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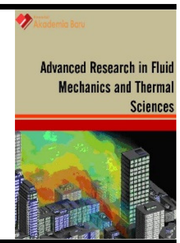

Unsteady Natural Convection of Sodium Alginate Viscoplastic Casson Based Based Nanofluid Flow over a Vertical Plate with Leading Edge Accretion/Ablation

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ABSTRACT

The present study explores the impact of viscous dissipation on unsteady two dimensional boundary layer flow of viscoplastic Casson ferrofluid over semi-infinite vertical plate with leading edge accretion/ablation. Tiwari-Das model is used to incorporates the effects of volumetric fraction of nanoparticles. Sodium alginate (SA) is taken as viscoplastic Casson based fluid containing Fe_2O_3 ferroparticles. Formulated differential equations along with relevant boundary conditions are solved numerically by Runge Kutta Fehlberg fourth-fifth order (RKF45) method. The effects of sundry parameters such as the Prandtl number, Eckert number, Casson parameter, accretion/ablation parameter, and nanoparticle volume fraction on velocity and temperature fields are investigated for both Rayleigh-Stokes and Blasius flat plate problems. Thermal boundary layer thicknesses for Blasius flat plate problem is thinner than Rayleigh-Stokes problem.

Keywords:
Ferrofluid, Ferroparticles, Viscous dissipation, Accretion/ablation

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1. Introduction

Creative and novel performance to perk up heat transfer by using solid particles in the conventional heat transfer fluids. Choi [1] is the first who introduced the term nanofluids to refer the fluids with suspended nanoparticles. Khanafer *et al*., [2] developed a model to study the heat transfer enhancement in solid particles dispersion nanofluids through enclosure and obtained numerical results with the help of finite volume method. Buongiorno [3] published a survey article on the convective transport in nanofluids. Tiwari and Das [4] investigated flow of nanofluids inside a two-sided lid-driven differentially heated square cavity. Ahmed and Pop [5] studied nanofluid mixed convection flow embedded in a porous medium. Hamad [6] presented analytical solution of nanofluid

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in the presence of magnetic field when the natural convection takes place over a linearly stretching sheet. Kandasamy *et al*., [7] used scaling group transformation and analyzed MHD flow of a nanofluid past a vertical stretching surface with wall suction or injection. Khan and Pop [8] used the Buongiorno model and studied the boundary layer flow of a nanofluid past a stretching sheet. Anwar *et al*., [9] investigated nanofluids flow over a nonlinearly stretching sheet. Qasim *et al*., [10] studied heat and mass transfer phenomenon in nanofluids with convective boundary conditions whereas Matin and Pop [11] used Brinkman model for the flow through porous channel and studied the force flow of a nanofluid in the presence of chemical reaction. Khairy *et al*., [12] obtained numerical solution for thermal boundary layer problem of nanofluid over a nonlinearly permeable stretching/shrinking sheet. Sun *et al*., [13] studied the heat transfer flow of Fe_2O_3-water nanofluids inside copper tubes. Aly and Ebaid [14] analyzed Marangoni boundary layer Cu/TiO_2-water nanofluids with magnetic field and thermal radiation effects. Unsteady MHD flow of some nanofluids past an accelerated vertical plate embedded are investigated by Hussanan *et al*., [15]. Exact analysis for the flow and heat transfer characteristics of CNTs-water nanofluids in the presence of convective condition are obtained by Saleh *et al*., [16]. Hussanan *et al*., [17] studied the microstructure and inertial characteristics of nanofluids over a vertical plate. Recently, Hussanan *et al*., [18] discussed magnetite microploar ferrofluid over a stretching/shrinking sheet using effective thermal conductivity model.

Casson fluid is a subtype of viscoplastic fluids which behaves like elastic liquid where no flow occurs with small shear stress. Casson [19] in his pioneering work introduced this model to simulate industrial inks. Later, numerous articles flooded the field of Casson fluid research and the area was widely explored due to its engineering applications. Mustafa *et al*., [20] have studied the heat transfer flow of a Casson fluid over an impulsive motion of the plate using the homotopy method. The exact solution of forced convection boundary layer Casson fluid flow toward a linearly stretching surface with transpiration effects are reported by Mukhopadhyay *et al*., [21]. In the same year, Rao et al. [22] considered the velocity and thermal slip conditions on the laminar boundary layer heat transfer flow of a Casson fluid past a vertical plate. Shehzad et al. [23] discussed the viscous chemical reaction effects on the MHD flow of a Casson fluid over a porous stretching sheet. Hussanan *et al*., [24] obtained the exact solution of free convection flow of a Casson fluid over an oscillating plate with Newtonian heating. Hussanan *et al*., [25] also developed exact solutions for suction and injection flow of a casson fluid in the presence of viscous dissipation over a stretching sheet. In another paper, Hussanan *et al*., [26] considered the magnetic field effects on unsteady flow of a Casson fluid.

