Numerical Investigation on the Influence of Gas Area Fraction on Developing Flow in a Pipe Containing Superhydrophobic Transverse Grooves

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ARTICLE INFO

Article history:
Received 20 March 2018
Received in revised form 14 May 2018
Accepted 15 May 2018
Available online 17 May 2018

ABSTRACT

This study presents a numerical investigation on the entrance length for developing flow in a pipe having alternating superhydrophobic grooves and ribs arranged transversely to the flow direction. Flows at low Reynolds number (i.e., \( Re = 1 \)) over superhydrophobic transverse grooves of \( L = 0.1 \) are considered. The influences of superhydrophobic surfaces on radial velocity profile development, centerline velocity distribution and hydrodynamic entrance length estimation are examined. Numerical results show that the hydrodynamic entrance length arising from flow over superhydrophobic transverse grooves are longer as compared to that of smooth wall. It is also found that the resulted entrance length is directly influence by the relative surface area occupied by grooves (i.e., gas area fraction). When the gas area fraction is larger, it would yield an increase in the hydrodynamic entrance length.

Keywords:
Entrance length, water-repellent, surface roughness, laminar flow, wall slip

1. Introduction

By definition, the length of the hydrodynamic entry region before the flow reaches a fully developed flow condition is often designated as the hydrodynamic entrance length (\( L_h \)). This length is an essential measurement that determines the distance needed for a flowing liquid before a fully developed flow condition could prevail. In most existing literatures, studies pertaining to the hydrodynamic entrance length have been focused on smooth surface [7, 12] where no-slip condition prevails. For an internal flow passing through smooth wall in a pipe of diameter \( D \), the hydrodynamic entrance length can be approximated using correlation presented by Durst et al., [5] given by

\[
\frac{L_h}{D} = [(0.619)^{1.6} + (0.0567Re)^{1.6}]^{1/1.6}.
\]
Apart from smooth wall, influences of slip wall on developing flow were investigated recently [4, 6, 10]. Muzychka and Enright [10] investigated developing flow in channels and tubes with slip walls using computational fluid dynamics (CFD). The numerical simulations are well compared with the analytical solutions for $Re_{D_h} > 100$. However, the solutions presented by Muzychka and Enright [10] can only be applied to flow with nearly uniform wall slippage (i.e., rarefied gas flows and liquid slip over small-texture superhydrophobic surface). Instead of enforcing arbitrary velocity slip length, the superhydrophobic surface features can be fully resolved. These surfaces can be constructed with textured patterns consisting of protruding structures at the micron/submicron-scale with regular profiles such as grooves, post, holes, etc [1]. A significant amount of research has been dedicated to study flows over these microstructures, especially in the fully developed flow region [2, 3, 8, 9, 11, 13-15]. However, the influence of these regular microstructures on developing flow is still largely unknown.

2. Methodology

In this study, a pipe configuration of diameter $D$ in the cylindrical coordinate system $(r, \theta, z)$ is considered, as depicted in Figure 1. The surface of the pipe contains a periodic array of alternating superhydrophobic grooves and ribs arranged transversely to the flow direction. The length of one period groove-rib combination and the groove width are denoted by $E$ and $e$, respectively. The scale of the superhydrophobic structures are governed by the dimensionless gas area fraction ($\delta = e/E$) and normalized groove-rib periodic spacing ($L = E/D$).

![Fig. 1. Schematic diagram depicting flow in tubes having transverse superhydrophobic grooves](image)

Since the surface profile is symmetry about the centerline, the flow is thus symmetry about $z$-axis, thereby the flow field is independent of $\theta$-direction. In this study, steady laminar flow of an incompressible Newtonian fluid passing through a pipe is considered. For a two-dimensional axisymmetric flow, the flow field is governed by continuity and momentum equations:

$$\frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{\partial u_z}{\partial z} = 0,$$

$$u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} \left\{ \frac{u_r}{r^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) + \frac{\partial^2 u_r}{\partial z^2} \right\}.$$

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The governing equations are solved numerically using the ANSYS FLUENT 18.1. The pressure-velocity coupling was solved via SIMPLE scheme. The numerical solution for pressure and momentum are based on second order and second order upwind schemes, respectively. The convergence criteria with scaled residuals of $10^{-8}$ for continuity and momentum equations are employed in this study. Based on the computational domain depicted in Figure 1, no-slip boundary condition is applied along the solid-liquid interface while shear-free condition is prescribed along the liquid-gas interface. At the inlet, uniform velocities are assumed. Based on diameter of the pipe and the average flow velocity, the flow Reynolds number is given by $Re = \frac{V_{avg}D}{\nu}$, where $\nu$ is the kinematic viscosity of the fluid. On the other hand, zero static pressure is prescribed at the outlet.

