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Numerical Simulation of High Reynolds Number Flow in Lid-Driven Cavity Using Multi-Relaxation Time Lattice Boltzmann Method

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Abstract – *The lattice Boltzmann method (LBM) is a potent numerical technique based on kinetic theory, which has been effectively employed in various complicated physical, chemical and fluid mechanics problems. In recent years, transient and turbulent flow simulation by using this new class of computational fluid dynamics method has attracted more attention. In this paper, a two dimensional lid-driven cavity flows at different Reynolds number (1,000-50,000) are simulated by using multi-relaxation (MRT) and LBGK (SRT) Lattice Boltzmann method. The results are compared with previous published papers, which solved the Navier-Stokes equation directly. Moreover, the effects of relaxation parameters variation in MRT model and spatial oscillation reduction in solution near geometrically singular points are highlighted. The comparisons between the simulated results showed that the multi-relaxation lattice Boltzmann method has the capacity to predict the flow characteristics, such as circulating flow and velocity profile with reasonable accuracy and reliability. The proper adjustment of the relaxation factors for non-conserved modes in MRT is the key point for achieving most verifiable results with Navier-Stokes solution.*

Keywords: Two-dimensional flows, Lid-driven cavity, Lattice Boltzmann method, Turbulent flow, MRT

1.0 INTRODUCTION

Turbulent flows are the most challenging topics which occur in many situations in nature and real engineering application. The turbulence problem is tough to be realized regarding to physical understanding and mathematical solutions or in terms of the engineering accuracy needed for different applications. Recently, the microscopic dynamics approaches have attracted significant attention. The idea of digital fluid dynamics lays on the fact that fluid hydrodynamics is not sensitive to the underlying details in microscopic physics. Hydrodynamics is the result of the collective behavior of numerous molecules in the system [1,2].

In recent years, considerable progress has been made to derive turbulence models from discrete kinetic theory [3]. The lattice Boltzmann method (LBM) adopts the kinetic theory of gases, which simulates the evolution of fluids based on the behavior at the molecular level [4,5]. Due to simplicity, efficiency and comfort of parallel programming implementation, LBM has achieved considerable success. One of the earliest simulations of turbulence in the frame of LBM was carried out by Benzi and Succi [6], which was a two-dimensional forced isotropic turbulence. To validate the capability of LBM, lid-driven cavity flow is a classical benchmark problem because of its simplicity and powerful vortices structure. Hou et al. [7] and Guo et al. [8] have reported detailed studies of cavity flow problem by using Bhatnagar-Gross-Krook (BGK) model. Liu et al. [9] investigated a 2D cavity natural convention turbulent flow simulation with an LBGK method coupled with LES. For turbulent flow simulation, LBM can be employed as a direct numerical simulation tool or can be combined with different subgrid scale method such as Smagorinsky [10] in large eddy simulation (LES) modeling [11]. The LES is a quite well-accepted alternative due to the balance between accuracy and computational efficiency [3].

The simplest and the most famous model of LBM is the single relaxation time BGK approximation. However, there are several concerns related to this model that should be considered such as the implementation of boundary condition [12,13], the capability to deal with complex geometry [14,15] and the simulation of high Reynolds number. Especially in turbulence flow, the single relaxation BGK model suffers from strong spatial oscillations near singularity point and instability [7]. The numerical stability improvement is achieved by repressing non-hydrodynamic modes that are not related to Navier-Stokes equation or by adjusting relaxation parameters according to linear stability analysis [16]. In multi-relaxation lattice Boltzmann method (MRT-LBM), hydrodynamic and non-hydrodynamic moments are relaxed with various relaxation times. It offers more stable and accurate results while it reduces unphysical oscillations [17,18]. Due to this fact, the researcher attempts to solve the flow in higher Reynolds number using generalized lattice Boltzmann method. Premnath et al. [19] carried out a study based on the generalized lattice Boltzmann equation via multiple relaxation times with forcing term. Moreover, Wu and Shao [17] and Patil et al. [20] recreated the results of numerical simulations of cavity flow by using MRT lattice Boltzmann method. Li et al. [21] have studied the structure of a lid-driven cavity flow of Reynolds number up to 7,500 in different cavity aspect ratios.

