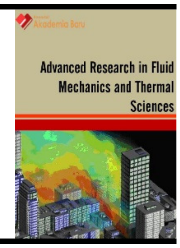
Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879



Analytical Solutions on the Flow of blood with the Effects of Hematocrit, Slip and TPMA in a porous tube

Open Access

Rajashekhar Choudhari[1], Manjunatha Gudekote[1,*], Naveen Choudhari[2]

[1] Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Udupi Karkala Road, Manipal, Karnataka 576104, India Karnataka, India
[2] Department of Mechanical Engineering, BMS College of Engineering, Bull Temple Road, Bangalore, Karnataka 586019, India

ARTICLE INFO

Article history:
Received 31 March 2018
Received in revised form 27 May 2018
Accepted 19 July 2018
Available online 23 July 2018

Keywords:
Angle of inclination, hematocrit, velocity slip, thermal slip

ABSTRACT

The purpose of the study is to investigate the role of hematocrit, slip and TPMA (Total Protein Minus Albumin) on the flow of blood in an axisymmetric inclined porous tube. The Walburn-Schneck equation is used to model the flow of blood and the resulting governing equations are solved by considering the long wavelength and small Reynolds number approximation. The closed form solutions are obtained for velocity, flow rate and temperature. The study reveals that an increase in the hematocrit and TPMA has a larger impact on both velocity and temperature. Furthermore, the impact of porous parameter and slip parameter play a vital role in controlling the blood flow.

1. Introduction

Mathematical studies on the non-Newtonian flow of blood are of interest to researchers due to its comprehensive applications in the field of Biomedical engineering and medicine. The Newtonian approach helps in understanding the flow of classical fluids through microchannels but fails to explain the complex behavior of blood [1,2]. The constitutive equations express the dependence between shear stress and shear rate for blood. Due to the complex macro-rheological behavior of blood, it is not possible to completely describe the various physiological parameters (e.g., hematocrit, TPMA, shear rate) through a single equation. Thus, many approaches exist to overcome this situation, some of which are a result of fitting a curve to the experimental data and others are based on some rheological models. Walburn and Schneck [3] introduced one such constitutive equation which gives the relationship between hematocrit and TPMA. Easthope [4] modified the Walburn-Schneck constitutive equation for blood by replacing scalar stress and shear strain rate by the stress and strain rate dyadics. This, when used to form a detailed kinematic model of blood flow gives rise to a new constitutive equation. Later, Rodkiewicz *et al*., [5] discussed the pulsatile blood flow in a conduit by using the constitutive equation for whole human blood, with the addition of chemical composition.

[*] *Corresponding author.*
E-mail address: manjunatha.g@manipal.edu (Manjunatha Gudekote)

Furthermore, Luo and Kuang [6] proposed the new three-parameter constitutive equation for whole human blood using Casson model and assumed that the study of blood is concerned with laminar flow in tubes with a diameter larger than 300 μm. This model (K-L model) can be used to properly describe the flow of blood when shear thinning behavior is concerned with wide shear range. Ballyk *et al*., [7] studied constitutive equation of whole human blood, the characteristics of Newtonian and non-Newtonian blood flows were compared in a two-dimensional study under both steady & unsteady flow conditions. Zhang and Kuang [8] investigated the blood flow using different constitutive equations. Singh and Singh [9] studied the role of hematocrit on the flow of blood in a tapered artery. Rajashekhar *et al*., [10] investigated the effect of hematocrit and magnetization on the FHD flow of blood. Further, Rajashekhar *et al*., [11] also studied the flow of blood in Y-bifurcation and obtained the velocity profiles for ICA (Internal Carotid Artery) and ECA (External Carotid Artery). Jahangiri *et al*., [12] compared the results of different non-Newtonian models to study the physiological behavior of blood. Recently, Manjunatha and Rajashekhar [13] carried out studies on the flow of blood using Casson equation by considering slip and porosity factors.