The above published data reveal that no work has yet to be conducted on non-Newtonian viscoplastic casson fluids with nanoparticles over a vertical plate with leading edge accretion/ablation. Therefore, present study investigates the behavior of sodium alginate viscoplastic Casson based fluid containing Fe_2O_3 ferroparticles and a comparison between the sodium alginate base fluids and the nanoparticle interaction is conducted. Based on comparisons, we are able to obtain a better understanding of how ferroparticles properties might alter flow patterns of viscoplastic Casson. The governing equations are solved numerically. Flow and convective heat transfer are discussed with corresponding figures.

2. Problem Formulation

Consider an unsteady two-dimensional boundary layer flow and heat transfer of a viscoplastic Casson ferrofluid past a semi-infinite vertical plate with leading edge accretion/ablation. Let the uniform free stream velocity be U and free stream temperature be denoted by T_∞. The x-axis is taken vertically up in direction of free stream, while y is the coordinate measured normal to it and the flow being confined to $y>0$. Governing boundary layer equations subjected to above assumptions, considering viscous dissipation are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K_{nf}}{\left(\rho c_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{\left(\rho c_p\right)_{nf}}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2. \tag{2}$$

The above equations are subjected to following initial and boundary conditions

$$t<0: u=v=0,\ T=T_\infty \text{ for all } x, y, \tag{3}$$

$$\begin{aligned} &t\geq 0:\ u=v=0,\ T=T_w \text{ at } y=0,\\ &\quad u\to U,\ T\to T_\infty \text{ as } y\to\infty, \end{aligned} \tag{4}$$

where u and v are the velocity components in the x and y directions, respectively, β is the Casson parameter. Further, μ_{nf}, ρ_{nf}, k_{nf} and $\left(\rho c_p\right)_{nf}$ are dynamic viscosity, density, thermal conductivity and heat capacitance of the ferrofluid, respectively, which are defined as [27, 28]

$$\begin{aligned} &\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s,\ \left(\rho c_p\right)_{nf}=(1-\varphi)\left(\rho c_p\right)_f+\varphi\left(\rho c_p\right)_s,\\ &\quad \mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\ \frac{K_{nf}}{K_f}=\frac{\left(K_s+2K_f\right)-2\varphi\left(K_f-K_s\right)}{\left(K_s+2K_f\right)+\varphi\left(K_f-K_s\right)}. \end{aligned} \tag{5}$$

The following similarity functions are introduced to translate the governing equations into its non-dimensional forms are

$$\begin{aligned} &\psi(x,y,t)=U\sqrt{\left(\nu_f t\right)\cos(\alpha)+\left(\nu_f x/U\right)\sin(\alpha)}F(\eta),\\ &\eta=\frac{y}{\sqrt{\left(\nu_f t\right)\cos(\alpha)+\left(\nu_f x/U\right)\sin(\alpha)}},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \end{aligned} \tag{6}$$

where ψ is the free stream function. The free stream function ψ defines the velocity components as

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}. \tag{7}$$

Using equation (6) into equation (7), velocity components u and v take the form

$$u = UF'(\eta),\ v = \left(\eta F'(\eta) - F(\eta)\right)\left[\frac{\nu_f \sin(\alpha)}{2\sqrt{(\nu_f t)\cos(\alpha) + (\nu_f x/U)\sin(\alpha)}}\right], \tag{8}$$

With the help of equations (6) to (8), equations (1) and (2) take the new dimensionless form as

$$\left(1+\frac{1}{\beta}\right)F'''(\eta) + \frac{1}{2}\left(\eta\cos(\alpha) + f\sin(\alpha)\right)(1-\varphi)^{2.5}\left((1-\varphi)+\varphi\frac{\rho_s}{\rho_f}\right)F''(\eta) = 0, \tag{9}$$

$$\begin{aligned}&\left(\frac{K_{nf}}{K_f}\right)\theta''(\eta) + \frac{\Pr}{2}\left((1-\varphi)+\varphi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(\eta\cos(\alpha)+f\sin(\alpha)\right)\theta'(\eta)\\ &+\Pr Ec(1-\varphi)^{-2.5}\left(1+\frac{1}{\beta}\right)\left(F''(\eta)\right)^2 = 0.\end{aligned} \tag{10}$$

