

Comparative Study on Heat Transfer Enhancement and Nanofluids Flow over Backward and Forward Facing Steps

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Abstract – *This review summarizes various researches and studies on two topics. The first section, focuses on the numerical and experimental studies using water and air over backward and forward facing steps (BFS and FFS). The second section, concentrates on the numerical and experimental studies using nanofluids over single, double and triples BFS and FFS, turbulators heat exchangers and microscale channels. The main purpose is to review the recent available results of the flow and heat transfer presented in the literature for single backward and backward facing steps. The second purpose is to understand the characteristics and functions of nanofluids, to expect their effects and heat transfer enhancement in such geometries. The third purpose is to present the recent published researches and propose suggestions that could lead to study new types of channels and heat exchangers using nanofluids. Copyright © 2016 Penerbit Akademia Baru - All rights reserved.*

Keywords: Nanofluids; Backward Facing Step; Forward Facing Step

1.0 INTRODUCTION

The flow separation, with subsequent reattachment occur due to a sudden expansion or compression in the geometry, such as the backward-forward facing steps. This phenomenon plays an important role the design of many engineering applications such as combustion chambers, environmental control systems, cooling systems for electronic equipment, high performance heat exchangers, cooling passages in turbine blades and chemical processes as well as energy system equipment. As a result, there is the occurrence of a significant amount mixture of high and low energy in the reattachment region. The separation of the backward facing step occurs at the upper sharp corner of the step causing the development of a recirculating region located behind the step. However, there are two recirculation regions for the forward-facing step geometry; the first region is located downstream and adjacent to the step while the second region is

located upstream the step and its location depends on the step height and the ratio of the boundary-layer thickness at the step [1]. Previous research studies investigated numerically and experimentally the problem of laminar flow over backward-forward facing steps geometries in forced, natural and mixed convection [2-5, 1, 6, 7]. Abu-Mulaweh [8] studied the effects of backward-forward facing steps on turbulent natural convection along a vertical heated flat plate. The results revealed that the maximum Nusselt number for the backward-facing step was approximately twice that of a flat plate, with occurrence of the vicinity of the reattachment region. However, the maximum Nusselt number for the case of the forward-facing step is two and a half times more than that of the flat plate.

Nanofluid is one of the techniques used to enhance the heat transfer rate whereby nanometer-sized particles are immersed in the traditional base fluids [9-11]. Most of the recent studies showed that, the enhancement of heat transfer coefficient can be increased by adding solid metallic or nonmetallic nanoparticles with a high thermal conductivity of the base fluid [12-14]. These nanoparticles can be for instance Al_2O_3 , SiO_2 , Cu, CuO, ZnO and TiO_2 [15]. Nanofluids effects have investigated by many researchers in the enhancement of heat transfer and the fluid flow [16-20].

Thermal behaviour investigation and characteristics of nanofluid flow conducted over backward-facing step by Abu-Nada (2008). He reported that the Nusselt number can be enhanced by increasing the nanoparticle volume fraction. Mohammed et al. [21] studied the effects of nanofluids on mixed convective heat transfer over a vertical and horizontal backward-facing step. Their findings revealed that the SiO_2 recorded the highest Nusselt number in the downstream primary recirculation region, while the diamond nanofluid recorded the highest Nusselt number in the primary recirculation region itself. Al-Aswadi et al. [17] conducted a numerical investigation on the laminar forced convection flow over a BFS in a duct using different nanofluids. They reported that the recirculation size and reattachment length increased with increasing the Reynolds number. Nanofluids with low dense nanoparticles such as SiO_2 demonstrated a higher velocity than those with high dense nanoparticles such as Au. Kherbeet et al. [22] implemented a numerical investigation on the effects of nanofluid on the laminar flow on a mixed convection heat transfer over 2D microscale backward-facing step. They used four types of nanoparticles; Al_2O_3 , CuO, SiO_2 and ZnO, with a volume fraction ranging from 1% to 4%. The results revealed that the fluid with SiO_2 nanoparticles showed the highest Nusselt number. The results also showed that the Nusselt number increased with increment in the volume fraction of nanoparticles in the base fluid.

Kherbeet et al. [23] presented a numerical study on the mixed convective flow over 3D horizontal microscale backward facing step (MBFS). The nanofluid EG- SiO_2 with a nanoparticle of 25 nm in diameter and volume fraction of 0.04, was used in this investigation. Their results revealed that the Nusselt number and skin friction coefficient increased with an increase in the step height. The findings also showed that the Reynolds number and pressure drop decrease with an increase in the step height. Recently Kherbeet et al. [24] implemented a numerical study on the laminar mixed convection flow of nanofluids over a 3D horizontal microscale forward-facing step (MFFS) using a finite volume method. Various types of nanoparticles materials namely SiO_2 , Al_2O_3 , CuO, and ZnO, were dispersed in a base fluid of ethylene glycol with

volume fraction ranged from 0 to 0.04. The step height maintained at 650 μm . The results showed that the SiO₂ nanofluid recorded the highest Nusselt number. The other finding was the Nusselt number increased with decreasing in the density of nanoparticle material, a decrease in diameter of nanoparticles and an increase in volume fraction.

This study starts with a review on the flow and heat transfer in backward and forward facing step channels. After that, nanofluids and its characteristics are described in such geometries. At the end, studies concentrate on the recent modifications over a backward and forward facing steps.

2.0 NUMERICAL AND EXPERIMENTAL STUDIES USING WATER AND AIR

Iwai et al. [25] performed a three-dimensional numerical simulation for flows over a backward-facing step at low Reynolds number to investigate the effects of the duct aspect ratio. Numerical results were compared with other investigations and found to agree well with experimental data. Close attention was paid to the distribution patterns of both Nusselt number and the skin friction coefficient on the bottom wall. It was found that an aspect ratio of as large as $AR=16$ at least was needed to obtain a 2D region near the centerline at $Re = 250$. This 2D region became wider for a lower Reynolds number. It was also found that the maximum Nusselt number did not appear on the centerline but near the two side walls in every case.

Iwai et al. [26] carried out three-dimensional numerical simulations for mixed convective flows over a backward-facing step in a rectangular duct. Reynolds number, expansion ratio and aspect ratio were kept constant at $Re = 125$, $ER = 2$ and $AR = 16$, respectively. Heat flux at the wall downstream of the step was kept uniform, while other walls were kept at adiabatic condition. Effect of the inclination angles, θ_1 , θ_2 , was the main objective in this study. It was found that when θ_1 was varied, the effect of buoyancy became prominent at $\theta_1 = 0^\circ$, 180° , while the effect was relatively small for the two horizontal cases ($\theta_1 = 90^\circ$, 270°). However, there was still small difference between $\theta_1 = 90^\circ$ and 270° in the region immediately after the step where the flow was relatively slow. When θ_2 was varied, flow and thermal fields became asymmetric about the duct centerline, except when $\theta_2 = 0^\circ$, 180° . The maximum Nusselt number, which appears symmetrically near the side walls in pure forced convection cases, was obtained at only one location close to the lower side wall.