Regulation of body temperature is one of the important functions of the human circulatory system. The heat in the body which is produced by the skeletal muscles is removed mainly by the convective heat transfer of blood. The principles of heat transfer have been employed by several researchers to explore information on how the human body transfers heat [14]. In 1943, Barcroft and Edholm [15] carried out experimental investigations to study the variation in blood flow due to changes in the temperature. The mean values of Nusselt number for the steady flow of diluted blood in a one-dimensional pipe of diameter 10mm were obtained by Mitvalsky [16]. Charm *et al*., [17] conducted experiments to collect more information on heat transfer coefficients in microcirculation. Victor and Shah [18] first showed the effects of blood rheology on the transfer of heat. They obtained the values of heat transfer coefficients for the flow of blood in a tube by considering blood as a Casson fluid. Barozzi and Dumas [19] carried out numerical studies on convective heat transfer in the blood vessels of the circulatory system. Craciunescu and Clegg [20] studied the effects of pulsatile nature of blood flow on heat transfer in four typical vessel sizes: arterioles, terminal arterial branches, large arteries, and aorta. They found that the velocity pulsations have a small influence on the transport of energy out of the cells for the thermally essential terminal arteries (0.04 *mm* – 1.00 *mm*). They also concluded that, for studies on bioheat transfer, it is reasonable to assume the non-pulsating velocity of blood flow. Many researchers have recently studied the blood flow in different physiological conditions as mentioned in the studies of [21-23].

To the best of author's knowledge, no attempts have been made in the literature which deals with the effects of slip, hematocrit and TPMA on the flow of blood in an inclined porous tube. The present investigation is helpful in filling the gap in this direction. The closed-form solutions are obtained by using long wavelength and small Reynold's number approximation for velocity, flow rate, and temperature. The outcomes of the present model help in providing a better understanding of the flow of blood in narrow arteries.

2. Mathematical Formulation and Closed Form Solutions

The flow of blood is modelled to be laminar, steady, incompressible, fully-developed, axisymmetric and inclined at an angle β with the horizontal axis (Figure 1).

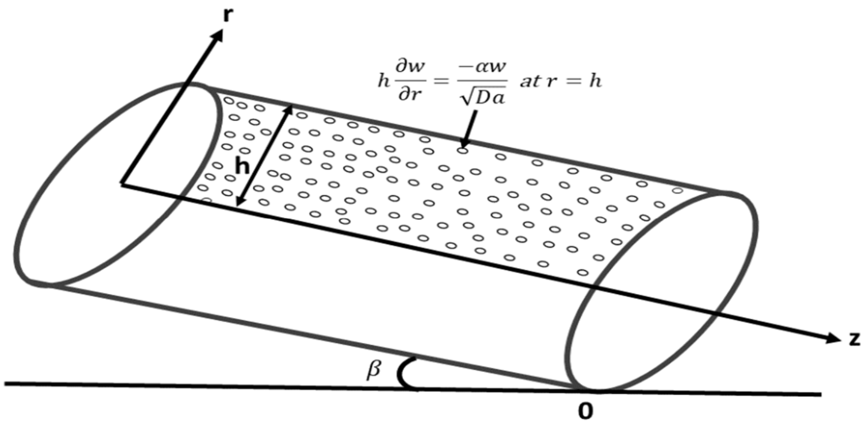


Fig. 1. Geometry of the porous tube

Under the assumption of long wavelength $\delta << 1$ and small Reynolds number the governing equation for the flow of blood may be considered in the form [13]

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = -\frac{\partial p}{\partial z}, \tag{1}$$

$$0 = \frac{\partial p}{\partial r}, \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) = Ec\,\mathrm{Pr}\left(-\frac{\partial w}{\partial r}\tau_{rz}\right). \tag{3}$$

Where Pr is the Prandtl number, Ec is the Eckert number, τ_{rz} is the shear stress along radial and axial coordinates. The constitutive equation for Walburn-Schneck equation is given by

$$\tau_{rz} = C_1 e^{C_2 H + \frac{C_4 \times TPMA}{H^2}} \left(\dot{\gamma}\right)^{1 - C_3 H}, \tag{4}$$

where, $C_1 = 0.00797$, $C_2 = 0.0608$, $C_3 = 0.00499$, $C_4 = 145.85$, H is the hematocrit percentage, $TPMA$ is the total protein minus albumin and $\dot{\gamma}$ is the strain rate.