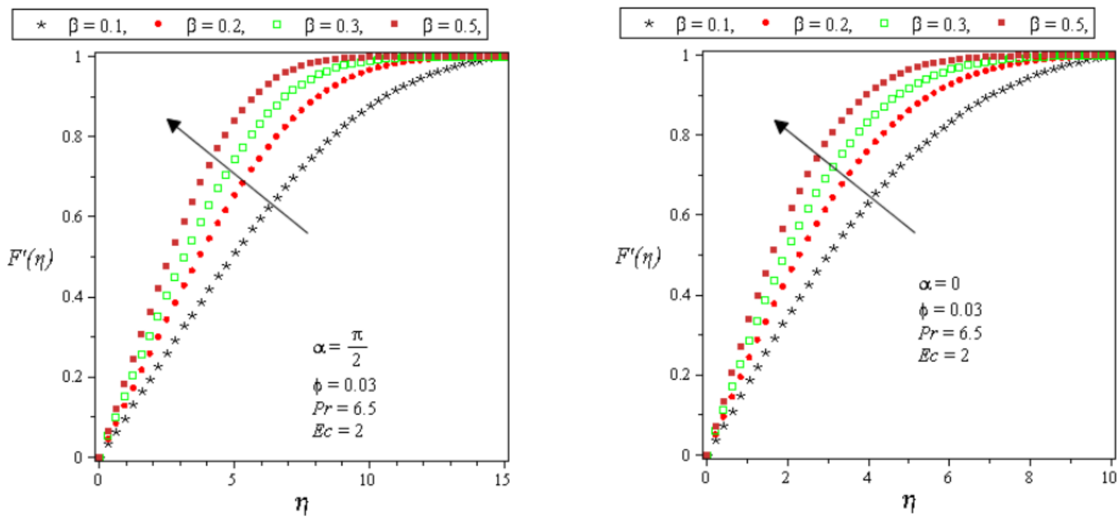
The corresponding boundary conditions are

$$\begin{aligned}&F(\eta)=0,\ F'(\eta)=0,\ \theta(\eta)=1,\ \text{at } \eta=0,\\ &\quad F'(\eta)\to 1,\ \theta(\eta)\to 0,\ \text{as } \eta\to\infty,\end{aligned} \tag{11}$$

Where $\Pr = \frac{\mu_f (c_p)_f}{K_f}$, $Ec = \frac{U^2}{(c_p)_f (T_w - T_\infty)}$, are the Prandtl number and Eckert number.

