Controller is an essential part in a system that playing a vital role especially dealing with the heavy machinery mechanism such as electrohydraulic actuator (EHA) system. It is proven in the past studies, the controller is necessary to be designed to reduce numerous defects such as nonlinearities and uncertainties which intrinsically exist in the EHA system. Therefore, different control approaches are presented and its performances are evaluated in this paper. Firstly, the conventional proportional-integral-derivative (PID) controller is designed, followed by the presentation of an improved PID controller called as fractional order PID (FO-PID) controller. Then the sliding mode controller (SMC) is designed, followed by the transformation on the sliding surface of the SMC controller with the PID control structure is carried out. The parameters of these four controllers are obtained by using the particle swarm optimization (PSO) tuning algorithm. The performances of these four different types of controllers applied in the EHA system are analysed. As a conclusion, it can be inferred that all of the controllers contain its own merit and demerit, which are considered as a trade-off between the controller's performances and its design complexity. The improvement of 31.41% has been obtained in the error analysis when the SMC-PID controller is applied to the EHA system.

Keywords: Electro-Hydraulic Actuator System, Fractional Order PID Controller, Sliding Mode Controller, PID Sliding Surface, Particle Swarm Optimization

1. Introduction

Nowadays, the development of electrohydraulic actuator (EHA) system for heavy engineering has been greatly raised the attention in both industrial and education fields. The EHA system has been implemented in various types of engineering applications, including oil and gas, material...
handling, mining, agriculture, and construction machineries. As reported in [1], agriculture and construction applications accounted the most with 75% component unit sales in 2014.

To understand the behaviour of the EHA system, the development of a mathematical model for this system based on the physical laws that represented in the differential equation usually begins with physically based including power supply, servo-valve, and hydraulic actuator, which taking the nonlinearities and the related dynamics into account [2]. The major sources of the nonlinearities exist in the EHA system including actuator friction, mechanism leakages, compressibility of the fluid, and nonlinear pressure characteristics [3]. These problems consequently increase the difficulties during the controller design, which simultaneously motivate academia and researchers to further examine and design a powerful controller that is able to overcome these problems before applied to the potential industrial applications.

As the proportional-integral-derivative (PID) controller, which is a famous controller that always applied in the industrial applications, many researchers have trying to investigated and modified this controller with the integration of various kind of methods, including the modification of the structure in this controller, for instance the gain-scheduling control, or the fractional order (FO-PID) controller that is proven to be more effective compared with the conventional PID controller [4-6].

Another famous robust nonlinear controller, which is the sliding mode controller (SMC) that has been commonly used in the control of various engineering applications [7-9], and also in the EHA system [10-12]. Various attempts have been done in order to enhance the capability of the SMC controller by improving or modified the sliding surface of the SMC, including the integration of the PID [13], fractional order [11,14] and integral augmented [15] control structure into the sliding surface of the SMC.

Recently, increasing studies with respect to the integration of the computational tuning methods into the controller design have been proposed. With the integration of the computational tuning method, including Particle Swarm Optimization (PSO), Gravitational Search Algorithm (GSA), Genetic Algorithm (GA), the performances of the PID controller [16-21], FO-PID controller [22-23], and SMC [24-26] are proven to be enhanced compared to the controller with conventional or without any proper tuning method.

In this paper, in order to produce a more insightful view of the performance and the capabilities of the controller, four different types of control approaches are presented and compared. The favourite controller in the industry field, which is the proportional-integral-derivative (PID) controller is first introduced. Follow by the improved PID controller, named Fractional Order (FO-PID) controller is designed. Then, the prominent robust controller in the control field, called a sliding mode controller (SMC) is established. Follow by the integration of the PID control structure to the SMC (SMC-PID) is carried out. Instead of obtaining the controller’s parameters without any appropriate technique, the well-known tuning technique in computer science, named particle swarm optimization (PSO) is utilized.

The examination of these controllers and methods in the simulation environment will be applied to the hardware that is in the development process. The organizations of this paper are, Section 2 is the summary of the basic structure of the paper, followed by the brief explanation of the FO-PID controller in Section 3. The derivation of the conventional SMC and the SMC-PID is explained in Section 4. The examination of the controller performance is pictured in Section 5, and finally the discussion and conclusion in Section 6.
2. Common Structure of the EHA System

As the modelling of the EHA system has been well developed in the past, therefore, the discussion of the detail mathematical model of the EHA system can be found in [27]. Generally, the common components consist of power unit, control unit, and actuator with sensing unit as illustrated in Figure 1.

