

## Piston Surface Pressure of Piston- Cylinder System with Finite Piston Speed

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### ABSTRACT

Understanding the thermodynamics' fundamental are important for the advance of energy and environmental technologies. The study aims to derive an expression of the average pressure on the piston surface,  $p_{ps}$  in an adiabatic piston-cylinder system during an irreversible process with finite piston speed using the kinetic molecular theory. Expressions for the average pressure on the piston surface,  $p_{ps}$  are derived from the change of momentum and change of energy. The Maxwell-Boltzmann distribution is used to estimate the gas molecule velocities. The piston surface pressure obtained from the change of momentum is 37.18% lower than that obtained by the previous study. The piston surface pressure obtained from the change of energy is 26.94% lower than that obtained by the previous study.

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## 1. Introduction

Design of heat engines, energy devices in a power plant and thermo-fluid devices have increased the need for fundamental understanding of thermodynamics for the advance of energy and environmental technologies [1-2]. A process of a compression or an expansion of a gas in a piston-cylinder system is common in many applications, however an irreversible process is not well understood [3-9]. In thermodynamics, state quantities at a final state in a reversible process can be determined. A reversible process may occur when a system is maintained continuously and thermally at an equilibrium state [9-12]. Therefore, the reversible process is also called a quasi-static or a quasi-equilibrium process [10]. The process is reversible when a piston moves with zero velocity in a piston-cylinder system. On the other hand, the thermal equilibrium state breaks in the system and the process becomes irreversible when the piston moves with infinite velocity. In general, we cannot determine the state quantities at a final state in an irreversible process. The only exception is a throttling process when a gas or a steam passes through a capillary tube or a

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porous material. For example, in an adiabatic throttling process, the specific enthalpy at the final state is identical to the specific enthalpy at the initial state. Then, the state quantities at the final state can be determined. This is strictly highlighted in a thermodynamic textbook [11] that we can make calculations only for a reversible process.

Expressions for the average pressure on the piston surface in an adiabatic piston-cylinder system during an irreversible process with finite piston velocity using the kinetic molecular theory has been derived by Petrescu *et al.*, [14]. However, the deriving process and the molecular velocity distribution they used are not clearly described. The study aims to derive an expression for the average pressure on the piston surface,  $p_{ps}$  in an adiabatic piston-cylinder system during an irreversible compression process with finite piston velocity using the kinetic molecular theory with Maxwell-Boltzmann distribution.

## 2. An Ideal Gas Laws

Various studies have investigated of a reversible process in closed system because it can be used as a medium to analyze the irreversible process occur in the system and as an important source in understanding the fundamental of thermodynamics [3-8,15]. A reversible process is the process in which the system and surroundings can be restored to the initial state from the final state without producing any changes in their equilibrium states. The piston moves with very slow movement will result in a uniform gas pressure. This phenomenon can be expressed in first laws of thermodynamics for a closed system [1-3, 10-11, 16].

$$dq = di + dw \quad (1)$$

where  $dw$  is the specific work done by the fluid to the piston. The specific work done by the fluid to the piston is expressed as

$$dw = p_{ps} dv \quad (2)$$

where  $p_{ps}$  is the average pressure on the piston surface. Note that Eqs. (1) and (2) are valid for both reversible and irreversible processes. Substituting Eq. (2) into Eq. (1) for the case of an adiabatic process ( $dq=0$ ), the change of internal energy is given as

$$di = -p_{ps} dv \quad (3)$$

For an ideal gas

$$di = C_v dT \quad (4)$$

If the process is a reversible process

$$dw = p_{ps} dv = RT \frac{dv}{v} \quad (5)$$

Substituting Eqs. (4) and (5) into Eq. (3), the following equation can be obtained

$$C_v dT = -RT \frac{dv}{v} \quad (6)$$

Note that for ideal gas,  $R = C_p - C_v$ ,  $\gamma$  is the ratio between the specific heats,  $C_p/C_v$ . For a diatomic molecule,  $\gamma = 1.4$ . Then, by integrating Eq. (6) between  $(T_i, v_i)$  and  $(T, v)$ , the following equation can be obtained