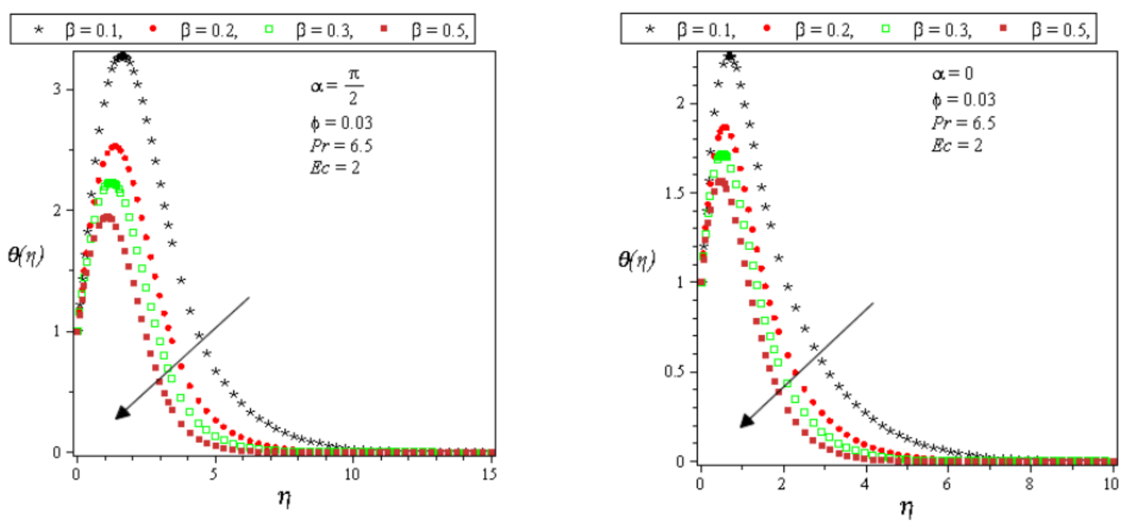
3. Results and Discussions

In this study we used Tiwari-Das model to investigate the impact of viscous dissipation on unsteady two-dimensional boundary layer flow of viscoplastic Casson ferrofluid over semi-infinite vertical plate with leading edge accretion/ablation. The effects of sundry parameters such as the Prandtl number $\Pr$, Eckert number Ec, Casson parameter β, accretion/ablation parameter α and nanoparticle volume fraction ϕ on velocity $F'(\eta)$ and temperature $\theta(\eta)$ fields are investigated for both Rayleigh-Stokes problem $(\alpha = 0)$ and Blasius flat plate problem $(\alpha = \pi/2)$ cases, separately. Figures 1(a) and 1(b) describe the effect of Casson parameter β on velocity field $F'(\eta)$ for both Rayleigh-Stokes problem $(\alpha = 0)$ and Blasius flat plate problem $(\alpha = \pi/2)$. The results show that the velocity field $F'(\eta)$ decreases with increase of β. It is also noticed that there is a sharp fall in the velocity field for Blasius flat plate problem $(\alpha = \pi/2)$ as compare to Rayleigh-Stokes problem $(\alpha = 0)$ within the layer $\eta < 10$ and then it becomes uniform for both cases as $\eta \to \infty$. The temperature field $\theta(\eta)$ with Casson parameter β for $\alpha = 0$ and $\alpha = \pi/2$ is plotted in Figures 2(a) and 2(b). It is found that temperature field $\theta(\eta)$ decreases with increasing β in both cases of Rayleigh-Stokes problem and Blasius flat plate problem. The effects of Eckert number Ec on temperature field $\theta(\eta)$ are illustrated in Figures 3(a) and 3(b) for stretching sheet with both Rayleigh-Stokes $(\alpha = 0)$ and Blasius flat plate problems $(\alpha = \pi/2)$, keeping the other parameters fixed. Based on the definition of Eckert number (relationship between a kinetic energy flow and the enthalpy), the increase in its value suggests a progressive increase in temperature $\theta(\eta)$. It is also seen that temperature of fluid increases for both cases.



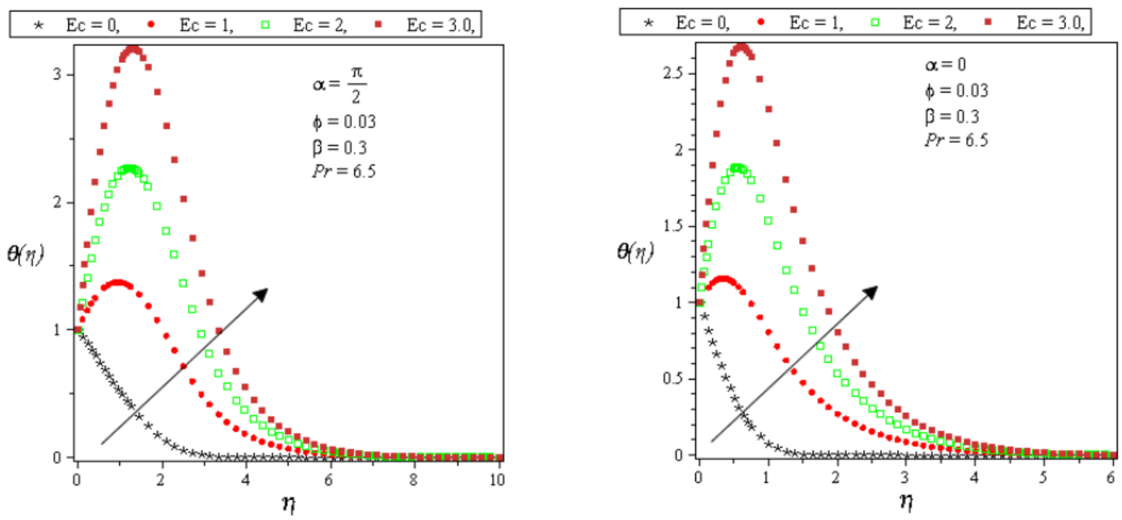
(a) (b)

Fig. 1. Velocity field for different β. (a) Rayleigh-Stokes problem, (b) Blasius flat plate problem



(a) (b)

Fig. 2. Temperature field for different β. (a) Rayleigh-Stokes problem, (b) Blasius flat plate problem



(a) (b)

Fig. 3. Temperature field for different Ec. (a) Rayleigh-Stokes problem, (b) Blasius flat plate problem

4. Conclusions

In this study, flow and heat transport of viscoplastic Casson ferrofluid over semi-infinite vertical plate with leading edge accretion/ablation is investigated numerically. Some of the interesting results of the present study can be epitomized as

i. Remarkable change occurs to velocity for Rayleigh-Stokes and Blasius flat plate problems.
ii. In the absence of viscous dissipation, the fluid has lower temperature along the plate.
iii. Thermal boundary layer thicknesses for Blasius flat plate problem is thinner than Rayleigh-Stokes problem.

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