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The Application of the Trigonometric Deformation Theory of Thick Plates

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Abstract – *The trigonometric shear deformation theory for thick orthotropic plates, is developed. The displacement field depends the three unknown's element. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness, and stress boundary conditions on the plate boundary surface, thus a shear correction factor is not required. Plate governing equations and boundary conditions are derived by employing the principle of virtual work. The exact solutions for static bending analysis are presented for sinusoidal and uniformly distributed loads. The accuracy of the present theory is ascertained by comparing it with various available results in the literature. The results show that the present model performs as good as the Reddy's and Kirchhoffs shear deformation theories for analyzing the static behavior of thick orthotropic plates.*

Keywords: Trigonometric Shear Deformation, Strain, Thick Orthotropic Plates, Boundary Conditions

1.0 INTRODUCTION

The laminated composite plats are basic structural components when dealing with a variety of engineering structures such as airplane wings, helicopter blades and turbine blades as well as many others in the aerospace, mechanical [1], and civil industries. The great possibilities provided by composite materials can be used to alter or change favorably the response characteristics of structures. Due to the outstanding engineering properties, such as high strength-to-weight and stiffness-to-weight ratios, the composite plat structures are likely to play a significant role in the design of structures when the weight and strength are of primary consideration Classical composites structures suffer from discontinuity of material properties at the interface of the layers and constituents of the composite [2]. Therefore, the stress fields in these regions create interface problems and thermal stress concentrations under high temperature environments [3].

Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material [4]. However, in most of the applications available in the literature, as in the present work, the variation is through the thickness only [5]. Therefore, the early state development of improved production techniques, new applications, introduction to effective micro mechanical models and the development of theoretical

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methodologies for accurate structural predictions, encourage researchers in this field. Plate theories can be developed by expanding the displacements in power series of the coordinate normal to the middle plane. In principle, theories developed by this means can be made as accurate as desired simply by including a sufficient number of terms in the series. These higher-order theories are cumbersome and computationally more demanding [6], because each additional power of the thickness coordinates, an additional dependent is introduced into the theory. This observation is more or less true for many other higher order theories as well [7]. And, thus there is a scope to develop simple to use higher order plate theory. To prove the efficiency of present theory, the problem of the cylindrical bending of especially orthotropic plates is considered. Numerical results are obtained and compared with results based on the corresponding exact, three dimensional solutions due to [8].

2.0 MATHEMATICAL FORMULATIONS

The assumed displacement field for thick orthotropic plate based on the trigonometric shear deformation theory can be written as follows:

$$U(x,y,z) = -z\frac{\partial w(x)}{\partial x} - f(z)\phi(x) \tag{1}$$

$$W(x,y,z) = w(x) + (\frac{h}{\pi})^2 f'(z)\zeta(x) \tag{2}$$

with

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi.z}{h}\right) \tag{3}$$

Where U and W are the axial transverse displacement of the plate center line. The sinusoidal function is assigned according to the shear stress distribution through the thickness of the plate. ϕ, and ζ represent rotations of the plate at neutral surface, which are unknown functions to be determined [9]. Normal and shear strains are obtained within the framework of linear theory of elasticity using displacement field given by Eqn. 1.

$$\varepsilon_x = \frac{\partial U}{\partial x} = -z\frac{\partial^2 w(x)}{\partial x^2} - f(z)\frac{\partial \phi(x)}{\partial x} \tag{4}$$

$$\varepsilon_z = \frac{\partial W}{\partial z} = -\zeta(x)\frac{\pi}{h} f(z) \tag{5}$$

$$\gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} = f'(z)[\frac{h\partial\zeta(x)}{\pi\partial x} + \phi(x)] \tag{6}$$

The stress strain relationship for the plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} \tag{7}$$

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where, Q_{ij} are reduced stiffnesses.

3.0 GOVERNING EQUATIONS

Using the expressions for stresses, strains, and principle of virtual work variational consistent differential equations and boundary conditions for the plate under consideration are obtained [10]. The principle of virtual work when applied to the plate leads to principle of virtual work variational.

$$\delta W_{ext}(\delta U) + \delta W_{\text{int}}(\delta U) = 0, \forall\, \delta U \tag{8}$$

The work of external effort written

$$\delta W_{ext}(\delta U) = \int_D f^V \delta U \, dV + \int_{\partial D_f} f^S \delta U \, dS \tag{9}$$