The corresponding boundary conditions are

$$h\frac{\partial w}{\partial r} = \frac{-\alpha w}{\sqrt{Da}}, \quad \theta + \phi\frac{\partial \theta}{\partial r} = 0 \text{ at } r = h, \tag{5}$$

$$\frac{\partial \theta}{\partial r} = 0,\ \tau_{rz} \text{ is finite at } r = 0. \tag{6}$$

Equation (5) corresponds to the velocity and thermal slip conditions respectively. Further, Da is the porous parameter (Darcy number), α is the velocity slip parameter, ϕ is the thermal slip parameter and θ is the temperature.

The closed form solution is obtained for the velocity expression (1) satisfying the boundary conditions (5) and (6). We obtain the expression for velocity (w) as

$$w=\left[\frac{P+f}{2C_1 e^{C_2 H+\frac{C_4 TPMA}{H^2}}}\right]^{\frac{1}{1-C_3 H}}\left\{h^{\frac{2-C_3 H}{1-C_3 H}}\frac{\sqrt{Da}}{\alpha}+\frac{1-C_3 H}{2-C_3 H}\left[h^{\frac{2-C_3 H}{1-C_3 H}}-r^{\frac{2-C_3 H}{1-C_3 H}}\right]\right\}, \tag{7}$$

where $P=-\dfrac{\partial p}{\partial z}$ and $f=\dfrac{\sin\beta}{F_1}$.

The expression for temperature (θ) is obtained by solving equation (3) together with the boundary conditions (5) and (6)

$$\theta=\frac{\Pr Ec\, P}{2}\left[\frac{P}{2C_1 e^{C_2 H+\frac{C_4 TPMA}{H^2}}}\right]^{\frac{1}{1-C_3 H}}\frac{1-C_3 H}{4-3C_3 H}\left\{\frac{1-C_3 H}{4-3C_3 H}\left[r^{\frac{4-3C_3 H}{1-C_3 H}}-h^{\frac{4-3C_3 H}{1-C_3 H}}\right]-\phi h^{\frac{3-2C_3 H}{1-C_3 H}}\right\}. \tag{8}$$

The instantaneous flow rate Q across any cross section of the artery is defined as given below:

$$Q=2\int_0^h w\, r\, dr. \tag{9}$$

$$Q=\left[\frac{P+f}{2C_1 e^{C_2 H+\frac{C_4 TPMA}{H^2}}}\right]^{\frac{1}{1-C_3 H}} h^{\frac{4-3C_3 H}{1-C_3 H}}\left\{\frac{\sqrt{Da}}{\alpha}+\frac{1-C_3 H}{4-3C_3 H}\right\}. \tag{10}$$

3. Results and Discussion

The present paper investigates the flow of blood using Walburn-Schneck equations. The role of hematocrit, TPMA, porous parameter, velocity slip parameter, thermal slip parameter, Eckert number and Prandtl number on velocity and temperature are analyzed graphically with the help of MATLAB and are presented in Figures 2 to 11.

Figure 2 shows the variation of hematocrit (H) on velocity (w). It is noticed from the figure that an increase in the percentage of hematocrit decreases the velocity in a porous tube. This is because, an increase in the percentage of hematocrit reduces the concentration of plasma and hence lowers the velocity of blood. This observation on hematocrit is in good agreement with the results obtained by Rajashekhar et al. [10]. The effect of total protein minus albumin $(TPMA)$ on velocity is plotted in Figure 3. It is found that an increase in the concentration of $TPMA$ decreases the velocity. This is mainly due to the fact that a rise in the level of TPMA increases the plasma concentration and there by decreases the velocity. Figure 4 illustrates the variation of a porous parameter (Da) on velocity. It is observed from the figure that an increase in the value of porous parameter increases the velocity in an inclined porous tube. The effect of velocity slip parameter (α) on velocity is illustrated in Figure 5. It is important to note that, an increase in the value of velocity slip parameter decreases the flow of blood. Furthermore, the impact of inclination (β) on velocity is shown in Figure

(6). It is found that an increase in the angle of inclination increases the velocity in an inclined porous tube.