Iwai et al. [27] performed two- and three-dimensional numerical simulations for mixed convective upward flows over a backward-facing step in a duct. The Reynolds number, expansion ratio, and aspect ratio (3-D simulations) were kept constant at $Re = 125$, $ER = 2$, and $AR = 16$, respectively. The heat flux at the wall downstream of the step was uniform. The straight wall, the step, and the side walls (3-D simulations) were assumed to be adiabatic. It was found that the reattachment point and the peak Nusselt number point moved upstream as Ri^* was increased. The secondary recirculation region, which developed at the corner of the step, became larger. A secondary flow was also found in a cross section immediately downstream of the step. Flow directed toward the center of the duct became more intensive as Ri^* increased, which possibly resulted from an increase in the level of three-dimensionality of the flow and thermal fields.

Chiang et al. [28] conducted a computational investigation on the effect of side wall on structure laminar incompressible fluid flow over a plane symmetric sudden expansion. In their analysis, 14 aspect ratios were used and varied from 3 to 48, with $Re = 60$ for three dimensional analyses and $Re = 60$ and 140 for two-dimensional analysis. Oliveira [29] performed numerical simulations on viscoelastic liquid flow in symmetric sudden expansion. The constitutive model follows FENE-MCR, employed in this simulation as three meshes with different sized cells, a Reynolds number ranging from 0 to 100 and an expansion ratio of 1:3. Due to asymmetric vortex shapes, a pitch-fork type bifurcation takes place beyond the critical Reynolds number ($Re_c = 64$), while it occurred at $Re_c=54$ with Newtonian fluid in the same expansion ratio. Hammad et al. [30] reported a numerical study of the laminar flow of non-Newtonian Herschel Bulkley fluid through axisymmetric sudden expansion. They used a finite difference method to solve governing fully elliptic momentum and continuity equations, and obtained the regime of laminar flow for a range of yield numbers, Reynolds number, and power-law index values, where the range of Re varied between 50 and 200, yielded numbers that varied between 0 and 2, and power-law index values varied between 0.6 and 1.2. They obtained significant dependence of the flow for large values of yield number only, while at lower yield number, the power law index value became effective with yield number on the flow field. Meanwhile, Miranda et al. [31] numerically studied the local loss coefficient for inelastic laminar fluids flow in axisymmetric sudden expansion by using the finite volume method, and the sudden expansion varied from 1 to 2.6. The results showed that the local loss coefficient varied inversely with Reynolds number at low Re .

Abu-Mulaweh et al. [32] reported measurements of turbulent mixed convection flow over a two-dimensional, vertical backward-facing step. Laser-Doppler velocimeter (LDV) and cold wire anemometer were used, respectively, to measure simultaneously the time-mean velocity and temperature distributions and their turbulent fluctuations. The experiments carried out for a range of free stream air velocities $0 \text{ m/s} \leq u_\infty \leq 0.41 \text{ m/s}$ for a step height of 22 mm, and a temperature difference, ΔT , of 30°C between the heated walls and the free stream air. The upstream and downstream walls were heated to a uniform and constant temperature. The results revealed that the introduction of a small free stream velocity causes a decrease in the turbulence intensity of both the streamwise and transverse velocity and temperature fluctuations. Also, it was found that the reattachment length increases while the heat transfer rate from the downstream heated wall decreases as the free stream velocity increases.

Abu-Mulaweh et al. [33] examined experimentally the effect of the backward-facing step heights on turbulent mixed convection flow along a vertical flat plate. The geometry consisted of an adiabatic backward-facing step, an upstream wall and a downstream wall. Both the upstream and downstream walls were heated to a uniform and constant temperature. Laser-Doppler velocimeter and cold wire anemometer were used, respectively, to measure simultaneously the time-mean velocity and temperature distributions and their turbulent fluctuations. The experiment was carried out for step heights of 0, 11, and 22 mm, at a free stream air velocity, u_∞ , of 0.41 m/s, and a temperature difference, ΔT , of 30°C between the heated walls and the free stream air. The present results revealed that the turbulence intensity of the streamwise and transverse velocity fluctuations and the intensity of temperature fluctuations downstream of the step increase as the step height increases. Also, it was found that

both the reattachment length and the heat transfer rate from the downstream heated wall increase with increasing step height.

Abu-Mulawah et al. [8] reported on the effect of step height on turbulent mixed convection flows over a backward-facing step. Abu-Mulawah [33] noticed that the highest local Nusselt number at the reattachment region for the backward- forward-facing steps in turbulent natural convection flow along a vertical flat plate. The turbulent fluid flow and heat transfer of a mixed convection boundary-layer of air flowing over an isothermal two-dimensional, vertical forward step was experimentally investigated by Abu-Mulaweh [34]. His results indicated that the Nusselt number increased with the increase of step height, and the highest value was obtained at the reattachment region. The present results indicated that the increase of step height leads to an increase in the intensity of temperature fluctuations, the reattachment length transverse velocity fluctuations and the turbulence intensity of the stream.

Nie and Armaly [35] presented a numerical analysis for a three-dimensional incompressible laminar forced convection flow adjacent to backward-facing step in rectangular duct. The effects of step height on the flow and heat transfer characteristics were the main objectives of this study. The geometry, thermophysical properties, and flow condition were based on the available measurements of Armaly et al. [36]. The simulations were performed at $Re = 343$ for different step heights ($s = 0.008, 0.010, \text{ and } 0.012 \text{ m}$), while the other geometries were kept constant. It was found that increasing the step height increases the reattachment length, the Nusselt number, the size of the side wall reverse flow region, and the general three-dimensional features of the flow. Furthermore, as the step height increases, the distance between the step and the point where the impingement flow reaches the stepped wall downstream increases, and a secondary recirculation flow region develops adjacent to the bottom corner of the step. For the case of small step height ($s = 0.008\text{m}$), the minimum Nusselt number was located near the bottom corner between the step and the center-line of the duct. As the step height increases, the minimum Nusselt number was appeared to move towards near the bottom corner of the step and the side wall. It was found that the friction coefficient increases in the stream wise direction along the center line of the duct inside the primary recirculation flow region, but it decreases outside the recirculation region with increasing step height.

Tylli et al. [37] studied experimentally and numerically the side wall effects in three-dimensional laminar water flow over a backward-facing step. Experiments were based on both digital and stereoscopic particle image velocimetry; a spectral element method was used for the laminar, transition, and turbulent simulations. The geometry has an expansion ratio, ER, and aspect ratio, AR, of 2 and 20, respectively. It was noticed that excellent agreement between the 3D simulation and the experiments was obtained when Reynolds number reaches $Re \approx 650$. Moreover, it was found that at low Reynolds numbers ($Re < 400$), side wall effects did not affect the structure of laminar flow in the channel mid plane; thus, the mid plane field was accurately predicted by two-dimensional simulations. In addition, it was found that at higher Reynolds numbers, laminar flow was characterized by side wall separation and the formation of a recirculation zone, which, however, did not penetrate up to the channel mid plane. The wall-jet intensity and flow three-dimensionality increased with Reynolds number for

laminar flow, and decrease for transitional flow; three-dimensional variation is hardly noticed in the time average field of turbulent flow.