The work of internal effort written

$$\delta W_{\text{int}}(\delta U) = -\int_D \sigma : \delta \in dV, \tag{10}$$

The principle of virtual work when applied to the plate leads to

$$\delta\pi = 0 = \int_{z=-h/2}^{z=h/2} \int_{x=0}^{x=a} [\sigma_x \delta\varepsilon_x + \sigma_z \delta\varepsilon_z + \tau_{xz} \delta\gamma_{xz}] dxdz - \int_{x=0}^{x=a} q(x)\delta w dx = 0 \tag{11}$$

Integrating the Eqn. 5 by parts and collecting the coefficients of δW, $\delta\phi$ and $\delta\zeta$ the following governing equations and boundary conditions are obtained in terms of displacement variables. The governing equations are as follows

$$\frac{Q_{11}h^3}{12}\frac{\partial^4 w}{\partial x^4} - \frac{Q_{11}2h^3}{\pi^3}\frac{\partial^3\phi}{\partial x^3} + \frac{Q_{13}2h^2}{\pi^2}\frac{\partial^2\zeta}{\partial x^2} = q \tag{12}$$

$$\frac{Q_{11}2h^3}{\pi^3}\frac{\partial^3 w}{\partial x^3} - \frac{Q_{11}h^3}{2\pi^2}\frac{\partial^2\phi}{\partial x^2} + \frac{Q_{55}h}{2}\phi + (\frac{Q_{13}h^2}{2\pi} + \frac{Q_{55}h^2}{2\pi})\frac{\partial\zeta}{\partial x} = 0 \tag{13}$$

$$\frac{Q_{13}2h^2}{\pi^2}\frac{\partial^2 w}{\partial x^2} - (\frac{Q_{13}h^2}{2\pi} + \frac{Q_{55}h^2}{2\pi})\frac{\partial\phi}{\partial x} - \frac{Q_{55}h^3}{2\pi}(\frac{Q_{13}h^3}{2\pi^2}\frac{\partial^2\zeta}{\partial x^2} + \frac{Q_{33}h}{2})\zeta = 0 \tag{14}$$

And the associated boundary conditions at edges $x = 0$ and $x = a$ are as follows

$$\frac{Q_{11}h^3}{12}\frac{\partial^3 w}{\partial x^3} - \frac{2Q_{11}h^3}{\pi^3}\frac{\partial^2\phi}{\partial x^2} + \frac{Q_{13}2h^2}{\pi^2}\frac{\partial\zeta}{\partial x} = 0 \tag{15}$$

$$\frac{Q_{11}h^3}{12}\frac{\partial^2 w}{\partial x^2} - \frac{2Q_{11}h^3}{\pi^3}\frac{\partial\phi}{\partial x} + \frac{Q_{13}2h^2}{\pi^2}\zeta = 0 \tag{16}$$

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$$\frac{2Q_{11}h^3}{\pi^3}\frac{\partial^2 w}{\partial x^2}-\frac{Q_{11}h^3}{2\pi^2}\frac{\partial \phi}{\partial x}+\frac{Q_{13}h^2}{2\pi}\zeta=0 \quad (17)$$

$$\frac{Q_{55}h^2}{2\pi}\phi+\frac{Q_{55}h^3}{2\pi^2}\frac{\partial \zeta}{\partial x}=0 \quad (18)$$

4.0 ILLUSTRATIVE EXAMPLE

A plate of length (span) a and thickness h is considered. It is assumed that the plate is of an infinite extent at the y direction while it is simply supported at its edges x = 0, x = a. The plate is subjected to single sine load, $q(x)$ on surface z = -h/2 acting in downward z-direction given by

$$q(x)=q_0\sin\left(\frac{\pi x}{a}\right) \quad (19)$$

where q_0 is the maximum intensity of sinusoidally distributed load. The following material properties for plate are used.

$$\frac{E_1}{E_2}=25,\frac{E_3}{E_2}=1,\frac{G_{12}}{E_2}=\frac{G_{13}}{E_2}=0.5,\frac{G_{23}}{E_2}=0.2; \qquad \mu_{12}=\mu_{13}=\mu_{23}=0.25 \quad (20)$$

The following is the solution form for w, ϕ and ζ satisfying the boundary conditions perfectly for a plate with all edges simply supported

$$w(x)=w_1\sin\left(\frac{\pi x}{a}\right) \quad (21)$$

$$\phi(x)=\phi_1\sin\left(\frac{\pi x}{a}\right) \quad (22)$$

$$\zeta(x)=\zeta_1\sin\left(\frac{\pi x}{a}\right) \quad (23)$$

where W_1, ϕ_1 and ζ_1 are coefficients, which can be easily evaluated after substitution of Eq.(9) in the set of three governing differential equations (6) and solving the resulting simultaneous equations[11]. Having obtained the values of W_1, ϕ_1 and ζ_1 one can then calculate all the displacement and stress components within the plate.