![Fig. 1. The basic structure of the EHA system](image1)

Basically, this paper is conducted with the focus of design and optimization of the controller as illustrated in the basic control structure in Figure 2.

![Fig. 2. Basic control structure](image2)

The EHA system is well-known to be intrinsically nonlinear. In order to improve its performance, especially when applied to the applications that required high precision factor, for example, aircraft and vehicle part pressing industry, different controllers emerge in the past. Hence, this study intends to design four different controllers, including the conventional PID, FO-PID, conventional SMC, and SMC-PID controllers to improve or to overcome the existing drawback in the EHA system.
The parameters of each controller are tuned by using the PSO tuning method, where the discussion regarding the PSO tuning method has been conducted in [28], with the equal parameters including the particle’s size (50), the number of iterations (30), the acceleration (2 for both $c_1$ and $c_2$), the inertia weight (decreased from 0.9 to 0.4) and most importantly the performance index that use to obtain the minimum error, which is the Integral Time Absolute Error (ITAE). The obtained parameters will be applied to the designed controllers, and the performance of each controller will be evaluated.

3. Fractional Order PID Controller

Fractional Order calculus involved in the control and the dynamic system was first proposed by Igor Podlubny in the 20th century [5]. By expanding the general differential equations into the fractional order differential equations [29], the flexibilities of the fractional order calculus will be employed into PID controller which yield the Fractional Order (FO-PID) controller.

Instead of the conventional PID controller, which is well-known in the control system that contains three parameters, two additional parameters which are the integrating order, $\lambda$ and the derivative order, $\mu$ have been integrated into the integral and derivative gains of the PID controller [29-31]. Commonly, the transfer function of the conventional PID controller is obtained by

$$G(s) = \frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

(1)

where $K_p$ is the proportional gain, $T_i$ is the Integral gain time in constant time, and $T_d$ is the derivative gain in constant time. While the additional order that integrated to the FO-PID controller yields the transfer function of

$$G(s) = \frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s^\lambda} + T_d s^\mu \right)$$

(2)

where the order $\lambda$ and $\mu$ are not necessarily the integer number [29]. If the order $\lambda$ and $\mu$ are assumed to be 1, the convention PID controller is formed.

The common system with a fractional order or known as a non-integer type of system can be found in the transmission line or the heat flow system. The closed-loop control system generally consists of an integer or fractional order system with integer or fractional order controller, or the interchangeable of these system and control structure [32].

It is proven in the previous study, the FO-PID controller, or known as $P^\lambda D^\mu$ controller is able to ameliorate the conventional PID controller capability with the introduction of the integral and derivation order $\lambda$ and $\mu$ respectively. However, in the computer science point of view, since additional parameters are added to the FO-PID controller, the process to obtain the controller parameters become more complex and simultaneously increase the computational time.

4. Conventional SMC and SMC-PID Controllers

The pioneering work of the sliding mode notion designed in continuous time had been established in the early 1960’s in Russia. The concept is not disseminated over a period of time when a book is published by Itkis [33], and a journal article was written by Utkin [34]. After that, an
insightful view regarding the introduction and the growth of the SMC control strategy has been carried out in [35]. Thereafter, a number of studies regarding the SMC have been proposed to deal with the uncertain and the nonlinear in the system. The design of the SMC is unique since its performance does not directly depend on the tracking state while depending on the design of the sliding surface. The concept of the SMC technique is to force the control signal moving toward the sliding surface and force the control signal to stay on that surface one the control signal is reached [36]. Generally, the design of SMC has a structure as demonstrated in Figure 3.