3. Result and discussions

A pipe of 0.001m diameter with 0.014m length is considered. The density and dynamic viscosity of the working fluid are assumed to be 1000 kg/m$^3$ and 0.001 kg/m.s, respectively. The fluid properties are assumed to be constant. Numerical simulation was first performed for flow through a pipe having smooth wall. As can be deduced from Figure 2, a good agreement is attained for both the normalized entrance length ($L_e/D$) computed via numerical simulations and that from correlation [5] given in Eq. 1 for Reynolds number ranging from $Re = 1 \times 10^{-4}$ to $Re = 1$. It is worth to note that the hydrodynamic entrance length $L_h$ is attained based on the axial length required by the centerline velocity from the inlet to reach 99% of its fully developed value.

In this numerical study, structured grid is employed. To investigate the grid dependence of the numerical solution for flow in a pipe having transverse grooves, grid independence study was conducted over four grid resolutions, $N_z \times N_r = 400 \times 20, 800 \times 40, 1600 \times 80$ and $3200 \times 160$. 

\[
\frac{u_r}{\partial r} + u_z \frac{\partial u_z}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \mu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{\partial^2 u_z}{\partial z^2} \right).
\]
$N_z$ and $N_r$ are the number of elements in the axial direction and radial direction, respectively. It is also worth to mention that only superhydrophobic transverse grooves of $L = 0.1$ is considered in this study and the liquid-gas interface is assumed to be ideally undeformed in the simulation. As can be seen from Figure 3, the solution converges at higher grid resolutions. Doubling the grid resolution from 1600 elements to 3200 elements in the axial direction and 80 elements to 160 elements in the radial direction gives rise to less than 0.2% difference for $L_h$. To ascertain the accuracy of the hydrodynamic entrance length computed for flow in pipe having transverse grooves, grid resolution of $N_z \times N_r = 1600 \times 80$ is thus employed throughout this study.

![Fig. 3. Grid independence test for flow through pipe having transverse grooves with $L = 0.1$ and $\delta = 0.5$ at $Re = 1$](image)

3.1 Effect on Development of Velocity Profile

The effect of superhydrophobic transverse grooves was first examined on the development of the radial velocity profile at successive axial locations. As depicted from Figure 4, the radial velocity profile changes rapidly from initially uniform velocity distribution at the inlet to a parabolic profile throughout the hydrodynamically fully developed flow region.

![Fig. 4. Development of axial velocity along the pipe having transverse grooves of $L = 0.1$ and $\delta = 0.5$ at $Re = 1$](image)
While the wall is assumed to be stationary, the fluid in close proximity with the wall is retarded. Away from the wall, the bulk of the flow accelerates. Since the scale of the superhydrophobic transverse grooves is relatively small (i.e., $L = 0.1$ where the groove-rib combination period width is one-tenth of the pipe diameter), the influence of these microstructures on the bulk flow is small. Therefore, parabolic velocity profile, similar with that of flow in a smooth wall pipe, is attained.

### 3.2 Effect on Centerline Velocity Distribution

Figure 5 presents the centerline velocity variation along the axial direction for different values of gas area fraction (i.e., $\delta = 0.25, 0.5$ and $0.75$). For flow scenario with pipe having smooth wall, it is denoted by $\delta = 0$. As can be deduced from this figure, the velocity magnitude at the centerline rises rapidly as the flow travels from the inlet before reaching its maximum velocity in the hydrodynamically fully developed flow region. For smooth wall, the maximum velocity is double of the inlet velocity. When $\delta$ becomes larger, the maximum velocity along the centerline decreases. This is similar to that observed by Davies et al. [3] for fully developed flow over superhydrophobic transverse grooves. By maintaining a constant flow rate through the pipe, the presence of superhydrophobic surface reduces the flow friction resistance, thereby allowing more flow in the vicinity of the superhydrophobic wall and thus reducing the velocity at the centerline.

![Fig. 5. Velocity magnitude along the centerline of the pipe for flow scenarios with $\delta = 0$, 0.25, 0.5 and 0.75. Simulations are conducted at $Re = 1$ for $L = 0.1$](image)

### 3.3 Effect on Hydrodynamic Entrance Length

The influence of gas area fraction is also visible on the resulting hydrodynamic entrance length, as illustrated in Figure 6. The numerical results predict that, in the presence of superhydrophobic transverse grooves and regardless of the size of the ribs, the hydrodynamic entrance length is longer than that of smooth wall. When $\delta$ becomes larger, the hydrodynamic entrance length is expected to be longer. For $\delta = 0.75$, the entrance length is predicted to be 3.6% longer as compared to that of smooth wall.
4. Conclusion

Numerical simulations were employed to investigate the developing flows in pipe having superhydrophobic transverse grooves, focusing on the influence on hydrodynamic entrance length arising from variation of gas area fraction. For fluid flowing at $Re = 1$ in a pipe over superhydrophobic transverse grooves of $L = 0.1$, the entrance length is expected to increase with the rise in gas area fraction.

Acknowledgement
The authors acknowledge Universiti Sains Malaysia Short-Term Grant No: 304/PMEKANIK/6315074 for the financial support.

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