5.0 NUMERICAL RESULTS AND DISCUSSIONS

The results are obtained for different degree of orthotropy and h/a ratios of plate. The results obtained are presented in Figure 1-13. The results of exact elasticity theory available in the literature are used as a basis for comparison of results obtained by various plate theories. The results obtained for displacements and stresses are presented in the non-dimensional parameters. The results are presented in the following nondimensional form [12].

$$\bar{u} = \frac{uE_2}{qh} \tag{24}$$

$$\bar{w} = \frac{w100h^3E_2}{qa^4} \tag{25}$$

$$\overline{\sigma_x} = \frac{\sigma_x, \sigma_y}{qS^2} \tag{26}$$

$$\overline{\tau_{zx}} = \frac{\tau_{zx}}{qS} \tag{27}$$

$$\%\,error = \frac{value\,by\,a\,particular\,model - value\,by\,exact\,elasticity\,solution}{value\,by\,exact\,elasticity\,solution} X100 \tag{28}$$

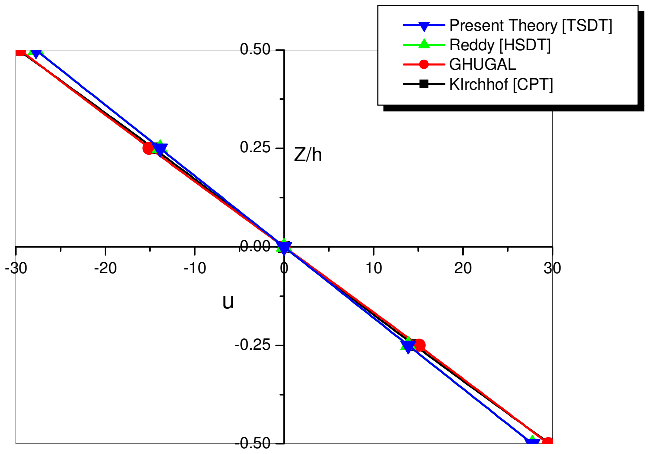

Figure 1: Through-thickness variation of axial displacement for $E_1/E_2 = 2$ and $h/a = 0.15$

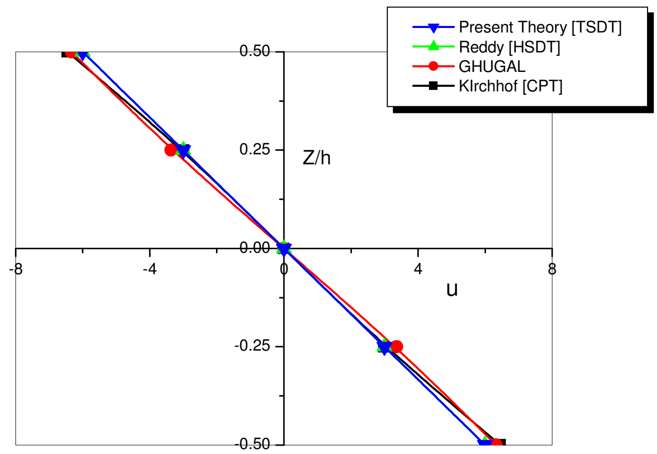

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Figure 2: Through-thickness variation of axial displacement for $E_1/E_2 = 2$ and $h/a = 0.25$

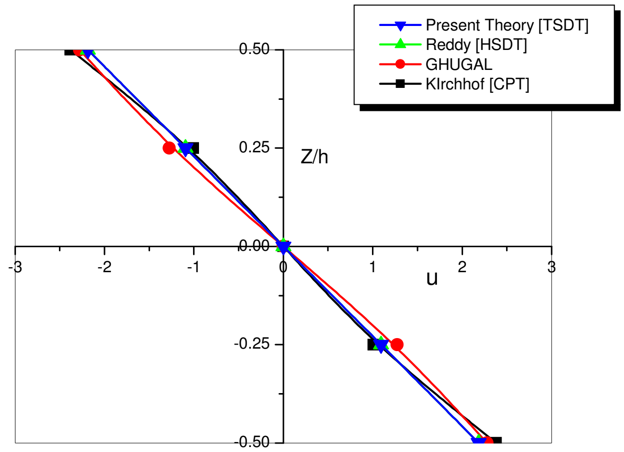

Figure 3: Through-thickness variation of axial displacement for $E_1/E_2 = 2$ and $h/a = 0.35$