Recently, Zhen et al. [22] and Sheng Chen [23] studied the lid-driven cavity flow in high Reynolds number up to 1,000,000 and 50,000, respectively. The first study uses the multi-relaxation method to solve the problem directly, while the latter applies the subgrid scale LES model combined with LBM. The presented results of Zhen et al. [22] for streamline plot of Reynolds number of 20,000 and above showed some instability even in the main vortex and more fluctuation near the corners of the cavity. Nevertheless, the results by Sheng Chen [23] reached to steady state for the main vortex, however the smaller vortices near the cavity's corners are inconsistent with the study by Erturk et al. [24]. To close this gap, therefore, the objectives of the current study are firstly, to simulate cavity flows using the multi-relaxation time lattice Boltzmann model towards higher Reynolds numbers up to 50,000, and secondly to inspect the migration of vortex structure in response to the variations of the Reynolds numbers. The flow structure and velocity profiles are verified with the results of Navier-Stoke solutions.

2.0 MRT LATTICE BOLTZMANN METHOD

Lattice Boltzmann method is a developing alternative to Navier–Stokes (NS) based methods for flow computation [25-28]. The practical approach interpreted in the LBM consists of solving the lattice Boltzmann equation for the evolution of a single distribution function $f(x, t)$ of particles as they move and collide on a lattice. The solution of the equation includes two main steps; the stream step propagates information through the lattice cells, while the collision

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step normalizes the distribution functions to the equilibrium distribution function. The number of discrete velocity directions standing for the lattice is chosen withrespect to certain symmetry requirements to recover the isotropy of the viscous stress tensor of the fluid flow [29]. Streaming step is similar in different models of LBM but researchers have been searching for the appropriate collision model for LBM since the collision step is more complicated. The proposed methods for collision operator are different in relaxation factor, numerical stability and adequate Galilean invariance.A particular D2Q9 lattice Boltzmann model wasconsidered in this study. In this model, space is discretized into square lattice and the nine possible velocities are c_i , $i \in \{1,2,3,4,5,6,7,8,9\}$:

$$c_i = \begin{cases} (0,0) & i = 1 \\ \left(\frac{\cos(i-1)\pi}{2}, \sin\frac{(i-1)\pi}{2}\right) c & i = 2,3,4,5 \\ \left(\frac{\cos(2i-9)\pi}{4}, \sin\frac{(2i-9)\pi}{4}\right)\sqrt{2}\, c & i = 6,7,8,9 \end{cases} \tag{1}$$

where $c = \Delta x/\Delta t$, Δx and Δt are the lattice grid space and time step, respectively. The macroscopic hydrodynamic variable could be obtained using known distribution function:

$$\text{Streaming: } f_i(x + c_i\Delta t, t + \Delta t) = f_i(x, t + \Delta t) \tag{2}$$

$$\text{Collision: } f_i(x, t + \Delta t) = f_i(x,t) - \frac{1}{\tau}\left(f_i(x,t) - f_i^{eq}(x,t)\right) \tag{3}$$

$$\text{Equilibrium DF: } f_i^{eq} = w_i\rho\left[1 + 3\frac{c_i u}{{c_s}^2} + \frac{9}{2}\frac{(c_i u)^2}{{c_s}^4} - \frac{3}{2}\frac{uu}{{c_s}^2}\right] \tag{4}$$

$$\rho = \Sigma f_i \;;\quad \rho u = \Sigma f_i c_i \;; \tag{5}$$

where w is the weighting factor in the lattice fluid density and the sound speed of the lattice is $c_s = \frac{1}{\sqrt{3}}$. Equations (3) and (4) covered the collision part of SRT method.In the MRT-LBM, the particle distribution function at each lattice point is calculated by:

$$f_i(x + c_i\Delta t, t + \Delta t) = f_i(x,t) - M^{-1}.S.[m - m^{eq}], \tag{6}$$

where f_i is the distribution function corresponding to c_i, while m_i and m_i^{eq} are velocity moments and their equilibrium functions, respectively. M is a 9×9 orthogonal transformation matrix, which converts f as the distribution function in velocity space to m in the momentum space, in which the collision relaxation is operated. S is the diagonal relaxation matrix that indicates the relaxation rates for non-conserved moments. The ordering of the moments in this study could be mentioned as follow:

$$m = (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9) = (\rho, j_x, j_y, e, p_{xx}, p_{xy}, q_x, q_y, \epsilon) \tag{7}$$

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ρ is the mean density in the system and is usually set to be unity in simulations, while j_x and j_y are the x and y component of momentum or mass flux. e is the energy mode, where as p_{xx} and p_{xy} match to the stress tensors. q_x and q_y are the third-order moments represent the energy flux in x and y direction and ϵ is the fourth-order moment of energy square.