$$T = T_i \left( \frac{v_i}{v} \right)^{\gamma-1} \quad (7)$$

Eq. (7) is the general equation that can be used to obtain the initial and final temperatures and volumes for an adiabatic reversible process in a piston-cylinder system. The pressure in a system is expressed as

$$p = p_i \left( \frac{v_i}{v} \right)^{\gamma} \quad (8)$$

Note that in case of a reversible process, the pressure in the system is uniform and is identical to the average pressure on the piston surface  $p = p_{ps}$ . In a case of an irreversible process, the pressure in the system is not uniform. The following correlation for the work done by the fluid is widely known when the average pressure in the system is denoted by  $p_{ave}$

$$dw < p_{ave} dv \quad (9)$$

However, it is not known how the work done by the fluid is less than  $p_{ave} dv$ . Note that Eq. (2) is available for both cases of reversible and irreversible processes. Substituting Eq. (9) into Eq. (2), we obtain

$$p_{ps} dv \leq p_{ave} dv \quad (10)$$

Therefore, the following correlation for a compression process is obtained

$$p_{ps} \geq p_{ave} \quad (\text{compression process}) \quad (11)$$

The average pressure on the piston surface,  $p_{ps}$  using the kinetic molecular theory will be derived in the next section.

### 3. Kinetic Molecular Theory

The kinetic molecular theory for an ideal gas simply expressed that all particles are moving in a random, constant and straight-line motion. The pressure of a gas then results from the collisions of the molecules of the gas with the walls of the container.

### 3.1 Maxwell (-Boltzmann) distribution

Figure 1 shows a stationary container in which single-atomic gas such as Ar, Ne or He is filled. Since the container is stationary, the gas molecule travels with the thermal velocity denoted by  $\vec{C}$ . The thermal velocity is not a unique value and obeys the Maxwell distribution [17-18]. The Maxwell distribution is expressed as

$$f(C) = 4\pi C^2 \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} \exp\left( \frac{-mC^2}{2kT} \right) \quad (12)$$

where  $m$ ,  $k$  and  $T$  are mass of a molecule in kg, the Boltzmann constant and temperature in K, respectively.

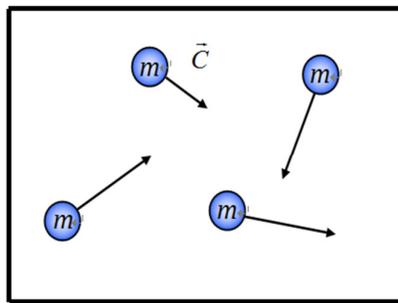


Fig. 1. Molecules in a container

The Maxwell distributions of Ar, Ne and He gases at room temperature (300 K) are plotted in Figure 2 as a form of the probability distribution. The effect of temperature on the molecular velocity of Ar gas is shown in Figure 3. Note that the y-axis in Figures 2 and 3 is in s/m so that the area under any section of the curve is dimensionless.

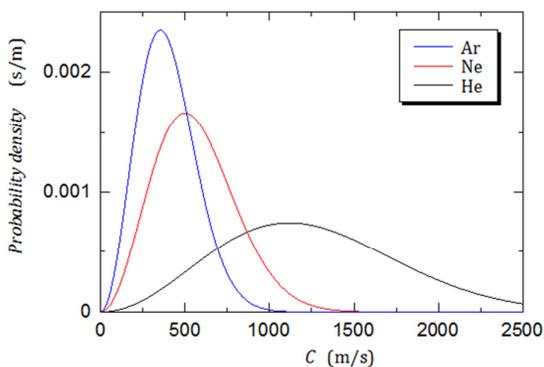


Fig. 2. Maxwell distribution of Ar, Ne and He gases at 300 K

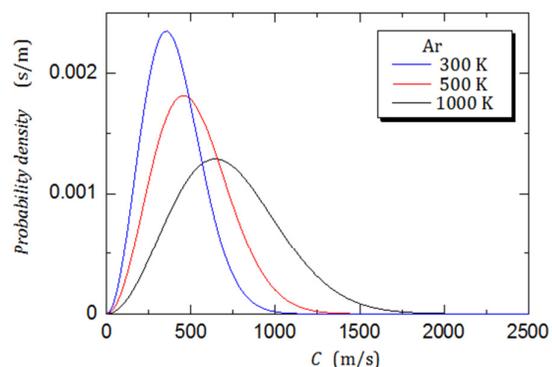


Fig. 3. Effect of temperature on molecular velocity of Ar gas

### 3.2 Boltzmann constant

Table 1 listed the parameters involved in obtaining Boltzmann constant.