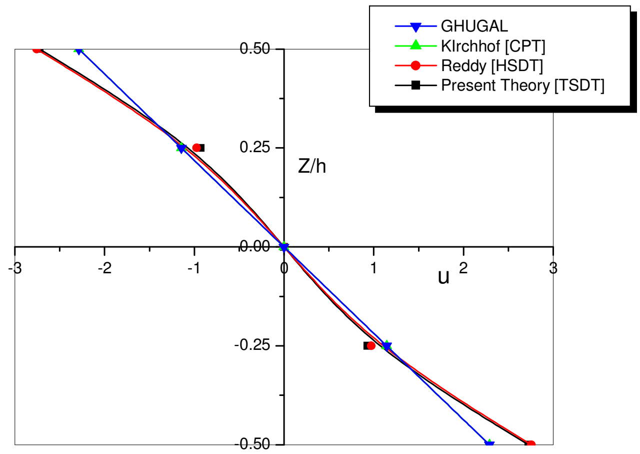

Figure 4: Through-thickness variation of axial displacement for $E_1/E_2 = 25$ and $h/a = 0.15$

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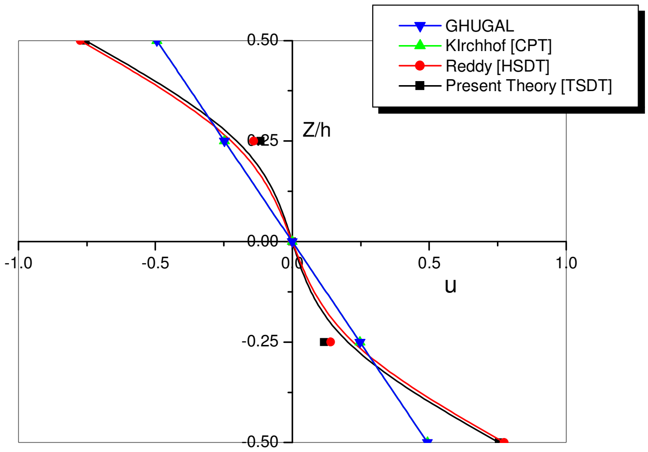

Figure 5: Through-thickness variation of axial displacement for $E_1/E_2 = 25$ and $h/a = 0.25$

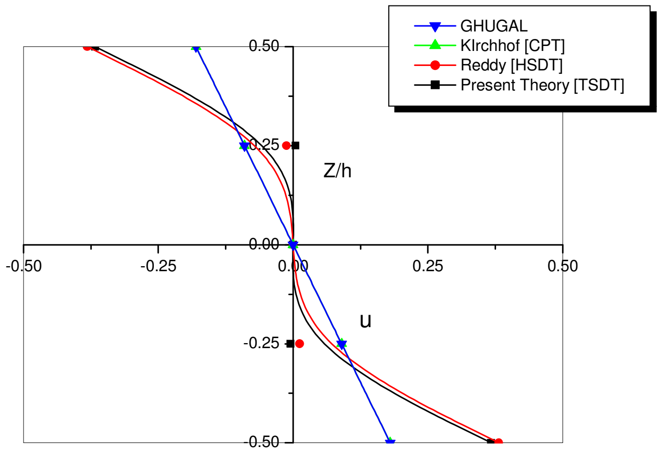

Figure 6: Through-thickness variation of axial displacement for $E_1/E_2 = 25$ and $h/a = 0.35$

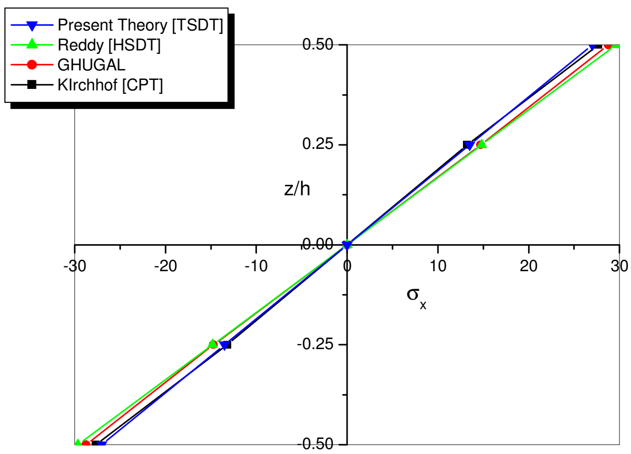

Figure 7: Through-thickness variation of normal bending stress at $E_1/E_2 = 2$ and $h/a = 0.15$.