A large eddy simulation model for the analysis of turbulent heat transfer in separated flow over a backward-facing step were represented by Labbe et al. [38], Avancha and Pletcher [39], and Zouhaier et al. [40]. These researchers showed significant improvement of heat transfer rate in the recirculation zone and obtained trends of heat transfer coefficients that agreed with previous experimental data. Numerical simulations for two-dimensional turbulent forced convection flows adjacent to backward-facing step were investigated by Chen et al. [41]. They paid attention to the effects of step height on turbulent separated flow and heat transfer. The researchers considered Reynolds number and duct height downstream from the constant step as $Re = 28,000$ and $H = 0.19$ m, respectively. Heat flux was uniformly maintained at $q_w = 270$ W/m² at the stepped wall downstream from the step, while other adjacent walls were treated as adiabatic. The velocity and temperature fields were calculated by using two equations at low Reynolds number and turbulence models.

The influence of buoyancy-assisting laminar flow and heat transfer over a 2D vertical backward-facing step was examined by Abu-Mulaweh [42]. The boundary conditions and general step geometry were taken from the literature except the downstream heated wall was maintained to a uniform heat flux in the range between $102 \leq q_w \leq 290$ W/m². As the buoyancy force increased, the reattachment length X_r decreases but the local Nusselt number increased and the location of its maximum values moves closer to the step. Moreover, it was found that the local Nusselt number to be lower in the region where $X < X_r$ and higher in the region where $X > X_r$ for higher free stream velocity.

Armaly et al. [43] measured the velocity for three-dimensional laminar separated air flow adjacent to a backward-facing step using two-component laser Doppler velocimeter. The backward-facing step, with a height of $s = 1.0$ cm has an aspect ratio of $AR = 8$, and an expansion ratio of $ER = 2.02$. The flow measurements were covered Reynolds number range between $98.5 \leq Re \leq 525$ to develop a clear view of both side wall and the step effects, and compared with the previous 2D studies. It was determined that for $Re \leq 98.5$ there was no recirculation flow region adjacent to the side wall and at $Re \leq 190$, a small recirculation flow region was detected in the upper corner of the side wall. It was concluded that the span wise and the transverse peak velocities move to the center of the duct as the distance from the step increases.

Nie and Armaly [44] presented Laser-Doppler velocity measurements adjacent to the bounding walls of (3-D) backward-facing step flow. The backward-facing step geometry was configured by a step height (S) of 1 cm, which was mounted in a rectangular duct having an aspect ratio (AR) of 8:1 and an expansion ratio (ER) of 2.02:1. Results were presented for a Reynolds number range between 100 and 8000, thus covering the laminar, transitional and turbulent flow regimes. The boundaries of the reverse flow regions were identified by locating the streamwise coordinates on a plane adjacent to the bounding walls where the mean streamwise velocity component was zero. The size of the reverse flow regions increased and moved further downstream in the laminar flow regime; decreased and moved upstream in the transitional flow regime; and remains almost constant or diminishes in the turbulent flow regime; as the Reynolds number increases. The span wise distribution of the boundary line for the

reverse flow region adjacent to the stepped wall develops a minimum near the sidewall in the laminar flow regime, but that minimum in the distribution disappears in the turbulent flow regime. Predictions agree well with measurements in the laminar flow regime and reasonably well in the turbulent flow regime.

Dejoan and Leschziner [45] used Large eddy simulation to investigate the effects of a periodic perturbation introduced into a separated shear layer that borders a recirculation bubble behind a backward-facing step in a high-aspect-ratio channel. The perturbation was provoked by the injection of a slot jet, at zero net mass-flow rate, uniformly along the spanwise edge at which separation occurs. Attention was focused on one particular jet-forcing frequency, at the Strouhal number 0.2, for which experimental data showed the perturbation to cause a maximum change to the properties of the unperturbed flow – in particular, the largest reduction in the time-mean recirculation length. Results are reported for time-mean and phase-averaged velocity and Reynolds stresses, and these were compared with experimental data. The time evolution of phase-averaged properties, including stream-function, pressure and turbulence energy, were investigated in an effort to identify the mechanisms responsible for the observed substantial changes to the time-mean properties. The simulations provided clear indications that the high level of sensitivity to the perturbation at the Strouhal number considered was due to a strong interaction between shear-layer instabilities, which are amplified by the perturbation, and shedding-type instabilities, which were induced by the interaction of large-scale structures developing downstream of the step with the wall, causing the shear-layer to flap.

Saldana et al. [46] studied numerically forced convective air flow over a three-dimensional backward-facing step. The horizontal backward-facing step channel was set to have a step height $s = 1$ cm, an aspect ratio of 8 and an expansion ratio of 2. The stepped wall was fixed to a constant heat flux $q_w = 50 \text{ W/m}^2$ while the other walls were considered as insulated. The Reynolds numbers were set to be in the range of 98.5 and 512. The local Nusselt number along the bottom wall was found to lie in the vicinity of the X_u -line and the X_w -line points of intersection where the shear stress along the bottom wall is equal to zero.

A numerical study of three-dimensional mixed laminar air flow over a horizontal backward step using the finite volume method conducted by Saldana et al. [47]. The bottom wall of a channel was heated with constant temperature and the other walls were adiabatic, the aspect ratio was equal to 4 and the range of Richardson number (Ri) varied between 0 and 3. The numerical results indicated a decrease in the size of the primary recirculation region with increase in Richardson number, and also moved the maximum value of the average Nusselt number. Li and Armaly [48] presented a numerical study of laminar mixed convection in a three-dimensional backward-facing step, where the full elliptic 3 coupled governing equation was solved using the finite volume method. They found that buoyancy force and temperature affect reattachment length.

Chen et al. [41] simulated numerically the turbulent forced convective flow adjacent to a two-dimensional backward-facing step. The main objective of this study was to explore the effects of step height on turbulent separated flow and heat transfer. The stepped wall downstream from the step was set to be uniform and constant heat flux q_w

= 270W/m², while other walls were treated as adiabatic. Reynolds number and duct's height downstream from the step were kept constant at $Re_0 = 28,000$ and $H = 0.19$ m, respectively. On the other hand, the expansion ratio was maintained to $ER = 1.11, 1.25,$ and 1.67 , respectively. It was noticed that as the step height increased; the primary and secondary recirculation regions, magnitude of the maximum turbulent kinetic energy increased, and the bulk temperature increased rapidly as well. Near the step and below the step height, it was found that the kinetic energy became smaller as the step height increased. Moreover, it was found that the maximum temperature becomes greater as the step height increases. The peak values of the transverse velocity component become smaller as the step height increased, but inside the recirculation region; the skin friction coefficient becomes less significant and smaller in magnitude with the increase of the step height.