**Table 1**  
 The parameters involved in obtaining Boltzmann constant

The number density, $N$ ( $N = N_A / V$ )	Volume, $V$	Pressure, $p$	Temperature, $T$	The Boltzmann constant, ( $k = p / NT$ )
$2.6868 \times 10^{25}$	$22.414 m^3$	$1.01325 \times 10^5 \text{ Pa}$	273.15 K	$1.3806 \times 10^{-23}$

where  $N_A$  is the Avogadro constant ( $N_A = 6.02214 \times 10^{26} \text{ 1/kmol}$ ) and  $V$  represents the volume of a gas of 1 kmol at arbitrary pressure and temperature.

### 3.3 Typical velocities

There are three typical velocities, they are the mean velocity, most probable velocity and root-mean-square velocity can be obtained from properties of the Maxwell distribution. The most probable velocity,  $C_{mp}$ , is the velocity most likely to be possessed by any molecule (of the same mass  $m$ ) in the system and corresponds to the maximum value. To find it, we calculate  $df/dC$ , set it to zero and solve for  $C$ .

$$\frac{df(C)}{dC} = 0 \quad (13)$$

which yields

$$C_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2R_0T}{M}} = \sqrt{2RT} \quad (14)$$

where  $R_0$  is the universal gas constant (8314.33 J/(kmol K)),  $M$  is the molar weight of a substance and  $R$  is the gas constant, J/(kg K). For diatomic nitrogen,  $N_2$ , at room temperature (300 K), this gives  $C_{mp} = 422 \text{ m/s}$ .

The mean velocity is the expected value of the velocity distribution

$$\bar{C} = \int_0^{\infty} C f(C) dC = \sqrt{\frac{8kT}{\pi m}} = \frac{2}{\sqrt{\pi}} C_{mp} \quad (15)$$

The root mean square velocity is

$$C_{rms} = \sqrt{\overline{C^2}} = \sqrt{\left( \int_0^{\infty} C^2 f(C) dC \right)} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3R_0T}{M}} = \sqrt{\frac{3}{2}} C_{mp} \quad (16)$$

These typical velocities are related as follows

$$C_{mp} < \bar{C} (=1.129 C_{mp}) < C_{rms} (=1.225 C_{mp}) \quad (17)$$

### 3.4 Number of Molecules which Collide with a Wall

Although the hard sphere model predicts more accurate one but using the molecular-mean-free-path model the pressure will be derived. The molecule is treated as a lumped mass. Besides, the following assumptions will be made.

- *The molecules are in random motion.*
- *The collisions with the walls and other molecules are perfectly elastic. This means, the kinetic energy is conserved during the collision.*
- *Except during collisions, the interaction among molecules is negligible. Note that intermolecular forces (e.g. Van der Waals force) are considered in the molecular dynamics (MD). In the kinetic theory, the interaction among molecules is not considered.*

Based on these assumptions, we can agree that mechanical energy is conserved in collisions between molecules. For the sake of simplicity, we should consider a rectangular container (unit area  $\times$  length  $C_{rms}$ , see Figure 4) and assume that *all* gas molecules travel with the root-mean-square velocity  $C_{rms}$ . Since the volume of this container is  $C_{rms}$ , the number of the molecules in the container is  $NC_{rms}$  where  $N$  is the number density which represents number of the molecules in unit volume. Since each  $\frac{1}{6}NC_{rms}$  molecules travel in  $\pm x$ ,  $\pm y$  and  $\pm z$  directions, the number of molecules which collide with one wall per unit area and per unit time is

$$\frac{1}{6}NC_{rms} \quad (18)$$

### 3.5 Pressure

When a gas molecule collides with the wall, the molecule bounces off in the opposite direction with the same velocity (an elastic collision). The change in momentum during the collision is

