1200]. Now using MATLAB we can get the gains and then the matrices A_0 & $\tilde{A}_0$

```
f1 =

   1.0e+07 *

  -3.7782   -0.0932    1.3158    0.0312


f2 =

   1.0e+10 *

    0.0000
    0.0005
    0.0245
    5.1689


A0 =

   1.0e+06 *

         0    0.0000         0         0
    3.0805    0.0760   -1.0728   -0.0255
         0         0         0    0.0000
    9.2810    0.2289   -3.2321   -0.0767
```

```
A02 =

   1.0e+10 *

  -0.0000    0.0000         0         0
  -0.0005   -0.0000    0.0000         0
  -0.0245         0         0    0.0000
  -5.1689   -0.0000    0.0000         0
```

Now let's consider the closed loop transfer functions of the system with the inputs taken as $U = [z_{ref}, f_P]$ (reference signal & pendulum disturbance) and the outputs are $Y = [z, u]$ (actual output of the plant & controller output). This will give the following transfer matrix

$$H(s) = \begin{bmatrix} h11(s) & h12(s) \\ h21(s) & h22(s) \end{bmatrix} \tag{7}$$

where *h11* is the transfer function from z_{ref} to *z*. This transfer function shows the tracking problem we are interested in. From figure 5.1 we can see that

$$h11(s) = \frac{K(s)P(s)}{1+K(s)P(s)} \tag{8}$$

and *h12* is the transfer function between F_P and *z*. This transfer function can be used with disturbance rejection problem.

$$h12(s) = \frac{P(s)}{1+K(s)P(s)} \tag{9}$$

h21 & *h22* are the transfer functions between (the reference & disturbance signals) and the controller output

$$h21(s) = \frac{K(s)}{1+K(s)P(s)}$$

$$h22(s) = \frac{-K(s)P(s)}{1+K(s)P(s)}$$

both of *h21* and *h22* can be used as an optimization constraints to limit the control action of the controller. Rearrange the equation of (9) and substitute with equations (6) and (7) result the following:

$$h11(s) = \frac{X(s)\tilde{D}(s)^{-1}+Q(s)\tilde{N}(s)}{Y(s)+X(s)\tilde{D}(s)^{-1}} \tag{10}$$

In the tracking problem one need from the output to follow the reference (command) signal that is

$$\lim_{t\to\infty}[z(t) - z_{ref}(t)] = 0$$

This is equivalent to making the DC gain equals 1 like so $|h11| = 1$. Parameter Q can be chosen as follows

a. $Q \in R$. This the simplest choice where Q is a scalar.

b. $Q(s) = \frac{a(s+b)}{(s+P_s)}$, where $P_s > \alpha_s$ is fixed value. α_s is shown in Fig. 1, and a, b, are free design parameters. This structure of Q can handle more constraints, but requires more optimization to be found.

c. $Q(s) = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}{(s+P_1)(s+P_2)\cdots(s+P_n)}$, where $P1, P2, \ldots Pn > \alpha_s \in R$ are fixed, and $a_0, a_1, \ldots a_n \in R$ are free design parameters.

In this work Q is chosen as a scalar.
From equation (10)

$$h11(s) = \frac{X(s)\tilde{D}(s)^{-1} + Q(s)\tilde{N}(s)}{Y(s) + X(s)\tilde{D}(s)^{-1}} = 1$$

$$|Q(s)| = \left|\frac{Y(s)}{\tilde{N}(s)}\right| \Rightarrow Q(0) = \frac{Y(0)}{\tilde{N}(0)}$$

Substituting the value of $Y(s), \tilde{N}(s)$ from the equation (5) at $s = 0$ and solving using the MATLAB code we can find
Q = 3.4459e+10
Substituting Q value into equation (12) and the resulting controller is K(s)

$$K \in \left\{ \left(Y - Q\tilde{N}\right)^{-1}\left(X + Q\tilde{D}\right), Q \in R, \left|Y - Q\tilde{N}\right| \neq 0 \right\} \qquad (12)$$

$$K(s) = \frac{-1.603e18\, s^4 - 5.644e17\, s^3 - 1.33e18\, s^2 - 4.65e17\, s - 1.58e19}{s^4 + 4320\, s^3 + 2.075e18\, s^2 + 7.301e17\, s}$$

5. Results and discussion

The simulation process is conducted after acquiring the controller K(s) for the TWIP. Fig. shows the Simulink block diagram of the system.