Khanafer et al. [49] analyzed mixed convection of laminar pulsatile flow and heat transfer past a backward-facing step in a channel. Fluid flow and heat transfer characteristics were examined in the domain of the Reynolds number, Richardson number and the dimensionless oscillation frequency as: $100 \leq Re \leq 1000$, $1.78 \times 10^{-3} \leq Ri \leq 10$, and $0.1 \leq \omega \leq 5$. It was found that the elimination of the separation region occurs for $Ri > 0.1$, and as Reynolds number increased, the recirculation zone along the heated downstream surface became larger. The thermal boundary layer decreased in thickness as Reynolds number increased and consequently increased the heat transfer rate. The average Nusselt number and the length of the recirculation zone decreased with an increase in Richardson number. It was also noticed that the dimensionless local variation of skin friction coefficient along the heated wall increases with increasing the Richardson number.

E. Erturk [50] presented Numerical solutions of 2-D laminar flow over a backward-facing step at high Reynolds numbers. The governing 2-D steady incompressible Navier–Stokes equations were solved with a very efficient finite difference numerical method which proved to be highly stable even at very high Reynolds numbers. Present solutions of the laminar flow over a backward-facing step are compared with experimental and numerical results found in the literature.

Lan et al. (2009) reported Results from (3-D) simulations of turbulent forced convection adjacent to backward-facing step in a rectangular duct using a $(k-\epsilon-\xi-f)$ turbulence model. This turbulence model was numerically robust near the wall, and it has been shown to predict turbulent heat transfer in separated and wall-bounded flows better than commonly used two-equation turbulence models. FLUENT-CFD code was used as the platform for these simulations and User Defined Functions (UDF) were developed for incorporating this turbulence model into the code. The UDF implementation was validated by simulating several 2-D separated flow/heat transfer benchmark problems. The resulting excellent agreements between the simulated results and benchmark data for these 2-D problems justified the use of this resource for simulating 3-D convection problems. Three-dimensional backward-facing step geometry with an expansion ratio of 1.48 and with a step height of 4.8 mm was used in this study. Three aspect ratios of 3, 8 and infinity (2-D simulation) were considered for studying its effect on the flow and heat transfer, and similarly the effect of the Reynolds number was examined by varying its magnitude in the range of 20,000–50,000. Simulated results are presented

for the general 3-D flow features, the reattachment lines, temperature and Nusselt number distributions that develop in this geometry.

Afshin and Peter [51] experimentally studied separated and reattaching laminar flows produced by a sudden inward expansion within confined annular geometries. Using Particle Image Velocimetry (PIV), combined with refractive index matching, the structure of fluid flow at the expansion region was investigated. Detailed measurement of the velocities, the reattachment length (L_r) and the relative eddy intensity (ξ) were obtained for two different expansion ratios of $\varepsilon = 1.4$ and 1.6 . The measured variation of the reattachment length as a function of Reynolds number ($50 < Re < 600$) was found to be non-linear for these expansion ratios. Eddy intensity was found to depend strongly on both Reynolds number and expansion ratio, with the relationship between ξ and Re being non-linear.

Kumar and Dhiman [52] investigated the augmentation in the laminar forced convection characteristics of the backward-facing step flow in a two-dimensional channel by means of introducing an adiabatic circular cylinder in the domain. The effects of various cross-stream positions (i.e., $y_c = 0-1.5$) of the circular cylinder on the flow and heat transfer characteristics of the backward-facing step flow has been numerically explored for the Reynolds number range 1-200 and Prandtl number of 0.71 (air). The governing continuity, Navier-Stokes and energy equations along with appropriate boundary conditions were solved by using FLUENT. The flow and thermal fields have been explained by streamline and isotherm profiles, respectively; however, no temperature dependency effects were considered for the flow viscosity and thermal conductivity. The engineering parameters like wake/recirculation length, total drag coefficient and average Nusselt number, etc. were calculated for the above range of conditions. The present results showed an enhancement in the peak Nusselt value of up to 155% using a circular cylinder as compared to the unobstructed case (i.e., without cylinder). Finally, simple correlations for total drag coefficient and peak Nusselt number were obtained for the above range of conditions.

Tihon et al. [53] investigated experimentally and numerically the backward-facing step flow at moderate Reynolds numbers. Different channel expansion ratios ($ER = 1.43, 2, 2.5, \text{ and } 4$) and inlet flow conditions (steady and pulsatile) were applied with the aim to analyze the structure and stability of flow behind the step. Electrodiffusion technique was used to measure the wall shear rate along the experimental water channel. Direction sensitive sensors detect the near-wall extent of different flow-recirculation regions (primary recirculation and secondary corner, roof, and bottom eddies). The results of 2D numerical simulations performed in commercial CFD software FLUENT provide additional information on global flow rearrangement caused by the change of operation parameters. As the channel expansion ratio was increased, the steady recirculation pattern observed in the laminar flow regime becomes more complex. The obtained experimental and numerical data suggested possible scaling for the reattachment length and roof eddy size. In the transitional regime the near-wall flow exhibits an unsteady character with a high sensitivity to external low-frequency perturbations. The inlet pulsatile forcing was found to affect strongly the overall flow structure behind the step. A significant reduction of the reattachment length and an intensification of pulsatile back flow can be achieved by applying an appropriate forcing at frequencies close to that of the global flow instability.

Wang et al. [54] proposed the Lattice Boltzmann-cellular automata (LB-CA) probabilistic model to simulate gas–solid flows, in which the two-way coupling between the carrier phase and the dispersed phase was considered. In the LB-CA model, the LB sub grid model for high Reynolds number flows was used to describe flow fields at the mesoscopic scale, and the CA probabilistic model utilizing the stochastic process was used to capture transport behavior of discrete solid particles among the same regular lattice nodes as fictitious fluid particles in the LB method. The transport probability of a solid particle to nearest neighboring node directly depends on its actual displacement under other external forces (e.g., drag force, gravity). The two-way coupling was realized by adding external force term for the feedback forcing of particles in the evolution equation of fluid particle density distribution function. The resultant LB-CA model with two-way coupling was then used to simulate gas-particle flows over a backward-facing step. By comparing the present results with experimental measurements and other simulation results from LES (large-eddy simulation)-Lagrangian model, LB-Lagrangian model and two-fluid model, it was found that the LB-CA model is capable of simulating mean and fluctuating velocities of the carrier and dispersed phases and gas-particle covariance with high precision. Generally, the LB-CA method achieves the similar precision with the LES-Lagrangian method, and performs better than some other macroscopic models (such as the two-fluid models).

3.0 NUMERICAL AND EXPERIMENTAL STUDIES USING NANOFLUIDS

3.1 Laminar/Turbulent Convection Nanofluids

Abu-Nada [16] considered a pioneer in the numerical study of heat transfer to nanofluid over a backward-facing step. Nanoparticles types in this study were represented by Cu, Ag, Al₂O₃, CuO, and TiO₂, with volume fractions between 0.05 and 0.2 and Reynolds numbers ranging from 200 to 600. Momentum and energy equations were solved by using the finite volume method, and an increase in Nusselt number was observed at the top and bottom of the backward-facing step. Also, the investigations found high thermal conductivity of nanoparticles outside of recirculation zones.