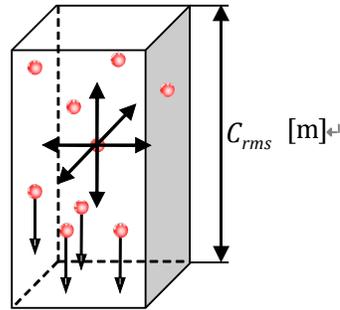
$$mC_{rms} - (-mC_{rms}) = 2mC_{rms} \quad (19)$$

The change in momentum per unit area and per unit time is

$$\left(\frac{1}{6}NC_{rms}\right)(2mC_{rms}) = \frac{1}{3}NmC_{rms}^2 = \frac{1}{3}Nm\overline{C^2} \quad (20)$$

The change in momentum per unit area and per unit time represents the force acting on the wall per unit area. Thus, the pressure

$$p = \frac{1}{3}NmC_{rms}^2 = \frac{1}{3}Nm\overline{C^2} = \frac{1}{3}\rho C_{rms}^2 = \frac{1}{3}\rho\overline{C^2} \quad (21)$$



**Fig. 4.** Collision of molecules in a rectangular container

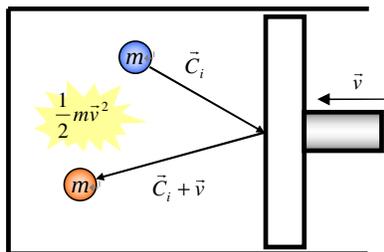
### 3.6 Collision with moving wall (uniform temperature case)

Consider a piston cylinder system in which single-atomic gas is filled and the piston is moving with constant velocity,  $\vec{v}$ , as shown in Figure 5. Here, we assume that the temperature in the piston-cylinder system including the piston surface is uniform. Each molecule in the piston-cylinder system travels with the thermal velocity,  $\vec{C}_i$  based on the *uniform temperature*. Attention will now be turned to the kinetic energy that the molecule brings to the piston when the molecule collides with the piston. The sum of the kinetic energy that the molecules bring to the piston per unit area and unit time is expressed as

$$E_{\text{arrive}} = \sum_{i=1}^{\frac{NC_{rms}}{6}} \frac{1}{2} m \vec{C}_i^2 \quad (22)$$

We further assume that the molecule which arrives on the piston surface is thermally fully accommodated with the surface. If the reflection on the piston surface is fully diffuse, molecules reflect with the thermal velocity plus the piston velocity as in Figure 5. Therefore, the total kinetic energy leaves from the wall is expressed as

$$E_{\text{leave}} = \sum_{i=1}^{\frac{NC_{rms}}{6}} \frac{1}{2} m (\vec{C}_i^2 + \vec{v}^2) \quad (23)$$



**Fig. 5.** Energy transport by molecules colliding with the wall

Consider the mean-square of the thermal velocity denoted by  $\overline{C^2}$ .  $\overline{C^2}$  can be obtained from Eq. (16) as

$$\overline{C^2} = \frac{6}{NC_{rms}} \sum_{i=1}^6 \overline{C_i^2} = C_{rms}^2 \quad (24)$$

Substituting Eq. (24) into Eqs. (22) and (23), then

$$E_{arrive} = \frac{NC_{rms}}{6} \frac{m}{2} C_{rms}^2 \quad (25)$$

$$E_{leave} = \frac{NC_{rms}}{6} \frac{m}{2} (C_{rms}^2 + v^2) \quad (26)$$

The difference of Eq. (25) and (26) is increase of the energy per unit time and it is equal to the work done by the piston as

$$\frac{NC_{rms}}{6} \frac{m}{2} (C_{rms}^2 + v^2) - \frac{NC_{rms}}{6} \frac{m}{2} C_{rms}^2 = \frac{NC_{rms}}{6} \frac{m}{2} v^2 = (p_{ps} - p_{ave})v \quad (27)$$

Eq. (27) can be rewritten as

$$\frac{p_{ps}}{p_{ave}} = \frac{NC_{rms}}{12} \frac{mv}{p_{ave}} + 1 \quad (28)$$