The effects of hematocrit, total protein minus albumin, thermal slip parameter, Prandtl number and Eckert number on temperature (θ) is plotted in Figures 7-11. Figure 7 shows the variation of hematocrit on temperature. It is seen that an increase in the concentration of hematocrit increases the temperature of blood. The effect of total protein minus albumin on temperature shows similar behavior as that of hematocrit (Figure 8). The impact of thermal slip parameter (ϕ) on temperature is plotted in Figure 9. It is observed that an increase in the value of thermal slip parameter decreases the temperature. Hence, the temperature of the fluid can be controlled by taking thermal slip into account.

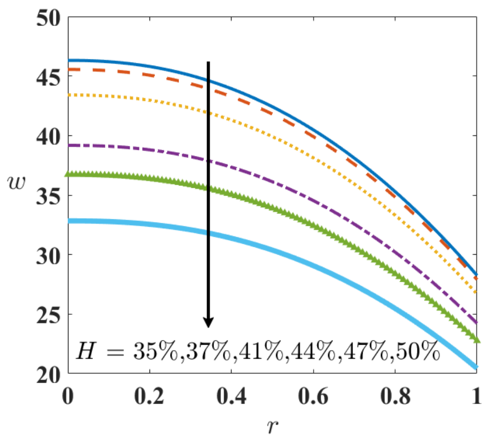


Fig. 2. w versus r for varying H with $\beta=\frac{\pi}{4}$, $TPMA=2.5$, $Da=0.02$, $P=1$ and $\alpha=0.2$.

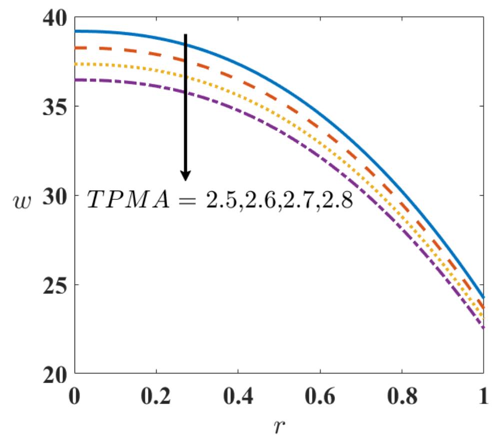


Fig. 3. w versus r for varying $TPMA$ with $\beta=\frac{\pi}{4}$, $H=45\%$, $Da=0.02$, $P=1$ and $\alpha=0.2$.

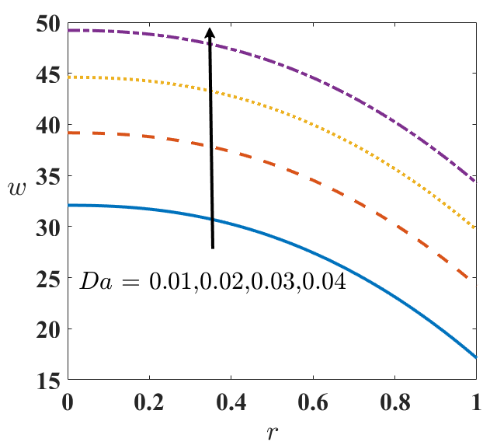


Fig. 4. w versus r for varying Da with $\beta=\frac{\pi}{4}$, $H=45\%$, $TPMA=2.5$, $P=1$ and $\alpha=0.2$.

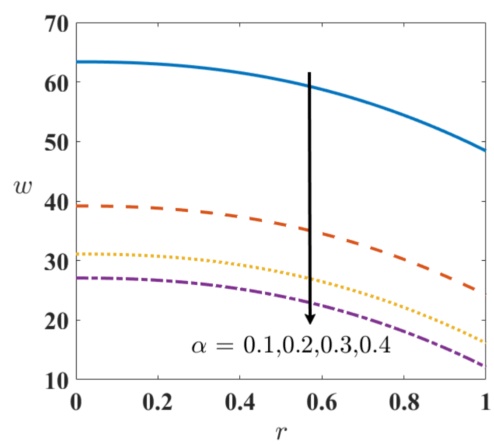


Fig. 5. w versus r for varying α with $\beta=\frac{\pi}{4}$, $H=45\%$, $TPMA=2.5$, $P=1$ and $Da=0.02$.