Al-aswadi et al. [17] numerically investigated laminar forced convection flow of nanofluids over a 2D horizontal backward facing step placed in a duct using a finite volume method. A 5% volume fraction of nanoparticles was dispersed in a base fluid besides using various types of nanoparticles such as Au, Ag, Al₂O₃, Cu, CuO, diamond, SiO₂, and TiO₂. The duct has a step height of 4.8 mm, and an expansion ratio of 2. The Reynolds number was in the range of $50 \leq Re \leq 175$. A primary recirculation region has been developed after the sudden expansion and it starts to change to become fully developed flow downstream of the reattachment point. The reattachment point was found to move downstream far from the step as Reynolds number increases. Nanofluid of SiO₂ nanoparticles was observed to have the highest velocity among other nanofluids types, while nanofluid of Au nanoparticles has the lowest velocity. The static pressure and wall shear stress increase with Reynolds number and vice versa for skin friction coefficient.

Mohammed [21] reported predictions for laminar mixed convection using various types of nanofluids over a horizontal backward-facing step in a duct, in which the upstream

wall and the step are considered adiabatic surfaces, while the downstream wall from the step was heated to a uniform temperature that was higher than the inlet fluid temperature. The straight wall that forms the other side of the duct was maintained at constant temperature equivalent to the inlet fluid temperature. Eight different types of nanoparticles, Au, Ag, Al₂O₃, Cu, CuO, diamond, SiO₂, and TiO₂, with 5% volume fraction were used. The conservation equations along with the boundary conditions are solved using the finite volume method. Results presented in this study were for a step height of 4.9 mm and an expansion ratio of 1.942, while the total length in the downstream of the step was 0.5 m. The Reynolds number was in the range of $75 \leq Re \leq 225$. The downstream wall was fixed at a uniform wall temperature in the range of $0 \leq \Delta T \leq 30$ °C which was higher than the inlet flow temperature. Results revealed that there was a primary recirculation region for all nanofluids behind the step. It was noticed that nanofluids without secondary recirculation region have a higher Nusselt number and it increases with Prandtl number decrement. Furthermore, nanofluids with secondary recirculation regions were found to have a lower Nusselt number. Diamond nanofluid has the highest Nusselt number in the primary recirculation region, while SiO₂ nanofluid has the highest Nusselt number downstream of the primary recirculation region. The skin friction coefficient increases as the temperature difference increases and the Reynolds number decreases.

Santosh Christopher et al. [55] studied numerically Laminar forced convection heat transfer from two-dimensional sudden expansion flow of different nanofluids. The governing equations were solved using the unsteady stream function-vorticity method. The effect of volume fraction of the nanoparticles and type of nanoparticles on heat transfer was examined and found to have a significant impact. Local and average Nusselt numbers were reported in connection with various nanoparticle, volume fraction, and Reynolds number for expansion ratio 2. The Nusselt number reached peak values near the reattachment point and asymptotic value in the downstream. Bottom wall eddy and volume fraction showed a significant impact on the average Nusselt number.

Mohammed [19] numerically simulated Laminar mixed convective buoyancy assisting flow through a two-dimensional vertical duct with a backward-facing step using nanofluids as a medium using finite volume technique. Different types of nanoparticles such as Au, Ag, Al₂O₃, Cu, CuO, diamond, SiO₂ and TiO₂ with 5 % volume fraction were used. The wall downstream of the step was maintained at a uniform wall temperature, while the straight wall that forms the other side of the duct was maintained at constant temperature equivalent to the inlet fluid temperature. The walls upstream of the step and the backward-facing step were considered as adiabatic surfaces. The duct has a step height of 4.9 mm and an expansion ratio of 1.942, while the total length in the downstream of the step is 0.5 m. The downstream wall was fixed at uniform wall temperature $0 \leq \Delta T \leq 30$ °C, which was higher than the inlet flow temperature. The Reynolds number in the range of $75 \leq Re \leq 225$ was considered. It was found that a recirculation region was developed straight behind the backward-facing step which appeared between the edge of the step and few millimeters before the corner which connect the step and the downstream wall. In the few millimeters gap between the recirculation region and the downstream wall, a U-turn flow was developed opposite to the recirculation flow which mixed with the uncirculated flow and travelled along the channel. Two maximum and one minimum peaks in Nusselt number were developed

along the heated downstream wall. It was inferred that Au nanofluid has the highest maximum peaks while diamond nanofluid has the highest minimum peak. Nanofluids with a higher Prandtl number have a higher peak of Nusselt numbers after the separation and the recirculation flow disappeared.

Togun et al. [56] presented a numerical study of heat transfer to turbulent and laminar Cu/water flow over a backward-facing step. Mathematical model based on finite volume method with a FORTRAN code was utilized to solve the continuity, momentum, energy and turbulence equations. Turbulence was modelled by the shear stress transport (SST) $K-\omega$ Model. In this simulation, three volume fractions of nanofluid (0%, 2% and 4%), a varying Reynolds number from 50 to 200 for the laminar range and 5000 to 20,000 for the turbulent range, an expansion ratio of 2 and constant heat flux of 4000 W/m^2 were considered. The results showed the effect of nanofluid volume fraction on enhancing the Nusselt number in the laminar and turbulent ranges. The effect of expansion ratio was clearly observed at the downstream inlet region where the peak of the Nusselt number profile was referred to as enhanced heat transfer due to the generated recirculation flow. An increase of pressure drop was evident with an increasing Reynolds number and decreasing nanofluid volume fraction, while the maximum pressure drop was detected in the downstream inlet region. A rising Reynolds number caused an increasing Nusselt number, and the highest heat transfer augmentation in the present investigation was about 26% and 36% for turbulent and laminar range, respectively compared with pure water.

Mohammed [57] numerically carried out two-dimensional (2D) laminar mixed convection heat transfer and nanofluids flows simulations over forward facing step (FFS) in a vertical channel. The continuity, momentum, and energy equations were solved by using a finite volume method (FVM). The wall downstream of the step was maintained at a uniform wall heat flux, while the straight wall that forms the other side of the channel was maintained at constant temperature equivalent to the inlet fluid temperature. The upstream walls for the FFS were considered as adiabatic surfaces. The buoyancy assisting and buoyancy opposing flow conditions were investigated. Four different types of nanoparticles, Al_2O_3 , CuO, SiO_2 , and ZnO with different volumes' fractions in the range of 1–4% and different nanoparticle diameters in the range of 25–80 nm, were dispersed in the base fluid were used. In this study, several parameters, such as different Reynolds numbers in the range of $100 < \text{Re} < 900$, and different heat fluxes in the range of $500 \leq q_w \leq 4500 \text{ W/m}^2$, and different step heights in the range of $3 \leq S \leq 5.8 \text{ mm}$, were investigated to identify their effects on the heat transfer and fluid flow characteristics. The numerical results indicated that the nanofluid with SiO_2 has the highest Nusselt number compared with other nanofluids. The recirculation region and the Nusselt number increased as the step height, Reynolds number, and the volume fraction increase, and it decreased as the nanoparticle diameter increases. This study has revealed that the assisting flow has higher Nusselt number than opposing flow.