Substituting Eq. (21) into Eq. (28), we obtain

$$\frac{p_{ps}}{p_{ave}} = \frac{v}{4C_{rms}} + 1 \quad (29)$$

Substituting Eq. (16) into Eq. (29), Eq. (29) can be rewritten as

$$\frac{p_{ps}}{p_{ave}} = \frac{v}{4\sqrt{3RT_{ave}}} + 1 = \frac{1}{4} \sqrt{\frac{\gamma}{3}} \frac{v}{\sqrt{\gamma RT_{ave}}} + 1 = \frac{1}{4} \sqrt{\frac{\gamma}{3}} Ma + 1 \quad (30)$$

However, Petrescu *et al.*, [14] obtained

$$\frac{p_{ps}}{p_{ave}} = 1 \pm \sqrt{3\gamma} \frac{v}{\sqrt{3RT_{ave}}} = 1 \pm \gamma Ma \quad (31)$$

When a gas molecule collides with the piston, the molecule bounces off in the opposite direction with the same velocity (an elastic collision). The change in momentum during the collision is

$$mC_{rms} - (-mC_{rms} - v) = 2mC_{rms} + v \quad (32)$$

The change in momentum per unit area and per unit time is

$$\left(\frac{1}{6}nC_{rms}\right)(2mC_{rms} + mv) = \frac{1}{3}NmC_{rms}^2 + \frac{1}{6}NmC_{rms}v \quad (33)$$

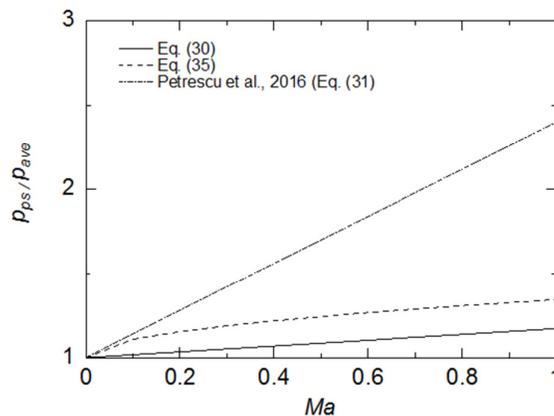
The change in momentum per unit area and per unit time represents the force acting on the piston surface per unit area. Thus, the average pressure on the piston surface

$$p_{ps} = \frac{1}{3}NmC_{rms}^2 + \frac{1}{6}NmC_{rms}v \quad (34)$$

Then,

$$\frac{p_{ps}}{p_{ave}} = 1 + \frac{v}{2C_{rms}} = 1 + \frac{v}{2\sqrt{3RT_{ave}}} = 1 + \frac{1}{2}\sqrt{\frac{\gamma}{3}}Ma \quad (35)$$

To compare the ratio of average pressure on the piston surface to the average pressure,  $p_{ps} / p_{ave}$  obtained using energy equation from Eq. (30) and the change of momentum from Eq. (35) with the results obtained by Petrescu *et al.*, [14] from Eq. (31), all the results are plotted as a function of Mach number,  $Ma$  in Fig. 6. Both the ratios of average pressure on the piston surface to the average pressure,  $p_{ps} / p_{ave}$  obtained from Eqs. (30) and (35) are lower than the results obtained by Petrescu *et al.*, [14]. In the case of Mach number, 0.5, the differences ratio of average pressure on the piston surface to the average pressure,  $p_{ps} / p_{ave}$  between Petrescu *et al.*, [14] with the Eqs. (35) and (30) are 37.18% and 26.94%, respectively.



**Fig. 6.** Comparison of  $p_{ps} / p_{ave}$  with previous results

The differences are because the previous studies [14] did not considered the energy conversion from the kinetic energy to thermal energy after the piston stops. And they also derive the Eq. (31) based on the following assumptions

- Clausius' kinetic-molecular model to determine the analytical expressions of temperature, internal energy, and pressure for a system of perfect gases in equilibrium, depending on the total number and the speed of the inside molecules.
- All molecules can have on a continuous scale, any speed value within this range (from a minimum speed threshold  $C_1$  and a maximum  $C_N$  ).
- After collision a certain number of molecules changes its trajectory and leaves the cluster, they are immediately replaced by an equal number of molecules migrated from other clusters.

#### 4. Conclusions

In this paper, a single compression process in an adiabatic piston-cylinder system has been analysed using kinetic molecular theory. The following conclusions are obtained.