![Diagram of Sliding Mode Controller](image)

**Fig. 3.** The fundamental idea and structure of the sliding mode controller

Commonly, the general equation of the sliding surface, \( s(t) \) in SMC can be obtained by referring to the system order, \( n \) as presented in the following equation.

\[
s(t) = \left( \lambda + \frac{d}{dt} \right)^{n-1} e(t)
\]  

(3)

Referring to the third order EHA system, the \( s(t) \) of the conventional SMC, which is proportional to the error, \( e \) and the control gain, \( \lambda \) can be obtained as

\[
s(t) = \ddot{e}(t) + 2\lambda \dot{e}(t) + \lambda^2 e(t)
\]  

(4)

By integrating the PID into the \( s(t) \) of the SMC and executes this structure to the EHA system with third order, the following equation can be obtained where \( k_p, k_i \) and \( k_d \) represent the PID parameters.

\[
s(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \dot{e}(t)
\]  

(5)

The error produced in a closed-loop environment can be acquired in equation 6 by subtracting the output of the desired tracking with the actual tracking.

\[
e(t) = x_r(t) - x_p(t)
\]  

(6)
The third order linearized EHA system will generate the error with the third derivative as expressed in Equation 7.

\[ \ddot{e}(t) = \dddot{x}_r(t) - \dddot{x}_p(t) \]  

(7)

When the \( s(t) \neq 0 \), switching control, \( u_{sw} \) will take place to leads the tracking error from the phase of reaching to sliding. While the \( s(t) = 0 \), equivalent control, \( u_{eq} \) will response to leads the tracking error on \( s(t) = 0 \) to the desired point. Thus, the SMC generally expressed as

\[ u_{smc}(t) = u_{eq}(t) + u_{sw}(t) \]  

(8)

For the conventional SMC, the \( u_{eq} \) will be acquired through the first derivative of the \( s(t) \) as

\[ \ddot{s}(t) = \dot{e}(t) + 2\lambda \dot{e}(t) + \lambda^2 \ddot{e}(t) \]  

(9)

When the PID control structure is applied to the sliding surface, the \( u_{eq} \) will be obtained through the second derivative of the \( s(t) \) as

\[ \dddot{s}(t) = k_p \dot{e}(t) + k_i \dot{e}(t) + k_d \ddot{e}(t) \]  

(10)

Some parameters exist in the EHA may impossible to be gathered and modelled. The simplified EHA model will be employed in the designed controller, where the EHA system will be represented through the perturbed linear model with third order, which has included the disturbances and uncertainties characters as indicated in the following equation.

\[ \dddot{x}_p(t) = -(A_n + \Delta A)\dddot{x}_p(t) - (B_n + \Delta B)\dddot{x}_p(t) + (C_n + \Delta C)u(t) + d(t) \]  

(11)

where \( d(t) \) is composed of the nonlinear leakage and friction, and the external load disturbance. The nominal system parameters are represented in \( A_n, B_n, \) and \( C_n \), while the uncertainties existed in the unmodeled dynamics are represented by the bounded uncertainties \( \Delta A, \Delta B, \) and \( \Delta C \). Then, the third-order EHA system will be organized as

\[ \dddot{x}_p(t) = -A_n \dddot{x}_p(t) - B_n \dddot{x}_p(t) + C_n u(t) + L(t) \]  

(12)

where \( L(t) \) is the lumped uncertainties that can be expressed as

\[ L(t) = \pm \Delta A \dddot{x}_p(t) \pm \Delta B \dddot{x}_p(t) \pm \Delta C u(t) + d(t) \]  

(13)

By assume \( L \) is neglected and substituting Equation 7 into 9, the \( u_{eq} \) for the conventional SMC will be indicated as

\[ u_{eq}(t) = \frac{1}{C} \left( \dddot{x}_r + A_n \dddot{x}_p + B_n \dddot{x}_p + 2\dot{\lambda} \dot{e}(t) + \lambda^2 \ddot{e}(t) \right) \]  

(14)
While the $u_{eq}$ for the SMC-PID will be obtained by substituting Equation 7 into 10 as

$$u_{eq}(t) = (k_d C_n)^{-1} \left( k_p \dot{e}(t) + k_i \dot{e}(t) + k_d (\ddot{x}_r + A_p \ddot{x}_p + B_p \dot{x}_p) \right)$$

(15)

For the conventional SMC, the switching control can be acquired by employing the signum function, $\text{sign}(s)$ into the sliding surface as expressed in Equation 16.

$$u_{sw}(t) = k_s \text{sign}(s)$$

(16)

where the signum function has a boundary as expressed in Equation 17, and $k_s$ is a positive constant value.

$$\text{sign}(s(t)) = \begin{cases} 1 & ; s(t) > 0 \\ 0 & ; s(t) = 0 \\ -1 & ; s(t) < 0 \end{cases}$$

(17)

For the SMC-PID controller, the following switching control function will be obtained.