With the ordering of the moments specified as the above, the transform matrix can be structured as follow:

$$M \equiv \begin{pmatrix} \langle\rho| \\ \langle j_x| \\ \langle j_y| \\ \langle e| \\ \langle p_{xx}| \\ \langle p_{xy}| \\ \langle q_x| \\ \langle q_y| \\ \langle\epsilon| \end{pmatrix} \equiv \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{8}$$

The equilibrium values of the velocity moments m^{eq} can be derived from the following equations:

$$\begin{gathered} \rho^{eq} = \rho\,, \qquad e^{eq} = -2\rho + 3\left(u_x^2 + u_y^2\right), \qquad \epsilon^{eq} = \rho - 3\left(u_x^2 + u_y^2\right), \\ j_x = \rho u_x\,, \\ q_x^{eq} = -u_x, \qquad j_y = \rho u_y\,, \qquad q_y^{eq} = -u_y\,, \qquad p_{xx}^{eq} = u_x^2 - u_y^2\,, \\ p_{xy}^{eq} = u_x u_y \end{gathered} \tag{9}$$

$$S = diag(s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9) \tag{10}$$

Due to the conservation of mass and momentum before and after particle collision, the total mass and momentum could be excluded from relaxation process. For non-conserved modes,$s_i \in (0,2)$ and it should be adjusted carefully. The consideration of numerical stability affects, the choices of the value of s_5 as follow:

$$s_5 = \frac{2}{6\nu + 1} \tag{11}$$

In this paper, different diagonal relaxation matrices were adjusted and the results were monitored. According to the proposed options for relaxation factor in [16] we set $s_7 = s_8$ by the following relation:

$$s_7 = s_8 = \frac{8(2 - s_5)}{8 - s_5} \tag{12}$$

By this definition, for higher Reynolds number, we always have very small relaxation factor for energy flux modes. Furthermore,$s_6 = s_5$ is assumed and s_9 is chosen near to unity while it has no effects on transport coefficient.

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It is well-known that the LBM approximates the near incompressible Navier-Stokes equations with compressibility error that grows as M^2[30],where $M = u/c_s$ is the Mach number. If $M < 0.2–0.3$ and the flow is steady and isothermal, compressibility effects will be small and a simplified incompressible flow model can be used. However, for the sake of Galilean invariant of the model, the Mach number was chosen near incompressible flow of $M<0.1$-0.2 in all cases.

3.0 RESULTS AND DISCUSSION

Our numerical simulations were carried out on a progressively increasing mesh numbers from 256^2 to 501^2 lattices for Reynolds number from 1,000 to 50,000. The cavity Reynolds number has been defined due to the depth of the cavity. By variation of driven wall velocity and mesh number related to each case, the relaxation time is adjusted.