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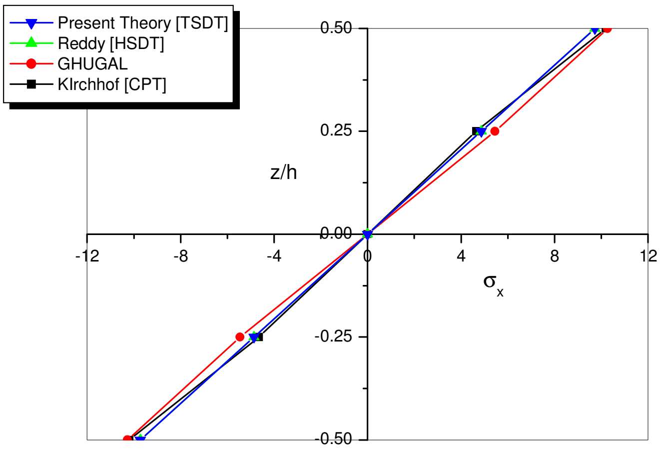

Figure 8: Through-thickness variation of normal bending stress at $E_1/E_2 = 2$ and $h/a = 0.25$

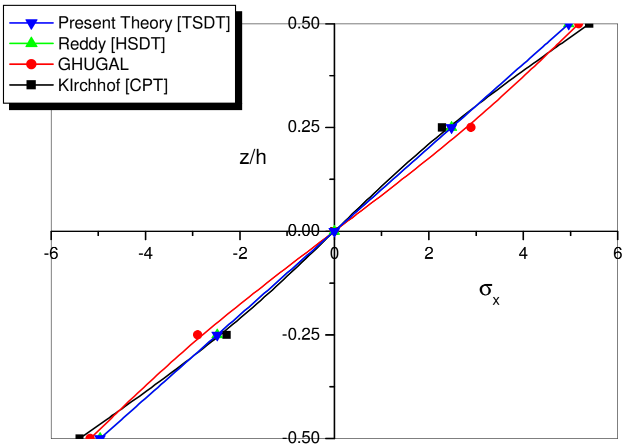

Figure 9: Through-thickness variation of normal bending stress at $E_1/E_2 = 2$ and $h/a = 0.35$.

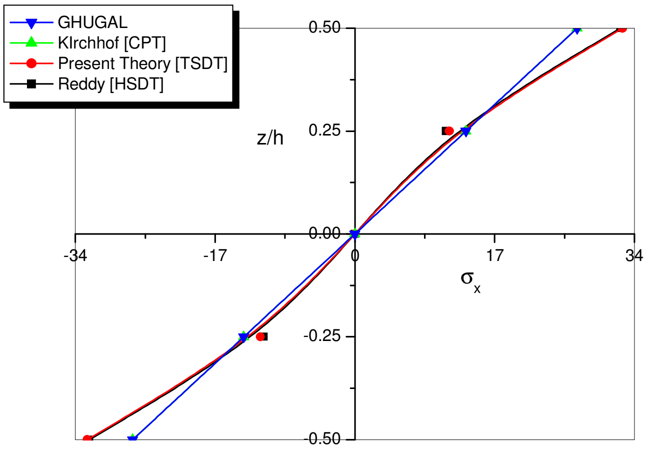

Figure 10: Through-thickness variation of normal bending stress at $E_1/E_2 = 25$ and $h/a = 0.15$

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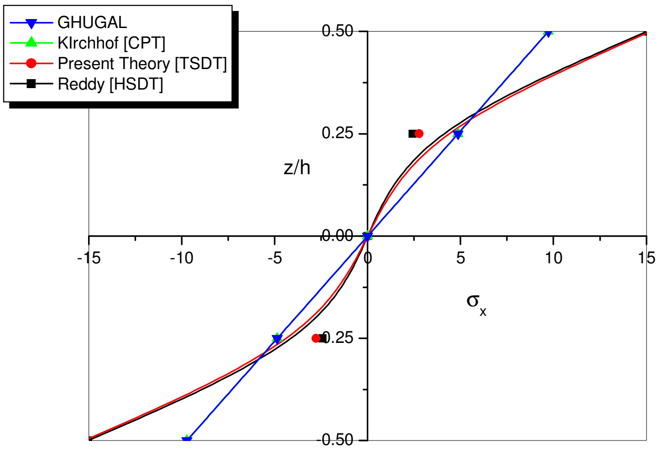

Figure 11: Through-thickness variation of normal bending stress at E_1/E_2 = 25 and h/a =0.25.