$$u_{sw}(t) = \lambda s(t) + k_s \text{sign}(\dot{s}(t))$$

(18)

where the signum function has a boundary as in Equation 19, and $\lambda, k_s \in \mathbb{R}^+$ is a positive constant value.

$$\text{sign}(\dot{s}(t)) = \begin{cases} 1 & ; \dot{s}(t) > 0 \\ 0 & ; \dot{s}(t) = 0 \\ -1 & ; \dot{s}(t) < 0 \end{cases}$$

(19)

In the conventional SMC, the Lyapunov theorem as adopted in [13, 15, 37-39] is used to analyse the stability of the controller when $s(t) \neq 0$ with the following function.

$$V(t) = \frac{1}{2} s^2(t)$$

(20)

The following reaching condition is required to be fulfilled to achieve the stable condition during the tracking from reaching to sliding phase.

$$V(t) = s(t) \dot{s}(t) < 0, \text{ for } s(t) \neq 0$$

(21)

By replacing Equations 7, 8, 9 into Equation 21, the following function will be obtained.

$$s(t) \ddot{s}(t) = s[\ddot{x}(t) + A_n \ddot{x}_n(t) + B_n \dot{x}_n(t) - C_n (u_{eq}(t) + u_{sw}(t))] + 2 \lambda \ddot{e}(t) + \lambda^2 \dot{e}(t)$$

(22)

The discontinuous function in Equation 16 might lead to the chattering effect, which can be minimized by replacing the hyperbolic tangent function as introduced in [13, 15, 39].
\[ u_{sw}(t) = k_s \tanh\left( \frac{s}{\phi} \right) \]  

(23)

For the SMC-PID controller, the controller stability when \( s(t) \neq 0 \) is analysed through the following Lyapunov function.

\[ V(t) = \frac{1}{2} s^2(t) + \frac{1}{2} \dot{s}^2(t) \]  

(24)

And the following reaching condition is required to be fulfilled to achieve the stable condition during the tracking from reaching to sliding phase.

\[ \dot{V}(t) = 0, \text{ for } s(t) \neq 0, \dot{s}(t) \neq 0 \]  

(25)

By replacing Equations 7, 8, 10 into Equation 25, the following equation will be obtained.

\[ \dot{V}(t) = s(t)\dot{s}(t) + \dot{s}(t)\ddot{s}(t) \]

\[ = s(t)\dot{s}(t) - k_d C_n \lambda \dot{s}(t)\dot{s}(t) - k_d C_n k_s |\dot{s}(t)| - k_d L(t)\dot{s}(t) \]

\[ \cdots \leq |\dot{s}(t)| \left[ s(t) - k_d C_n \lambda \dot{s}(t) - k_d C_n k_s - k_d L(t) \right] \]

\[ \cdots \leq |\dot{s}(t)| \left[ |s(t)| - k_d C_n \lambda \dot{s}(t) - k_d C_n k_s - k_d L(t) \right] \]

\[ \cdots \leq -|\dot{s}(t)| \left[ k_d C_n \lambda - 1 + k_d C_n k_s - k_d \max(t) \right] \]

(26)

And the discontinuous function in Equation 18 is replaced by the hyperbolic tangent function as

\[ u_{sw}(t) = \lambda s(t) + k_s \tanh\left( \frac{s}{\phi} \right) \]  

(27)

5. Performances Analyses

The real-time engineering applications commonly required the assistant of the controller to achieve the desired response for example lifting, clamping and bending. The controller is especially useful in dealing with the system with major uncertainties and disturbances. To overcome the existing drawback in the system, the designed controller must be able to reduce the actual required elements, for example, the voltage or power that generates torque to actuate the load or the application. Apart of reducing the actual effort, the powerful controller can perform surprisingly, even with the parameters changes along the operation.

Figure 4 demonstrates the controller effort generated from the designed conventional PID, FO-PID, conventional SMC and SMC-PID controllers. Different device contains different limitation in the practical system. The common practical system has the minimum and maximum voltage of -10 Volts and 10 Volts. As depicted in Figure 4, the conventional PID, FO-PID and conventional SMC controllers illustrated the requirement of substantial effort in order to achieve the required response. In term of the energy consumption, the FO-PID controller required the higher energy, followed by the conventional SMC and PID controller. While the SMC-PID controller is outperform compared to the others.
To analyse the controller’s performance in the field of control, the steady-state error, root mean square error (RMSE), and transient response analyses are the commonly used methods and very useful in generating numerical data for the comparison purpose. Based on the numerical data as tabulated in Table 1, the SMC-PID controller demonstrated the capabilities in providing the convenient performances in the control of the positioning tracking.