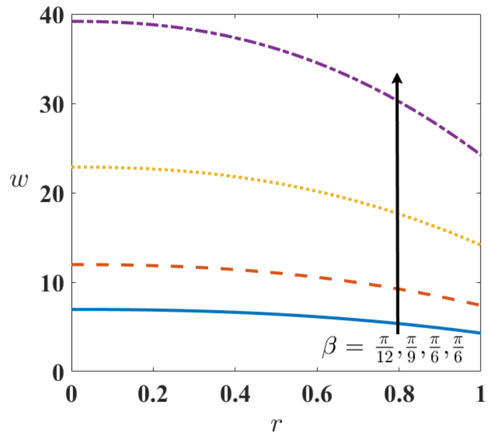


Fig. 6. w versus r for varying β with $\alpha = 0.2$, $H = 45\%$, $TPMA = 2.5$, $P = 1$ and $Da = 0.02$.

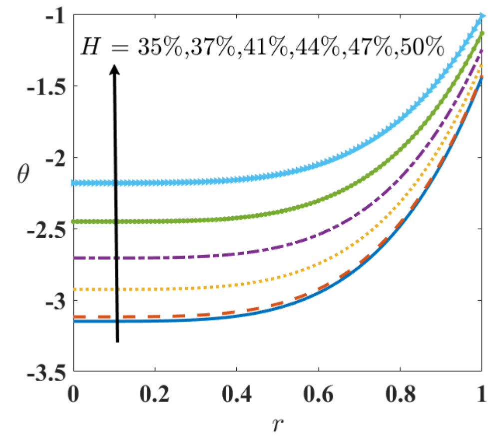


Fig. 7. θ versus r for varying H with $\Pr = 0.5$, $Ec = 0.5$, $TPMA = 2.5$, $P = 1$ and $\phi = 0.2$.

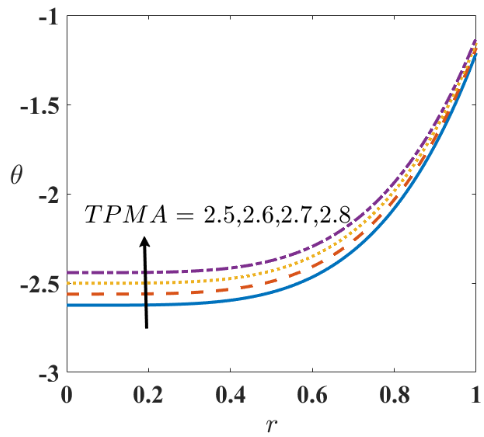


Fig. 8. θ versus r for varying $TPMA$ with $\Pr = 0.5$, $Ec = 0.5$, $H = 45\%$, $P = 1$ and $\phi = 0.2$.

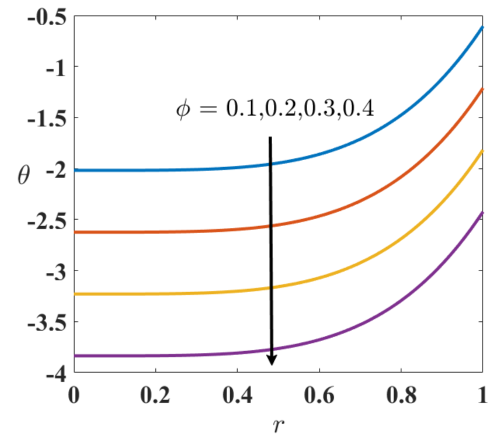


Fig. 9. θ versus r for varying ϕ with $\Pr = 0.5$, $Ec = 0.5$, $H = 45\%$, $P = 1$ and $TPMA = 2.5$.

