

Review of the ‘Flow through a Circular Tube with a Permeable Navier Slip Boundary’; the Double-Slip Challenge

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Abstract- *This note presents some comments concerning the paper entitled ‘Flow through a circular tube with a permeable Navier slip boundary’ by Barry James Cox and James Murray Hill which was published in Nanoscale Research Letters in 2011. The comments are mainly related to the assumption of constant and arbitrary slip length values along the tube. Copyright © 2015 Penerbit Akademia Baru - All rights reserved.*

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1.0 BACKGROUND

In the interesting paper entitled ‘Flow through a circular tube with a permeable Navier slip boundary’ [1], the authors have presented a framework for numerical and analytical investigation of the fluid flow in a right circular nanotube with a linear Navier slip boundary condition. The governing equations are derived by simplifications of the Navier-Stokes equations in cylindrical coordinates. The constitutive simplifying assumptions are steady state, Newtonian, incompressible, axisymmetric flow with radial velocities that are only functions of the radius $u = u(r)$. The exact solution for the assumed quadratic pressure distribution exists only for a specific prescribed permeability value. They have brought some justifications for some of the high flow rates reported in the nanotubes. This note deals with the assumption $u = u(r)$ near the Navier slip boundary on channel walls and adoption of constant and arbitrary slip length values throughout the nanotube.

1.1 Double-slip Challenge

The dynamics of the fluid flow is drastically changed near the walls of nanotubes due to increased kinetic energy of the molecules after their inelastic bouncing-back collisions with the walls. The molecules transfer their momentum through this non-linear mechanism to the bulk fluid in the central region. This phenomenon described as double-slip theory is considered as a correction to the classical Klinkenberg theory which presents a linear relation between apparent permeability and the pore pressure [2].

Here, the focus is on this feature of the double-slip phenomenon that it imposes a non-linear and nonconstant molecular streaming influence on the bulk fluid flow especially for diameters less than 50 nm when the average pressure is reduced below 500 psi [2]. However,

in the considered paper [1] instead of the Klinkenberg theory the direct slip length concept is used. The underlying result of the Klinkenberg theory is borrowed to show that for the test cases of [1], the inclusion of the $u = u(r)$ assumption for the prescribed parameter range is questionable.

Permeability tensor is traditionally defined in terms of averaged velocities [3,4]. It is in detail a complex function of tube radius, fluid nature, potential solid diffusion rate and average pressure [2].

Furthermore, the Klinkenberg equation states that the apparent permeability significantly changes from entrance to the outlet of the tube length. These features violate the adopted assumption of constant permeability measure all over the tube in citeCox. Thus, constant values of slip length and consequently the simplifying assumption of $u = u(r)$ in reducing the solenoidality equation (3) to an ODE is challenged. Since the derived ODE is fundamental in reducing the governing equations to the form suitable to be solved via analytical methods, a deeper look at the mentioned assumption is suggested in the analysis. Moreover, the corresponding physical modeling statement leads to unphysical flow rate estimations due to remarkable variation of slippage on the walls within the channel.

2.0 DISCUSSION

To satisfy the requirements for the assumption of $u = u(r)$, the authors have arbitrarily chosen the constant slip lengths all over the tube length, while slip length is basically function of the capillary pressure and the radius [2]. In this context, the slip length can be interpreted as the ratio of Length scale of kinetic energy of bouncing back molecules by the wall to the Gas molecules mean free path. Thus, if the average nano-capillary pressure changes then the slip length would change leading to different velocity profile. In fact, for $r = 2 \text{ nm}$ and $\Delta P = 14:5 \text{ psi}$ the dependence is clearly non-linear and the derivative of the velocity profile across the tube cross section is not continuous [2]. This issue could be a source of partial slip [5] which effectively deviates from the assumption of full slip in [1]. [6] has also reported the remarkable effect of Knudsen number of the flow and the surface roughness pattern and root mean squares value of the roughness on slip length.

Furthermore, the slip length value has been shown to increase considerably with increasing the volume flow rates [7] a periori. This effect necessitates more careful attention since, the authors has considered a fixed arbitrary range of non-dimensional permeability values for all slip lengths. Also, the issue of solid diffusion could affect the permeability values especially in large fluxes of atoms passing through a unit area in the x-direction. The analysis would be more standard if the Klinkenberg theory is explicitly considered in the estimated flow rates together with considering the solid diffusion [8,9].

On the other hand, the effect of trapped layers of external contaminant or gas molecules on the surface [10] and the atomic roughness [11] would not be eliminated in capturing the slippery physics of the tough test case chosen. For real system applications one needs some practical manipulation such as heat treatments in vacuum media to free the trapped layers of external molecules to provide certain slip condition [12–14] and/or detailed chemical and structural investigations on the specific material at the nanoscale to distinguish between partial slip and full slip [5,15]. The difficulty in preparing such conditions could be the reason that some investigations show that the no-slip condition is the governing physical description even for surface polished walls [10,16]. Therefore, also from practical aspects, choosing

some formal and constant slip length values seems unreasonable especially for the parameters ranges claimed in [1].

3.0 CONCLUSION

It is shown that adopting $u = u(r)$ assumption near the slip boundaries or choosing arbitrary slip length values is in contrast to the well-documented double-slip phenomenon and other observations in nanotubes for the small characteristic length scales and low pressure differences in the considered test cases of [1].

Although, the analysis is mentioned to be limited to where a quadratic pressure corresponding to the radial boundary flow is prescribed, the extremely narrow width and low average pressure assumptions require some serious reconsiderations in the basic solution strategy.

The authors may employ the modified Klinkenberg relation for this extremely narrow channel to take into account the effect of double-slip phenomenon reliably or alternatively they may apply their differential equation framework for problems with larger widths and more average pressures with the possibility of partial slip. After all, the inclusion of the effects of molecular forces, Knudsen number of the flow and surface roughness in the analysis on the considered parameter values seems to be essential.

In addition, the model of [1] lacks the adsorption effect and also solid diffusion of adsorbed phase on nano-carbon capillary tube surface. Furthermore, a robust explanation of the value or range of the so-called specific permeability measure (as a need to reproduce the results) for which the exact solutions are available is absent in the paper.

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