1. The ratio of average pressure on the piston surface to the average pressure,  $p_{ps} / p_{ave}$  obtained from the change in momentum, Eq. (35) and energy equation, Eq. (30) are lower than the results obtained by Petrescu et al., [14].
2. The differences ratio of average pressure on the piston surface to the average pressure,  $p_{ps} / p_{ave}$  between Petrescu et al., with the Eqs. (35) and (30) are 37.18% and 26.94%, respectively.
3. The results show differences between previous studies because they did not consider the energy conversion from the kinetic energy to thermal energy after the piston stops.

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#### References

- [1] Abdel-Aziz Farouk Abdel-Aziz Mohamed, "Hybrid Nanocrystal Photovoltaic/Wind Turbine Power Generation System in Buildings", *J. Adv. Res. Mater. Sci.* 40, no. 1 (2018): 8-19.
- [2] Ny, G., N. Barom, S. Noraziman, and S. Yeow. "Numerical study on turbulent-forced convective heat transfer of Ag/Heg water nanofluid in pipe." *J. Adv. Res. Mater. Sci.* 22, no. 1 (2016): 11-27.
- [3] Anacleto, Joaquim, and J. M. Ferreira. "On the representation of thermodynamic processes." *European Journal of Physics* 36, no. 3 (2015): 035006.
- [4] Anacleto, Joaquim, J. M. Ferreira, and A. A. Soares. "When an adiabatic irreversible expansion or compression becomes reversible." *European Journal of Physics* 30, no. 3 (2009): 487.
- [5] Gislason, Eric A., and Norman C. Craig. "First law of thermodynamics; Irreversible and reversible processes." *Journal of chemical education* 79, no. 2 (2002): 193.
- [6] Plotnikov, L. V., and B. P. Zhilkin. "The gas-dynamic unsteadiness effects on heat transfer in the intake and exhaust systems of piston internal combustion engines." *International Journal of Heat and Mass Transfer* 115 (2017): 1182-1191.
- [7] Sieniutycz, Stanislaw. "Variational setting for reversible and irreversible fluids with heat flow." *International Journal of Heat and Mass Transfer* 51, no. 11-12 (2008): 2665-2675.
- [8] Zhao, Yingru, and Jincan Chen. "An irreversible heat engine model including three typical thermodynamic cycles and their optimum performance analysis." *International Journal of Thermal Sciences* 46, no. 6 (2007): 605-613.
- [9] Norton, John D. "The impossible process: Thermodynamic reversibility." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 55 (2016): 43-61.
- [10] Paul, D. R., and J. W. Barlow. "Basic thermodynamics." *J Macromol Sol (C)* 18 (1980): 109-116.
- [11] Cengel, Yunus A., and Michael A. Boles. "Thermodynamics: an engineering approach." *Sea* 1000 (2002): 8862.
- [12] Klein, Martin J. "Thermodynamics in Einstein's thought." *Science* 157, no. 3788 (1967): 509-516.

- [13] Christians, Joseph. "Approach for teaching polytropic processes based on the energy transfer ratio." *International Journal of Mechanical Engineering Education* 40, no. 1 (2012): 53-65.
- [14] Petrescu, S., B. Borcila, M. Costea, E. Banches, G. Popescu, N. Boriaru, C. Stanciu, and C. Dobre. "Concepts and fundamental equations in Thermodynamics with Finite Speed." In *IOP Conference Series: Materials Science and Engineering*, vol. 147, no. 1, p. 012144. IOP Publishing, 2016.
- [15] Sommerfeld, Arnold. *Lectures on Theoretical Physics: Thermodynamics and statistical mechanics*. Vol. 5. Academic press, 2012.
- [16] Sobel, Michael I. "Kinetic theory derivation of the adiabatic law for ideal gases." *American Journal of Physics* 48 (1980): 877-878.
- [17] Semat, Henry, and Robert Katz. "Physics, Chapter 16: Kinetic Theory of Gases." (1958).
- [18] Gruber, Christian, and Gary P. Morriss. "A Boltzmann equation approach to the dynamics of the simple piston." *Journal of statistical physics* 113, no. 1-2 (2003): 297-333.