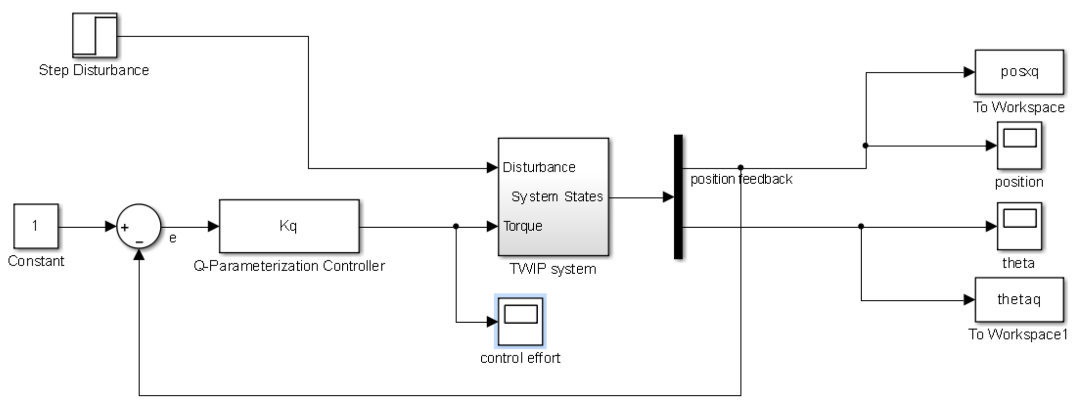


Fig. 4. TWIP Simulink block diagram with the Q-Parameterization controller

The Fig. **2** and Fig. shows the results of the simulation first without reference signal with only the disturbance on the upright pendulum. Secondly the step reference input is introduced and the results are shown below. From these two figures we can see clearly that the Q-parameterization controller performs better against a step disturbance.