Figures 10 and 11 represent the variation of Prandtl $(\Pr)$ and Eckert (Ec) number on temperature. It is seen that an increase in the values of Prandtl number results in a decrease of temperature. Physiologically, an increase in Prandtl number means a decrease in k which is responsible for the drop in temperature. Hence, in general, cooling of the heated tube can be improved by choosing a coolant with a large Prandtl number (Figure 10). Further, the impact of Eckert number on temperature shows a similar behavior as that of Prandtl number.

4. Summary and Conclusion

The present paper investigates the effects of hematocrit, velocity slip, thermal slip and TPMA on the flow of blood. The flow of blood is modelled by Walburn-Schneck equation by taking porous tube into account. Further, there is a possibility of extending the present model by considering variable thermal conductivity and convective boundary conditions into account. The present study plays a major role in the field of medicine to understand the flow of blood in narrow arteries. Some of the interesting findings are

- The increase in the percentage of hematocrit and TPMA significantly affects the blood flow.
- Darcy number and slip parameters play a vital role in the flow of blood.
- The velocity of blood increases with an increase in the angle of inclination.
- Temperature of the blood increases with an increase in the value of Hematocrit and TPMA.
- Temperature of the blood can be controlled by increasing the thermal slip parameter.
- An increase in the values of Prandtl and Eckert number decreases the temperature in a porous tube.

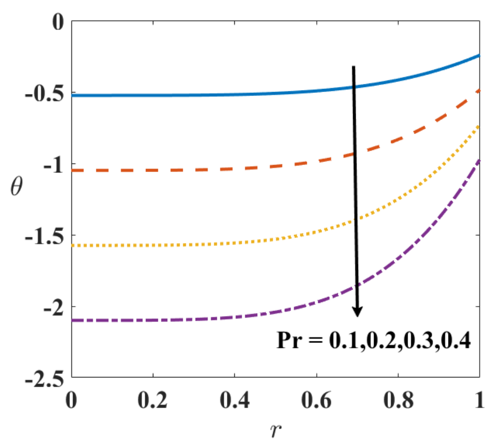


Fig. 10. θ versus r for varying $\Pr$ with $\phi = 0.2$, $Ec = 0.5$, $H = 45\%$, $P = 1$ and $TPMA = 2.5$.

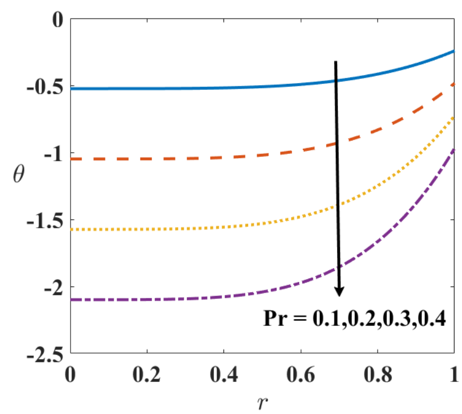


Fig. 11. θ versus r for varying Ec with $\phi = 0.2$, $\Pr = 0.5$, $H = 45\%$, $P = 1$ and $TPMA = 2.5$.

Acknowledgement
The authors appreciate the constructive comments of the reviewers which led to the definite improvement in the paper.

References
[1] Abubakar, S. B., and NA Che Sidik. "Numerical prediction of laminar nanofluid flow in rectangular microchannel heat sink." *J. Adv. Res. Fluid Mech. Therm. Sci.* 7, no. 1 (2015): 29-38.

[2] Soo Weng Beng and Wan Mohd. Arif Aziz Japar. "Numerical analysis of heat and fluid flow in microchannel heat sink with triangular cavities". *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 34, no. 1 (2017): 01-08.

[3] Walburn, Frederick J., and Daniel J. Schneck. "A constitutive equation for whole human blood." *Biorheology* 13, no. 3 (1976): 201-210.

[4] Easthope, Peter. "A three-dimensional dyadic Walburn-Schneck constitutive equation for blood." *Biorheology* 26, no. 1 (1989): 37-44.

[5] Rodkiewicz, C. M., Prawal Sinha, and J. S. Kennedy. "On the application of a constitutive equation for whole human blood." *Journal of Biomechanical Engineering* 112, no. 2 (1990): 198-206.