Safaei et al. [58] studied the turbulent forced convection heat transfer of water/functionalized multi-walled carbon nanotube (FMWCNT) nanofluids over a forward-facing step. Turbulence was modelled using the shear stress transport $K-\omega$ model. Simulations were performed for Reynolds numbers ranging from 10,000 to 40,000, heat fluxes from 1,000 to $10,000 \text{ W/m}^2$, and nanoparticle volume fractions of 0.00% to 0.25%. The two-dimensional governing equations were discretized with the

finite volume method. The effects of nanoparticle concentration, shear force, heat flux, contraction, and turbulence on the hydraulics and thermal behavior of nanofluid flow were studied. The model predictions were found to be in good agreement with previous experimental and numerical studies. The results indicate that the Reynolds number and FMWCNT volume fraction considerably affect the heat transfer coefficient; a rise in local heat transfer coefficient was noted when both Reynolds number and FMWCNT volume fraction were increased for all cases. Moreover, the contraction of the channel passage leads to the formation of two recirculation regions with augmented local heat transfer coefficient value.

Amiri et al. [59] performed an experimental study on thermophysical properties of ethylene glycol-functionalized graphene nanoplatelets/water–ethylene glycol nanofluids (EGGNP-WEG) and a numerical study on the convective heat transfer over a backward-facing step. Accordingly, EGGNP was first synthesized covalently to achieve a stable colloidal solution in water–ethylene glycol mixture. Some characterizations were applied to analyze the surface functionality and morphology of EGGNP-flakes. To study the convective heat transfer coefficient in turbulent regime, a numerical study was performed at different weight fractions of EGGNP. According to the results, a higher weight concentration of EGGNP in basefluid indicates a greater extent of convective heat transfer coefficient and thermal conductivity, implying higher heat transfer rate over a backward-facing step.

3.2 Baffles, Cylinders and Blockages (Turbulators)

Mohammed et al. [60] analyzed numerical simulations of two dimensional laminar combined convection flows using nanofluids over forward facing step with a blockage. The continuity, momentum and energy equations were solved using finite volume method (FVM) and the SIMPLE algorithm scheme was applied to examine the effect of the blockage on the heat transfer characteristics. In this study, several parameters such as different types of nanofluids (Al_2O_3 , SiO_2 , CuO and ZnO), different volume fraction in the range of 1% - 4%, different nanoparticles diameter in the range of 25nm-80nm were used. Effects of different shapes of blockage (Circular, Square and Triangular) were studied. The numerical results indicated that SiO_2 nanofluid has the highest Nusselt number. The Nusselt number increased as the volume fraction and Reynolds number increased, while it decreased as the nanoparticles diameter increased. Circular blockage produced higher results compared to triangular and square one.

Selimefendigil and Oztop [61] presented the application of the system identification method for forecasting the thermal performance of forced pulsating flow at a backward facing step with a stationary cylinder subjected to nanofluid. The governing equations were solved with a finite volume based code. The effects of various parameter frequencies (0.25 Hz–8 Hz), Reynolds number (50–200), nanoparticle volume fraction (0.00–0.06) on the fluid flow and heat transfer characteristics were numerically studied. Nonlinear system identification toolbox of Matlab was utilized to obtain nonlinear dynamic models of data sets corresponding to different nanoparticle volume fractions at frequencies of 1, 4 and 8 Hz. It was observed that heat transfer was enhanced with increasing the frequency of the oscillation, nanoparticle volume fraction and Reynolds number. The level of the nonlinearity (distortion from a pure sinusoid) decreases with increasing ϕ and with decreasing Reynolds number. It was also shown that nonlinear

dynamic models obtained from system identification toolbox could produce thermal output (length averaged Nusselt number) as close to as output from a high fidelity CFD simulation.

Heshmati et al. [62] numerically investigated mixed convection heat transfer over a 2-D backward facing step with an inclined slotted baffle by using nanofluids. Continuity, momentum and energy equations were solved by using Finite Volume Method with SIMPLE algorithm to link the pressure and velocity fields. Different Reynolds numbers from 50 to 400 were applied. In addition, the downstream wall of the step from $10 \leq X \leq 15$ was subjected to a uniform heat flux of $10,000 \text{ W/m}^2$ while the upper wall and the baffle are kept insulated. Five different geometries (without baffle, with a vertical solid baffle, with a solid inclined baffle, with two inclined slotted baffle) were compared to find the best for heat transfer enhancement. Different nanoparticles such as Al_2O_3 , CuO , ZnO and SiO_2 with different volume fractions from 0% to 4% and different nanoparticle diameter from 20 to 50 nm were considered with water as a base fluid to explore the best nanofluid for heat transfer enhancement. It was clearly shown that nanofluids with more nanofluid volume fraction and small nanoparticle diameter affect the heat transfer considerably. Results clearly illustrated that SiO_2 with 4% volume fraction and 20 nm nanoparticle diameter showed the best performance for heat transfer enhancement in compared with other nanoparticles. It was also found that the inclined baffle has the maximum average Nusselt number along the heated wall with high pressure drop and skin friction coefficient. However, by increasing Reynolds number, the inclined slotted baffle at $D = 0.5$ had an appropriate average Nusselt number and minimal changes of pressure drop and skin friction which can be considered as the appropriate geometry.

Mohammed et al. [63] numerically investigated the effects of two dimensional laminar and turbulent combined convection nanofluids flows over backward facing step in a channel having a blockage. The continuity, momentum and energy equations were solved using finite volume method (FVM) with the SIMPLE algorithm scheme. The duct has a step height of 0.01, and an expansion ratio of 2. The Reynolds number was in the range of 100–1900 (laminar flow) and in the range of 4000–10000 (turbulent flow). The effect of the blockage shape (circular, square and triangular) on the flow and heat transfer characteristics was examined. The effects of various types of nanoparticles such as Al_2O_3 , SiO_2 , CuO , and ZnO dispersed in a base fluid, volume fraction of nanoparticles in the range of 1% to 4% and nanoparticle diameter in the range of 25 nm to 80 nm were also studied. It was inferred that the circular blockage has the highest Nusselt number compared to other two shapes. The reattachment point was found to move downstream far from the step as Reynolds number increases. Nanofluid of SiO_2 was observed to have the highest Nusselt number and skin friction coefficient among other nanofluids types, while nanofluid of CuO nanoparticles has the lowest Nusselt number and skin friction coefficient.

Alawi et al. [64] numerically simulated laminar mixed convection flow using nanofluids over backward facing step in a heated rectangular duct having a baffle mounted on its wall. The continuity, momentum and energy equations were solved using finite volume method (FVM) and the SIMPLE algorithm scheme was applied to examine the effects of the baffle on heat transfer characteristics. In this study, several parameters such as different types of nanoparticles (Al_2O_3 , CuO , SiO_2 and ZnO), different volume fractions in the range of 1% to 4%, different nanoparticles diameter in

the range of 25 to 80 nm, and wall flux in the range of $10 \leq q_w \leq 70 \text{ W/m}^2$ were used. The effects of the baffle height H_b , baffle thickness W_b , and distance between the backward-facing step and baffle D on Nusselt number variation were numerically investigated. The numerical results indicated that the nanofluid with SiO_2 has the highest Nusselt number compared with other nanofluids types. The Nusselt number increases as the volume fraction of nanoparticles and the Reynolds number increase, while it decreased as the nanoparticles diameter increases. Effects of baffle distances baffle heights and baffle widths on heat transfer characteristics were significant, while, effects of wall flux are slightly insignificant.