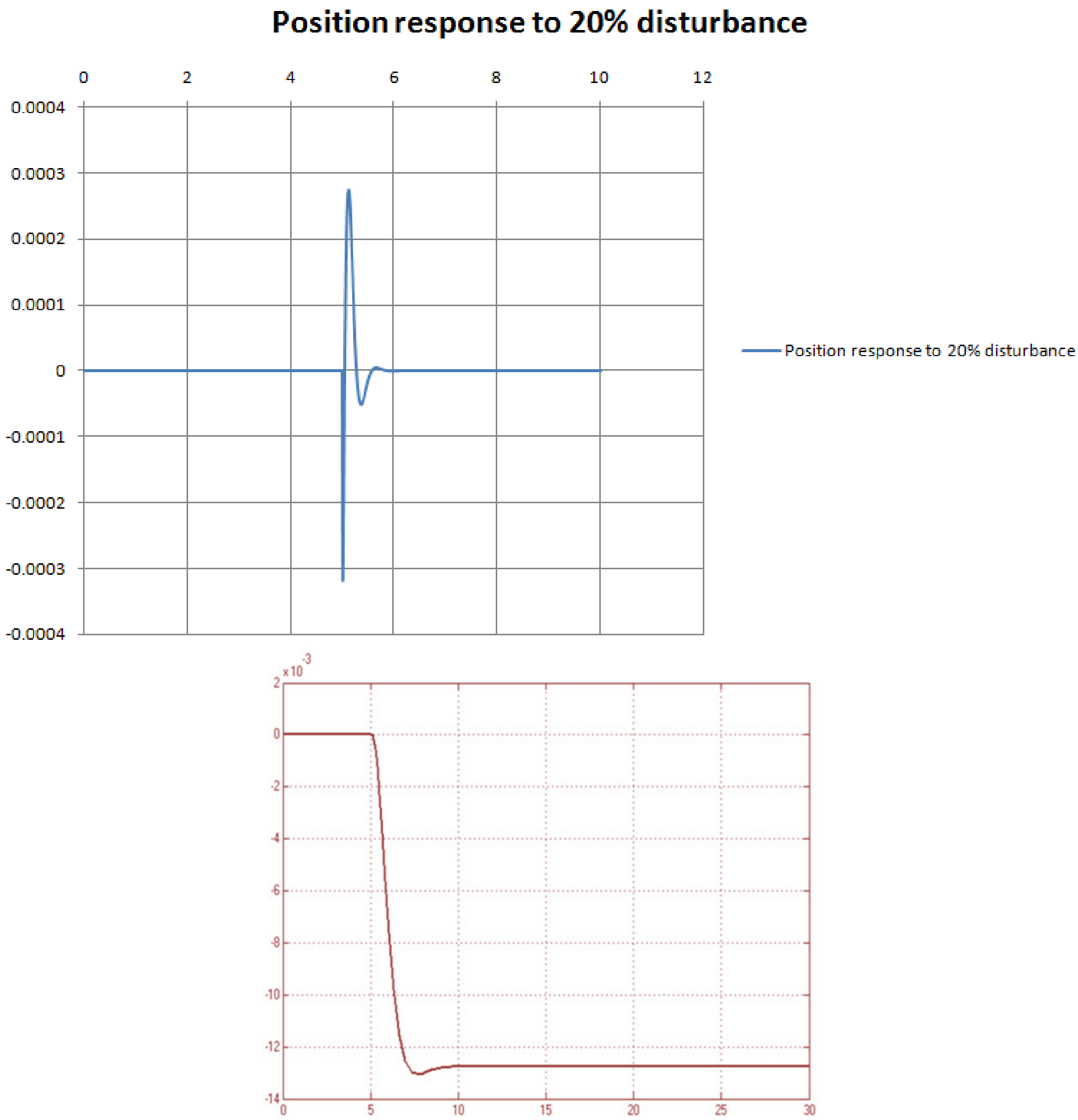
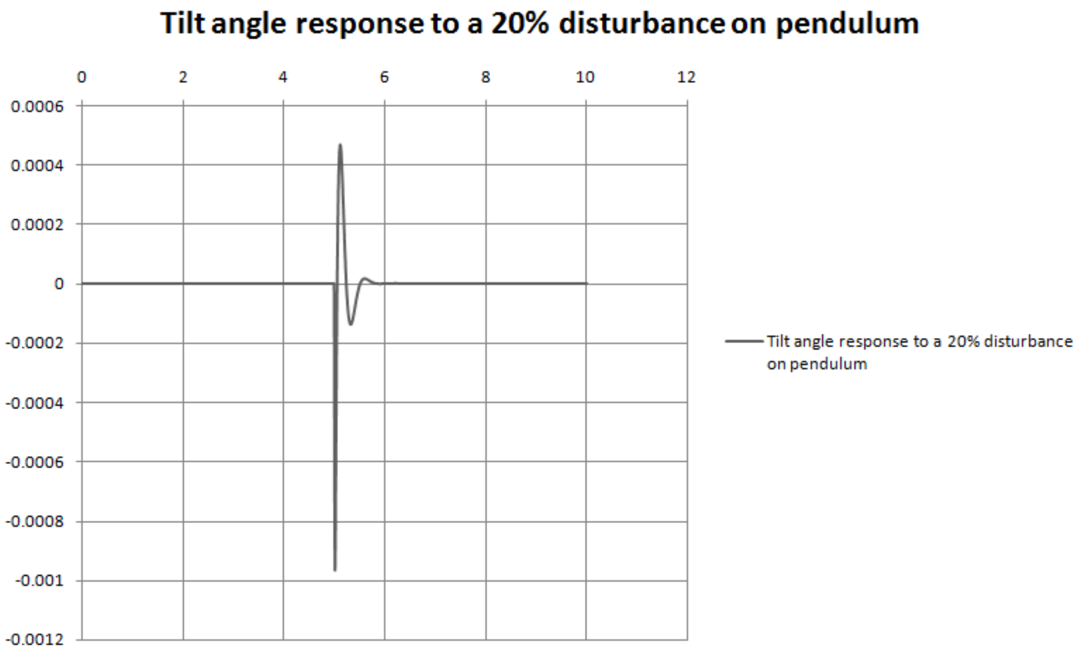


Fig. 2. Position of TWIP when 20% Step disturbance is introduced on the pendulum at time *t = 5 seconds* (Left: Q-Parameterization controller. Right: LQR controller)