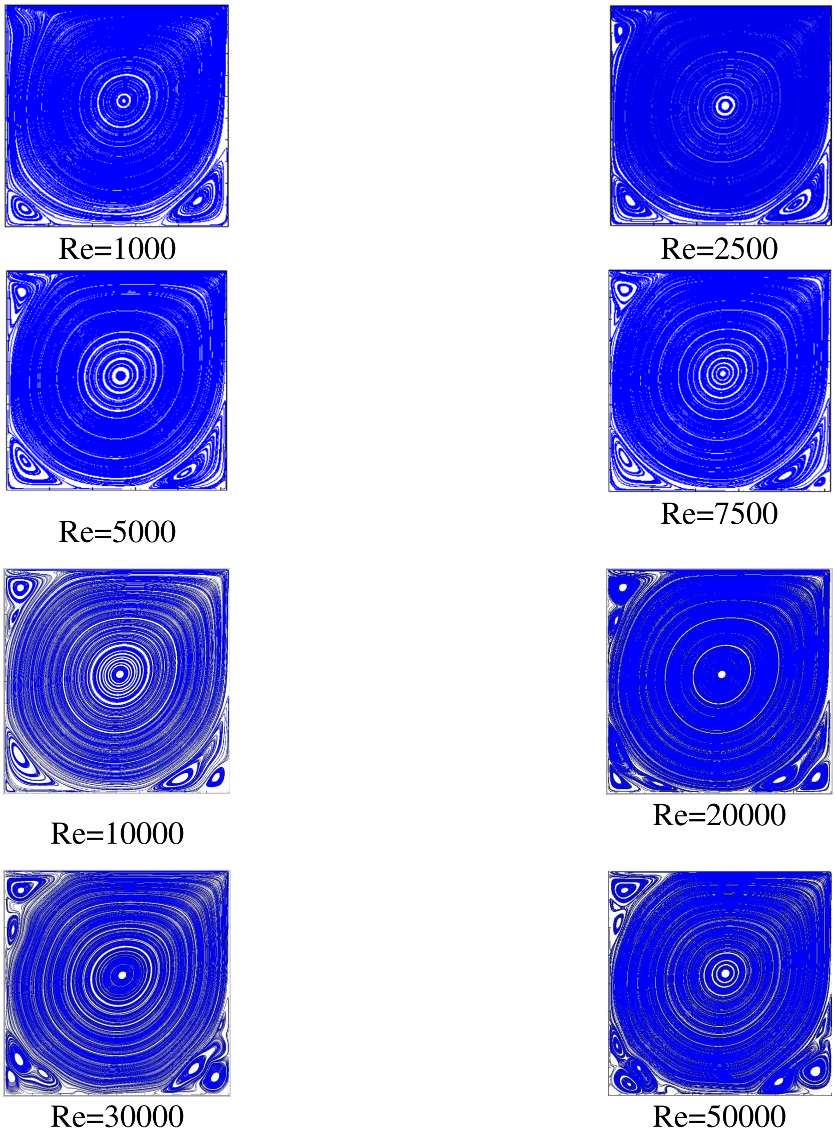

Figure1: Streamline contours of primary and secondary vortices for different Reynolds number

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To reach the numerically stable results, we set the iterative time step to be long enough, which is fixed at 700,000 iterations. The type of the numerical boundary conditions used in the computation is an important factor for the accuracy of the numerical solution. Therefore, a second order interpolation bounce-back boundary condition was utilized in this study [31]. The stream function contours are illustrated in Figure 1. This figure shows the formation of the counter-rotating secondary vortices which appeared as the Reynolds number increased. According to this figure, the centre of the primary vortex moves toward the center of the cavity as Reynolds number was increased. We can also see that the vortices which appeared in the corner grew with the increased of the Reynolds number. At the bottom right corner of the cavity in the Reynolds number beyond 5,000, other secondary vortices were observed while at the top left corner, the secondary vortex could be seen at Reynolds number beyond 2,500.

The streamlines illustrate that the flow region near the wall changed to turbulence when the Reynolds was increased. The plot of streamline for Reynolds numbed >10,000 showed a relatively stable primary vortex. However, the secondary vortices still have some oscillation and the solutions exhibited a periodic behavior beyond this Reynolds number. For solving this problem, the mesh density has been increased. As a result, periodic character of the flow showed more stability in the construction of the right corner vortices, while the instability in the left corner of the cavity could still be seen. It could be indicated that fine grid mesh was needed in order to achieve a steady solution as the Reynolds number increased. When Re was further increased, many small vortices appeared at the corner of the cavity. For Reynolds number greater than 20,000, the vortices for both corners of the cavity have shown periodic behavior. From the streamline plots, the results of the current research and Erturk et al. [24] studies are in very good agreement.

Table1 shows the predicted location of the primary and secondary vortices and compares the results with the solution of Navier-Stokes equation by Erturk et al. [24] and the results of Sheng Chen [23]. Well-agreed results were achieved at different Reynolds number. LBGK model may experience numerical instability to simulate large Reynolds number flow when the relaxation time approached to 0.5. One solution is to increase the grid density to make up the excessive reduction of the relaxation time. For the Reynolds number of 20,000 with the current mesh definition, the SRT lattice Boltzmann method is unstable and for solving the problem, a finer mesh should be chosen. Meanwhile, MRT could reach the stable results with the selected mesh even for higher Reynolds number.

Table1: Comparison of the location of the vortices at different Reynolds number

Re	Primary vortex		Left secondary vortices		Right secondary vortices	
	x	y	x	y	x	y
1,000						
LBM-SRT	0.5312	0.5625	0.0820	0.0779	0.8554	0.1093
LBM-MRT	0.5312	0.5625	0.0820	0.0779	0.8554	0.1093
Erturk et al. [24]	0.5300	0.5650	0.0833	0.0783	0.8633	0.1117
Sheng Chen [23]	0.5310	0.5650	0.0901	0.0800	0.8501	0.1100
2,500						
LBM-SRT	0.5195	0.5351	0.0859	0.1132	0.8320	0.0898
LBM-MRT	0.5195	0.5390	0.0859	0.1132	0.8320	0.0898