Table 1
Numerical analysis of each controller based on transient and RMSE analysis

<table>
<thead>
<tr>
<th>Controller</th>
<th>Transient Response</th>
<th>Steady-state Error ($e_{ss}$)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>4.1178 0.1602 0.5236</td>
<td>0.0004</td>
<td>17.8588</td>
</tr>
<tr>
<td>FO-PID</td>
<td>0 0.3399 0.7140</td>
<td>0.0002</td>
<td>17.4633</td>
</tr>
<tr>
<td>SMC</td>
<td>1.5383 0.0532 0.1852</td>
<td>0.0001</td>
<td>14.1417</td>
</tr>
<tr>
<td>SMC-PID</td>
<td>1.3877 0.0195 0.1334</td>
<td>0.0001</td>
<td>12.2491</td>
</tr>
</tbody>
</table>

Each controller has their own strength and weakness and performed differently as shown in Figure 5.

Overshoot is acceptable in some situation, especially the small overshoot that might not lead to any inconvenience for example in the pushing and rotating processes. But unlike the process such as lifting, pressing, or bending that require precision, the overshoot situation might damage the product or cause a hazard in the real environment. Briefly speaking, the smaller the error in any practical process, the better the end result. Figure 6 demonstrates the error produced by the designed conventional PID, FO-PID, conventional SMC and SMC-PID controller.
Fig. 5. The tracking performance of each controller

Fig. 6. Error generated by the conventional PID, FO-PID, conventional SMC and SMC-PID controllers

Rise time is an important factor that increases the productivity for example, in the production line that requires lifting process, where the quick processing time is important in the process. Generally, the high-rise time will lead to high overshoot and simultaneously slow down the settling time of the system. As tabulated in Table 1, the SMC-PID controller has overcome the former condition which produced the fastest rise time with 0.0195 second, fastest settling time with
0.1334 second, lowest steady-state error and RMSE compared to the conventional PID, FO-PID and conventional SMC controllers. The performance produced by the SMC-PID controller fulfil the requirement of various practical engineering applications, especially the application where high precision is required.

Instead of using the conventional tuning techniques, for example, the try and error, and the Ziegler-Nichols tuning technique, the PSO computational tuning technique, which is very time saving and convenient has been used to acquire the controller’s gains as listed in Table 2.

### Table 2
Parameters achieved through PSO tuning method

<table>
<thead>
<tr>
<th>Controller</th>
<th>Parameter</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$\lambda$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td></td>
<td>10.0910</td>
<td>0.0013</td>
<td>-4.6985</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FO-PID</td>
<td></td>
<td>34.8991</td>
<td>0.7052</td>
<td>8.5401</td>
<td>2.0296</td>
<td>8.1205</td>
</tr>
<tr>
<td>SMC</td>
<td></td>
<td>-</td>
<td>-</td>
<td>51.6241</td>
<td>358.7009</td>
<td></td>
</tr>
<tr>
<td>SMC-PID</td>
<td></td>
<td>1118.2151</td>
<td>0.000073</td>
<td>4.0390734</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

### 6. Conclusions

The comprehensive study of the complex controller design is required which usually produces a satisfactory outcome, especially dealing with the system that is complex with uncertainties such as EHA system. When it’s come to the industrial field, the PID controller is usually used, which is much easier and simple to be designed. Depends on the required outcome, if the high precision result is required, the PID controller might be unable to achieve the required objective. This paper intends to assess the performance of the common use PID controller, the improved PID controller named fractional order PID controller, the conventional SMC, and also the SMC-PID controller applied to the EHA system. It can be inferred from the results, although the FO-PID controller and the conventional SMC are capable to outperform the conventional PID controller, it is still unable to surpass the capability of the SMC-PID controller. Apart from using the PSO computational tuning method, the performance of these controllers might be enhanced through different computational tuning methods. Therefore, further investigation regarding the computational tuning algorithm which can be applied in the practical system will be carried out. Whether to enhance the performance of the particular system, or to enhance the capability of the controller that is able to apply to numerous applications.

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