Mohammed et al. [65] numerically simulated two-dimensional laminar and turbulent mixed convection flows using nanofluids over backward facing step in a heated rectangular duct having a baffle mounted on its wall. The continuity, momentum and energy equations were solved using Finite Volume Method (FVM) and the SIMPLE algorithm scheme was applied to examine the effects of the baffle on flow and heat transfer characteristics. The bottom wall of the duct was being heated with a constant heat flux, while other walls were being thermally insulated. In this study, several parameters such as different types of the nanoparticles (Al_2O_3 , CuO , SiO_2 and ZnO), different volume fractions in the range of 1% to 4%, and different nanoparticle diameters in the range of 25 to 80 nm were used. The Reynolds number of laminar flow was in the range of $100 \leq \text{Re} \leq 400$, while for turbulent flow it was in the range of $7500 \leq \text{Re} \leq 15,000$. The effects of baffle distances in the range of $\infty \leq D \leq 4$, baffle widths in the range of $0.01 \leq w_b \leq 0.04$, and baffle heights in the range of $0.005 \leq h_b \leq 0.015$ were studied. Baffle locations at the top wall and at bottom wall of the duct, and the number of baffles from 1 to 3 were also examined. The numerical results indicate that the nanofluid with SiO_2 has the highest Nusselt number compared with other nanofluid types. The Nusselt number increased as the volume fraction of the nanoparticles and the Reynolds number increase, while it decreased as the nanoparticle diameter increased. The effects of baffle distance heights, and baffle locations on fluid flow and heat transfer characteristics were significant, while the effects of baffle widths and baffle numbers were slightly insignificant.

Selimefendigil and Oztop [66] conducted a numerical investigation of mixed convection at a backward facing step with a rotating cylinder subjected to nanofluid was by using the proper orthogonal decomposition method. The governing equations were solved with a finite element based commercial solver. The effects of various pertinent parameters, Reynolds number, cylinder angular velocity and nanofluid volume fraction on the fluid flow and heat transfer characteristics were numerically studied. It was observed that flow field and thermal patterns change for different parameters and heat transfer enhancement was obtained for some combinations of parameters. Length averaged Nusselt number plots indicate that there was almost a linear increase in the heat transfer enhancement with increasing Reynolds number and nanoparticle volume fraction. Heat transfer enhancement was obtained for cylinder angular velocities of $\Omega = -4:5$ and $\Omega = 1:5$. A reduced order model of the system with proper orthogonal decomposition method is obtained and it provides accurate results when compared to high fidelity CFD model of the system.

Selimefendigil and Oztop [67] performed a numerical study of laminar forced convection of nanofluid flow over a backward facing step with a corrugated bottom

wall in the presence of different shaped obstacles placed behind the step. The bottom corrugated wall of the channel downstream of the step was isothermally heated and the other walls of the channel and obstacle surface are assumed to be adiabatic. The governing equations were solved with finite element method. The influence of the Reynolds number (between 10 and 200), solid volume fraction of the nanoparticle (between 0 and 0.05) and obstacle type (circular, square and diamond shaped) on fluid flow and heat transfer were numerically investigated. It was observed that among different obstacles, diamond shaped obstacle provides better local heat transfer enhancement characteristic in the vicinity of the step compared to circular or square obstacle at high Reynolds number. Heat transfer enhancement of 6.66 % was achieved in terms of maximum values with diamond shaped obstacle compared to no-obstacle case of corrugated channel. Adding an obstacle deteriorates heat transfer in terms of averaged values for the backward facing step geometry with a corrugated wall. When the solid volume fraction of nanoparticle was increased, maximum and averaged heat transfer rate increase. Heat transfer enhancements of 7.45 %, 7.42 %, 6.94 % and 6.64 % were obtained for the averaged values for circle, diamond, square and no-obstacle cases, respectively when solid volume fraction of 0.05 is compared to pure fluid.

3.3 Double and Triple BFS and FFS

Taher and Adam [68] investigated numerically the turbulent flow over a triple forward facing step configuration. To that end, numerical simulations were performed for 4 Re numbers and 36 different geometries. Turbulence was modelled in all simulations using the Standard $k-\epsilon$ model. The resulting flow field was examined in terms of pressure coefficient, vortices and turbulent kinetic energy. Furthermore, statistical analysis was performed on the results to reveal the quantitative effects of varying the Re number and geometric parameters on the resulting flow field.

Togun et al. [69] investigated numerically heat transfer and turbulent water flow over a double forward-facing step. The finite volume method was used to solve the corresponding continuity, momentum, and energy equations using the $K-\epsilon$ model. Three cases, corresponding to three different step heights, were investigated for Reynolds numbers ranging from 30,000 to 100,000 and temperatures ranging from 313 to 343K. The bottom of the wall was heated, whereas the top was insulated. The results show that the Nusselt number increased with the Reynolds number and step height. The maximum Nusselt number was observed for case 3, with a Reynolds number of 100,000 and temperature of 343 K, occurring at the second step. The behavior of the Nusselt number was similar for all cases at a given Reynolds number and temperature. A recirculation zone was observed before and after the first and second steps in the contour maps of the velocity field. In addition, the results indicate that the coefficient pressure increased with increasing Reynolds number and step height. ANSYS FLUENT 14 (CFD) software was employed to run the simulations.

Abdulrazzaq et al. [70] presented numerical study of heat transfer and fluid flow over vertical double forward facing step using ANSYS FLUENT 14. The $k-w$ model with finite volume method was employed to solve continuity, momentum, and energy equations. Different step heights were adopted for range of Reynolds number varied from 10000 to 40000, and range of temperature varied from 310K to 340 K. The straight side of duct was insulated while the side of double forward facing step was heated. The

result showed augmentation of heat transfer due to the recirculation region created after and before steps. Effect of step length and Reynolds number observed on increase of local Nusselt number particularly at recirculation regions. Contour of streamline velocity was plotted to show recirculation regions after and before steps.

Togun et al. [71] investigated numerically the turbulent heat transfer to nanofluid flow over double forward-facing steps. The duct geometry and computational mesh were developed with ANSYS 14 ICEM. Two-dimensional governing equations were discretized and integrated using finite volume technique. The $k-\varepsilon$ turbulence model was used in the analysis. Al_2O_3 and CuO nanoparticles at volume fractions varying from 1% to 4% with water as the base fluid were employed for turbulent flow in a passage with a double forward-facing step. The effects of volume fraction and step height were compared with the base fluid thermal performance. The obtained results showed an increase in the Nusselt number with the increase in volume fraction of nanofluid, Reynolds number, and step height. A higher local Nusselt number value was found at the second step compared to the first step for all cases. Velocity contours were developed to visualize the recirculation regions before and after the first and second steps. The results also demonstrated enhanced heat transfer with the increase of nanoparticle concentration, and the largest thermal enhancement factor occurred for the highest nanoparticle volume fraction (4%) of Al_2O_3 considered in this investigation.