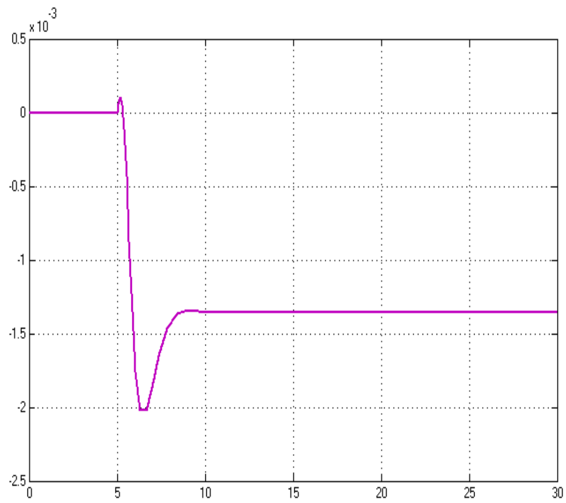


Fig. 6. Tilt angle (θ) of TWIP when 20% Step disturbance is introduced on the pendulum at time *t = 5 seconds* (Left: Q-Parameterization controller. Right: LQR controller).

Figure 7 10 shows the impulse response of the TWIP with both controllers. As one can see that the Q-parameterization controller is faster in response than the LQR, on the other hand LQR in the tilt angle was smoother.

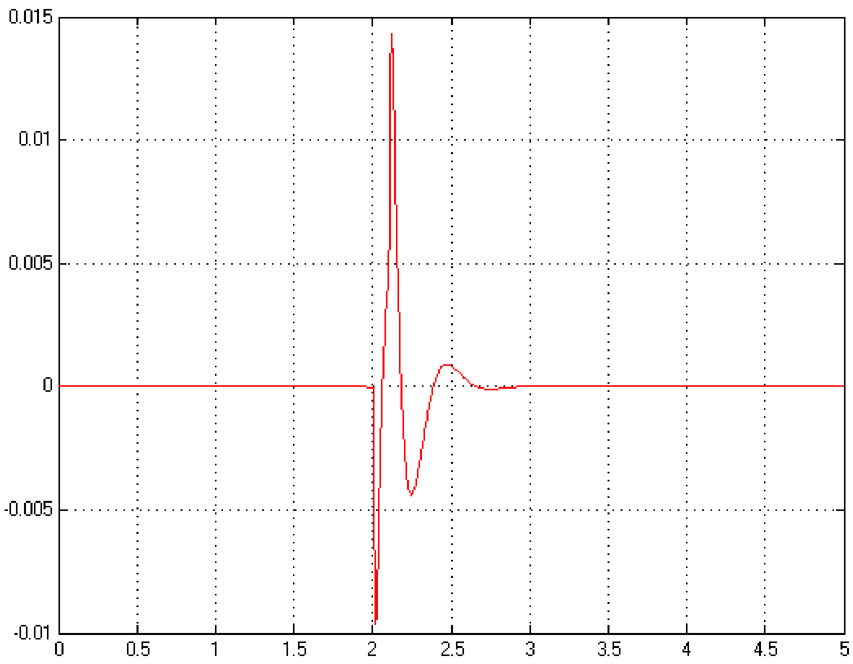


Fig. 7. Tilt angle (θ) of TWIP when impulse disturbance is introduced on the pendulum at time *t = 2 seconds* Q-Parameterization controller.