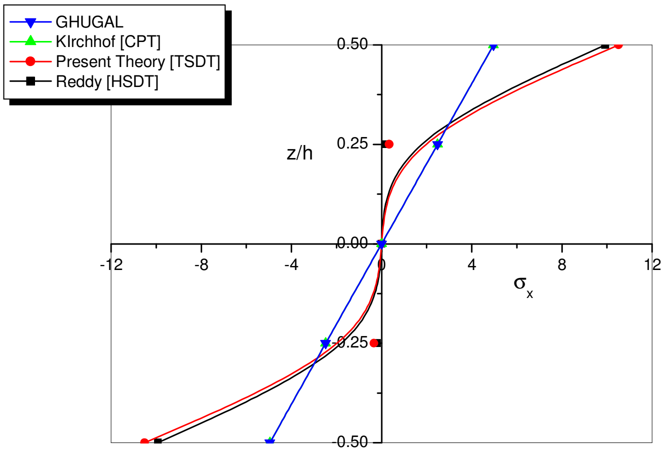

Figure 12: Through-thickness variation of normal bending stress at E_1/E_2 = 25 and h/a = 0.35

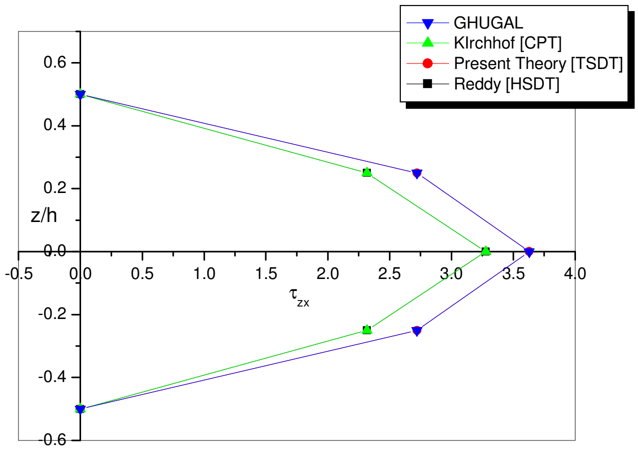

Figure 13: Through-thickness variation of transverse shear stress at E_1/E_2 = 2 and h/a = 0.15.

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Through thickness variation of axial displacements are shown in Figure 1 through 6. It is observed from the variation of axial displacement that the effect of nonlinearity becomes more pronounced with the increase in degree of orthotropy and the thickness to span ratio [13]. Fig. 7 and Figure 8 show that maximum normal bending stress predicted by present theory is in excellent agreement with that of exact solution for $E_1/E_2 = 2$ and $h/a = 0.15$ and $h/a = 0.25$. Minimum error predicted by present theory is 0.03 % for $E_1/E_2 = 2$ when $h/a = 0.35$ (see Figure 9). The normal bending stress predicted by present theory is in excellent agreement for all aspect ratios when $E_1/E_2 = 25$ (see Figure 10 through 12). Theory of Reddy shows marginal error in maximum inplane normal stress for all aspect ratios and degree of orthotropy [14].

Maximum error predicted by theory of Reddy is 6.28 % for $E_1/E_2 = 50$ when $h/a = 0.25$. Maximum in plane displacement predicted by FSDT and CPT is independent of degree of orthotropy and underestimates for all aspect ratios [15]. For high degree of orthotropy, the variation of normal bending stress is non-linear in nature (can be seen from Figure 11, Figure 12 and Figure 13).

6.0 CONCLUSION

The trigonometric shear deformation theory of thick orthotropic plates is carried out in the present study. Recently developed of this theory has been extended for the analyses of plat. A refined trigonometric shear deformation theory is developed for the bending response of thick orthotropic plates all comparison studies demonstrated that the bending obtained using the present theory. Hence, it can be said that the proposed theory is accurate and simple in solving the bending behavior of thick plates. From the numerical comparison made it is observed that, unless the plate is thick or highly reinforced, present theory provides accurate transverse displacement. The present theory is capable of producing reasonably good transverse shear stresses using constitutive relation, and better values of these stresses can still be obtained by integration of equilibrium equation with additional efforts.

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