[6] Luo, X. Y., and Z. B. Kuang. "A study on the constitutive equation of blood." *Journal of biomechanics* 25, no. 8 (1992): 929-934.

[7] Ballyk, P. D., D. A. Steinman, and C. R. Ethier. "Simulation of non-Newtonian blood flow in an end-to-side anastomosis." *Biorheology* 31, no. 5 (1994): 565-586.

[8] Zhang, Jian-Bao, and Zhen-Bang Kuang. "Study on blood constitutive parameters in different blood constitutive equations." *Journal of Biomechanics* 33, no. 3 (2000): 355-360.

[9] Singh, A. K., and D. P. Singh. "Effect of hematocrit on wall shear stress for blood flow through tapered artery." *Applied Bionics and Biomechanics* 10, no. 2, 3 (2013): 135-138.

[10] Rajashekhar, C., G. Manjunatha, and K. S. Basavarajappa. "Analytical Solution of FHD Flow of Blood Through Two Layered Model in the Presence of Magnetic Field." *Journal of Mechanical Engineering Research and Developments* 40, no. 2 (2017): 365-370.

[11] Rajashekhar, C., Manjunatha, G. and Fabian, B. "Finite element simulation of blood flow through an artery: A mathematical model". *Malaysian Journal of Mathematical Sciences* 11, no. 2 (2017): 165-179.

[12] Jahangiri, Mehdi, Ahmad Haghani, Reza Ghaderi, and Seyyed Mohammad Hosseini Harat. "Effect of Non-Newtonian Models on Blood Flow in Artery with Different Consecutive Stenosis." *International Journal of Advanced Design & Manufacturing Technology* 11, no. 1 (2018).

[13] Manjunatha, G., and Rajshekhar V. Choudhary. "Slip Effects on Peristaltic Transport of Casson Fluid in an Inclined Elastic Tube with Porous Walls." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 43, no. 1 (2018): 67-80.

[14] Yunus, Cengel A., and J. Ghajar Afshin. "Heat and mass transfer: fundamentals and applications." Tata McGraw-Hill, New Delhi, India (2011).

[15] Barcroft, Ha, and O. G. Edholm. "The effect of temperature on blood flow and deep temperature in the human forearm." *The Journal of physiology* 102, no. 1 (1943): 5-20.

[16] Mitvalský, V. "Heat Transfer in the Laminar Flow of Human Blood through Tube and Annulus." *Nature* 206, no. 4981 (1965): 307.

[17] Charm, S., B. Paltiel, and G. S. Kurland. "Heat transfer coefficients in blood flow." *Biorheology* 5, no. 2 (1968): 133-145.

[18] Victor, S. A., and V. L. Shah. "Steady state heat transfer to blood flowing in the entrance region of a tube." *International Journal of Heat and Mass Transfer* 19, no. 7 (1976): 777-783.

[19] Barozzi, Giovanni Sebastiano, and Antonio Dumas. "Convective heat transfer coefficients in the circulation." *Journal of biomechanical engineering* 113, no. 3 (1991): 308-313.

[20] Craciunescu, Oana I., and Scott T. Clegg. "Pulsatile blood flow effects on temperature distribution and heat transfer in rigid vessels." *Journal of biomechanical engineering* 123, no. 5 (2001): 500-505.

[21] Xie, Haiwei, and Yan Zhang. "The effect of red blood cells on blood heat transfer." *International Journal of Heat and Mass Transfer* 113 (2017): 840-849.

[22] Rahbari, A., M. Fakour, A. Hamzehnezhad, M. Akbari Vakilabadi, and D. D. Ganji. "Heat transfer and fluid flow of blood with nanoparticles through porous vessels in a magnetic field: A quasi-one dimensional analytical approach." *Mathematical biosciences* 283 (2017): 38-47.

[23] Zhang, Yan, and Haiwei Xie. "The effect of a bifurcation structure on the heat transfer and temperature distribution of pulsatile blood flow." *International Journal of Heat and Mass Transfer* 118 (2018): 663-670.