Hamdi et al. [72] executed a numerical investigation of laminar mixed convection flow through a water–alumina nanofluid in a microscale vertical duct preceded with a double-step expansion. The governing equations were solved by using Lattice Boltzmann equation (LBE) with multiple-relaxation-time (MRT) collision model. The thermal conductivity and effective viscosity of nanofluid have been calculated by Brinkman and Maxwell models, respectively. To examine the effects of nanoparticles concentrations on the heat transfer and the flow behavior, the study has been carried out for the Reynolds number $\text{Re}=10$ to 40 , Richardson number $\text{Ri}=0$ to 1.0 and the solid volume fraction 0 to 20% . The results obtained from Lattice Boltzmann modelling clearly show that the inclusion of nanoparticles into the base fluid produces a significant enhancement of the convective heat transfer, especially in the channel entry region. This enhancement increased as function of growing Reynolds number. In addition, the increase in Richardson number led to decrease the solid concentration effect. Results also showed that adding solid particles decreases significantly the fanning friction factor in mixed convection case.

3.4 Microscale BFS and FFS

Kherbeet et al. [22] numerically investigated Laminar mixed convection flow over a 2D horizontal microscale backward-facing step (MBFS) placed in a duct. The governing equations along with the boundary conditions were solved using the finite volume method (FVM). The upstream wall and the step wall were considered adiabatic, while the downstream wall was heated by uniform heat flux. The straight wall of the duct was maintained at a constant temperature that was higher than the inlet fluid temperature. Different types of nanoparticles such as Al_2O_3 , CuO , SiO_2 and ZnO , with volume fractions in the range of 1–4% were used. The nanoparticles diameter was in the range of $25 \text{ nm} \leq d_p \leq 70 \text{ nm}$. The expansion ratio was 2 and the step height was $0.96 \mu\text{m}$. The Reynolds number was in the range of $0.05 \leq \text{Re} \leq 0.5$. The results revealed

that the Nusselt number increased with increasing the volume fraction and Reynolds number. The nanofluid of SiO₂ nanoparticles was observed to have the highest Nusselt number value. It was also found that the Nusselt number increased with the decrease of nanoparticle diameter. However, there was no recirculation region was observed at the step and along the duct.

Kherbeet et al. [24] presented experimental and numerical studies to reveal the flow and heat transfer characteristics of nanofluid laminar flow over the microscale backward-facing step (MBFS). The duct inlet and the step height were 400 μm and 600 μm respectively. All the walls considered adiabatic except the downstream wall was heated by uniform heat flux. The experiment was conducted at the Reynolds number range from 280 to 470. The distilled water was considered as a base fluid with two types of nanoparticles SiO₂ and Al₂O₃ immersed in the base fluid. The particle diameter was 30 nm and the range of nanoparticles volume fraction in the base fluid varied from 0 to 0.01. The measurement results revealed that the water–SiO₂ nanofluid has the highest Nusselt number. It was found also that the Nusselt number increased with increases volume fraction. The water–SiO₂ nanofluid with higher volume fraction has the highest Nusselt number. The friction factor of water–Al₂O₃ was higher than of water–SiO₂ mixture. The numerical results were in good agreement with the measurement results.

Kherbeet et al. [73] presented experimental and numerical investigations to illustrate the nanofluid flow and heat transfer characteristics over microscale forward-facing step (MFFS). The duct inlet and the step height were 400 μm and 600 μm respectively. All the walls were considered adiabatic except the downstream wall was exposed to a uniform heat flux boundary condition. The distilled water was utilized as a base fluid with two types of nanoparticles Al₂O₃ and SiO₂ suspended in the base fluid. The nanoparticle volume fraction range was from 0 to 0.01 with an average nanoparticle diameter of 30 nm. The experiments were conducted at a Reynolds number range from 280 to 480. The experimental and numerical results revealed that the water–SiO₂ nanofluid has the highest Nusselt number, and the Nusselt number increases with the increase of volume fraction. The average friction factor of water–Al₂O₃ was less than of water–SiO₂ mixture and pure water. The experimental results showed 30.6% enhancement in the average Nusselt number using water–SiO₂ nanofluid at 1% volume fraction. The numerical results were in a good agreement with the experimental results.

Kherbeet et al. [23] presented a numerical study on the mixed convective flow over 3D horizontal microscale backward facing step (MBFS). The nanofluid EG-SiO₂ with a nanoparticle of 25 nm in diameter and volume fraction of 0.04, was used in this study. Their results revealed that the Nusselt number and skin friction coefficient increased with an increase in the step height. The findings also showed that the Reynolds number and pressure drop decreased with an increase in the step height. Thereafter, Kherbeet et al. [73] implemented a numerical study on the laminar mixed convection flow of nanofluids over a 3D horizontal microscale forward-facing step (MFFS) using a finite volume method. Various types of nanoparticles materials namely SiO₂, Al₂O₃, CuO, and ZnO, were dispersed in a base fluid of ethylene glycol with volume fraction ranged from 0 to 0.04. The step height maintained at 650 μm. The results showed that the SiO₂ nanofluid recorded the highest Nusselt number. The Nusselt number increased with decreasing in the density of nanoparticle material, a decrease in diameter of nanoparticles and an increase in volume fraction.

Kherbeet et al. [74] presented the numerical implementation of laminar mixed convective flow over a 3-D horizontal microscale backward-facing step (MBFS) to explore the effect of base fluid on the flow and heat transfer characteristics. The energy equations and momentum were discretized by means of a finite volume method (FVM). The SIMPLE algorithm used for the pressure correction and velocity fields in the entire domain in the procedure. The straight wall of the duct was at a constant temperature ($T_w = 323$ K). While the downstream wall maintained at uniform heat flux ($Q_w = 12$ Watt). The SiO_2 nanoparticles with diameter of 25 nm and volume fraction of 0.04 were immersed in four kinds of base fluids (glycerin, engine oil, ethylene glycol and water). The Reynolds number was maintained at $Re = 35$, and step height was 650 μm . The results revealed that the highest Nusselt number was with using glycerin as a base fluid. It was also revealed that the skin friction coefficient was higher with using water as a base fluid.

4.0 CONCLUSION

In this article, a comprehensive review of previous efforts is presented for different convective flow regimes and heat transfer through a duct having backward and forward facing steps. The effects of several parameters in geometry, boundary conditions, and types of fluids were extensively introduced and investigated.

New configurations of backward and forward facing steps such as single, double, triple, baffles, blockages and microscale have been discussed in this study to give an enough knowledge and information in such trend.

Heat transfer and nanofluid flow over a backward-forward facing step has been presented in this article. The effect shape of geometry on thermal performance is clearly seen in experimentally and numerically studies in the literature. The result show that heat transfer rate increases with an increase in step height and Reynolds number. More augmentation of heat transfer was found using nanofluid, due to an increase of heat transport in the fluid. The enhanced thermal conductivity and viscosity of nanofluids, as well as the random movement of nanoparticles, effect the increased enhancement of heat transfer and stream functions. Also, using nanofluids could offer a positive effect on the energy crisis that is happening in the world.

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