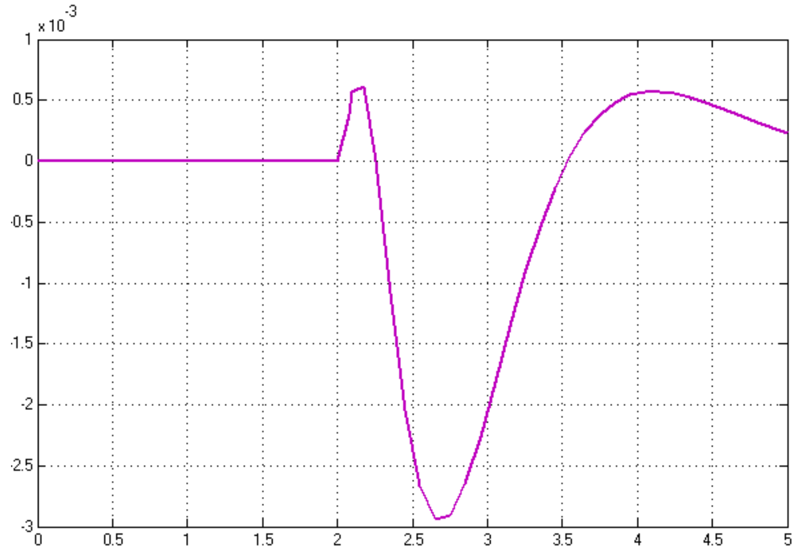


Fig. 8. Tilt angle (θ) of TWIP when impulse disturbance is introduced on the pendulum at time *t = 2 seconds* LQR controller.

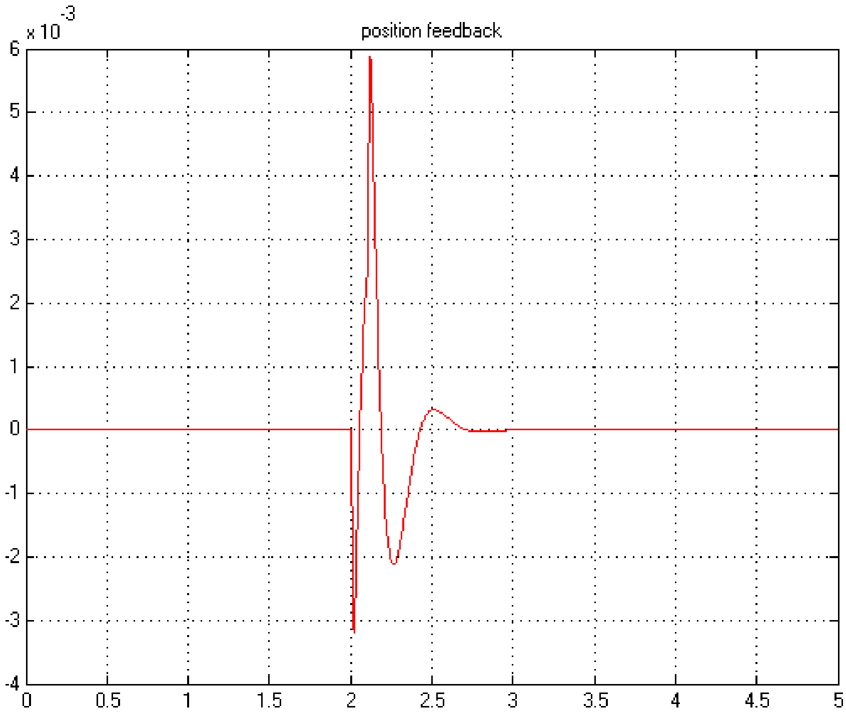


Fig. 9. Position of TWIP when impulse disturbance is introduced on the pendulum at time *t=2 seconds* Q-Parameterization controller.