Erturk et al.[24]	0.5200	0.5433	0.0850	0.1100	0.8350	0.0917
Sheng Chen[23]	-	-	-	-	-	-
5,000						
LBM-SRT	0.5195	0.5351	0.0820	0.1250	0.8125	0.0731
LBM-MRT	0.5156	0.5351	0.0820	0.1250	0.8085	0.0731
Erturk et al.[24]	0.5150	0.5350	0.0733	0.1367	0.8050	0.0733
Sheng Chen[23]	0.5040	0.5001	0.0950	0.1100	0.8285	0.0745
7,500						
LBM-SRT	0.5165	0.5321	0.0662	0.1520	0.7836	0.0682
LBM-MRT	0.5126	0.5302	0.0662	0.1520	0.7875	0.0662
Erturk et al.[24]	0.5133	0.5317	0.0650	0.1517	0.7900	0.0650
Sheng Chen[23]	-	-	-	-	-	-
10,000						
LBM-SRT	0.5069	0.5349	0.0558	0.1616	0.7744	0.0618
LBM-MRT	0.5109	0.5329	0.0618 0.0179	0.1716 0.0259	0.7804 0.9381	0.0638 0.0698
Erturk et al.[24]	0.5117	0.5300	0.0583 0.0167	0.1633 0.0200	0.7767 0.9350	0.0600 0.0667
Sheng Chen[23]	0.5117	05313	0.0585	0.1655	0.7813	0.0625
20,000						
LBM-SRT	-	-	-	-	-	-
LBM-MRT	0.5089	0.5988	- 0.0463	- 0.0642	0.7700 0.9340	0.0625 0.0790
Erturk et al.[24]	0.5100	0.5267	0.0483 0.0567	0.1817 0.0533	0.7267 0.9300	0.0450 0.1033
Sheng Chen[23]	0.5078	0.5313	-	-	-	-
30,000						
LBM-MRT	0.5291	0.5333				
50,000						
LBM-MRT	0.5270	0.5446				

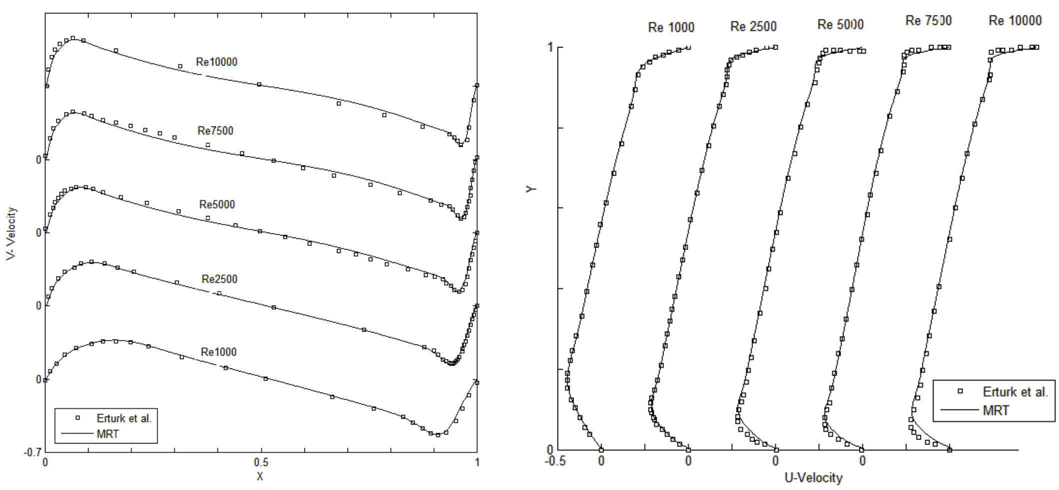

Figure2: U and V velocity profiles along a vertical line passing through the centre of the cavity

Figure 2 shows the computed u_x and u_y velocity profiles of a vertical line passing the geometrical center of the cavity at different Reynolds numbers. The results were closely agreed with the results of Navier-Stokes solution.

4.0 CONCLUSION

In this study, a multi-relaxation lattice Boltzmann model wasapplied to compute two-dimensional lid-driven cavity flows in a wide range of Reynolds number between 1,000 and 50,000. A range of parameters, such as grid density, maximum velocity and relaxation factors were explored to reach the stable and precise results. At lower Reynolds numbers, the numerical solutions convergence criteria have been adjusted to maximum absolute residuals of 10^{-8} while for high Reynolds numbers, iterative time step was fixed to 700,000. Near steady solutions were obtained and the predicted results of velocity profile and flow structure were compared with the results of Navier-Stokes equation. The results demonstrate that the results of cavity flow structure could reached to a near steady solution up to Reynolds number of 20,000 and forgreater Reynolds number, it will change to complete turbulence flow. The mentioned MRT model is acceptable for turbulent flow computation in comparisonwith previous research, which solves the Navier-Stokes equation directly or employs other arrangements of MRT lattice Boltzmann method.The current solutionis more flexible for mesh size than the LBM-SRT and computationally more efficient for turbulent flow simulation.

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