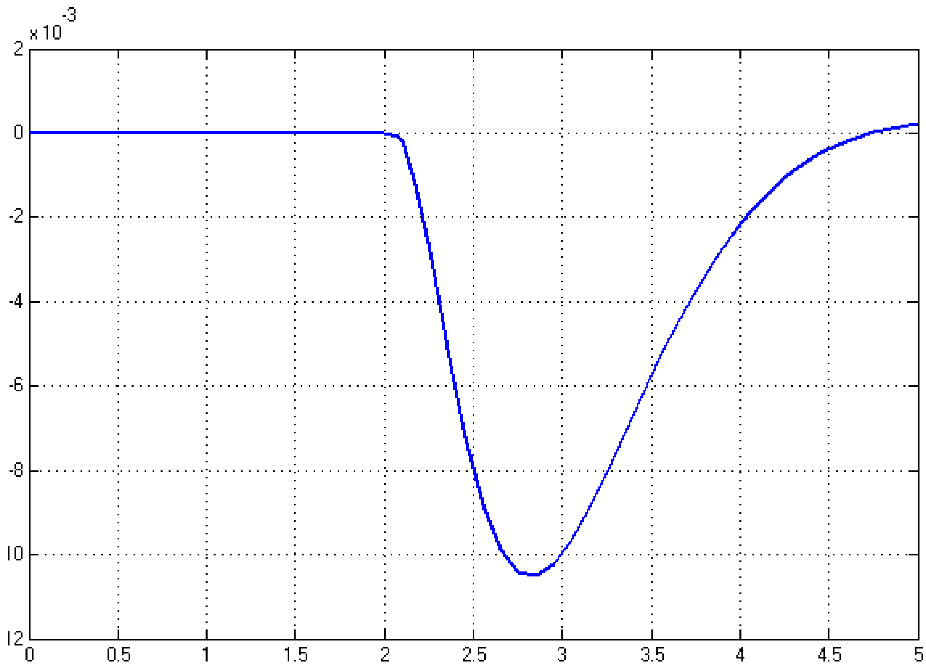


Fig. 10. Position of TWIP when impulse disturbance is introduced on the pendulum at time *t=2 seconds* LQR controller.

Fig. 3 shows the response to a reference step signal on the system as shown in the block diagram Figure 5.3. We can see that the lack of control effort constraints (where we can generate from *h21 & h22*) is very noticeable as it generates a spike at the beginning of the response. But the controller follows the reference signal.

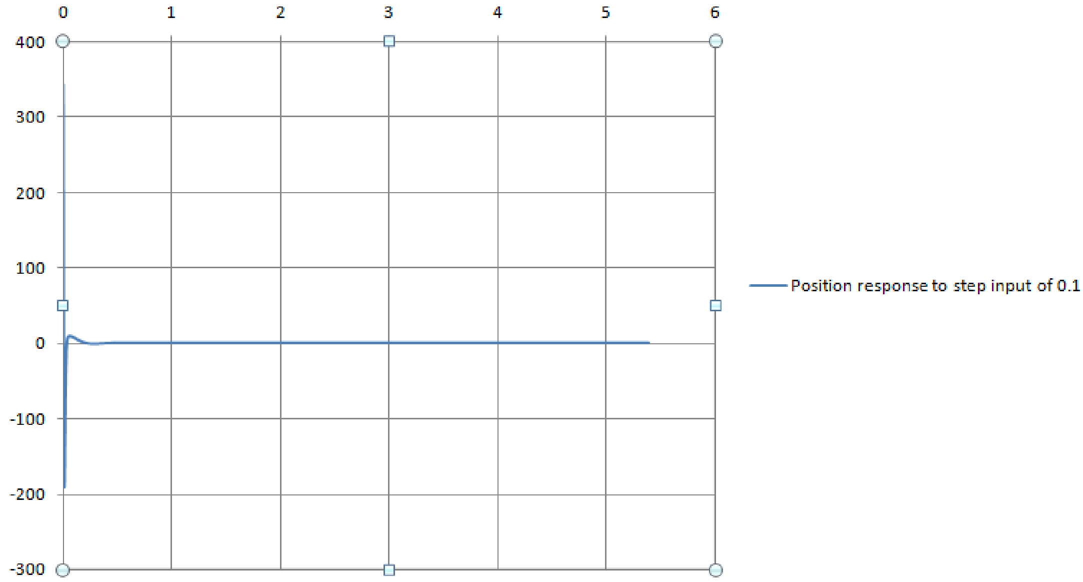


Fig. 3. Position of TWIP with step reference signal.

6. Conclusions

The mathematical model of a two wheeled inverted pendulum (TWIP) system in the state space equation has been presented. Besides that, the Q-Parameterization theory and controller design methodology has also been explained. The Q-Parameterization controller has been successfully designed (based on the fact that the free parameter were chosen as a scalar)by using linear state space equation to control system.

The performance of the TWIP control system using the Q-parameterization controller is illustrated by simulation. The result is compared to LQR control technique. From the results that had been shown in Chapters 5 & 4, the Q-parameterization controller showed a potential of better performance than LQR technique because of two reasons.

First, it proves that the controller design based on Q-parameterization theory can reject the disturbances more efficiently compared to LQR technique. Secondly, the Q-parameterization controller offer more work area than the LQR controller, where the LQR technique is limited to minimizing the quadratic cost function.

On the other hand, the choice done here for the free Q parameter was not enough for the controller to perform optimally. By adding more constraints on the system, one can find via optimization techniques the optimum controller for this plant.

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