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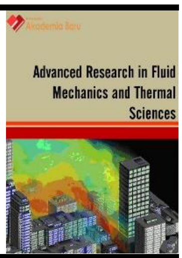

Study of Maxwell Nanofluid Flow Over a Stretching Sheet with Non-uniform Heat Source/Sink with External Magnetic Field

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ABSTRACT

In this paper we study the effect of Brownian motion and Thermophorosis on Maxwell nanofluid flow over a linearly stretching sheet under the influence of oblique external magnetic field. The impact of cross diffusion, frictional heat and irregular heat are considered to be prominent. The effect of inclination of magnetic field and non-uniform heat source/sink on fluid velocity, temperature and nanoparticle concentration are analyzed and depicted in graphs. It is observed that as inclination of magnetic field increases, temperature and nanoparticle concentration also increases whereas fluid velocity decreases. The effect of Dufour number D, Eckrect number Ec, Prandtl number Pr, Brownian motion parameter N_b, Thermophorosis parameter N_t, and Lewis number Le are also studied.

Keywords:
MHD, Thermophorosis, Brownian motion, Maxwell fluid, Heat transfer, nanofluids, Stretching sheet, Heat source/sink

1. Introduction

Recently nanotechnology has invented nanofluids discovered by Choi and Jeffrey [5] which are made of nanoparticles (<100nm) suspended in a base fluids such as water, oil and ethylene glycol in which the nano layer acts as a thermal bridge between solid nanoparticle and base fluid. Due to the presence of nanoparticles Brownian diffusion and heat transfer takes places in nanofluids. Hence nanofluids are highly conducting heat transfer fluids which enhance the efficiency of large scale heat exchangers used in chemical processing plants, smaller scale heat exchangers used in automotives. Ahmadreza [27] noted that due to their higher thermal conductivity nanofluids are used for industrial cooling applications resulting in to energy savings. Nura Mu'az Muhammad and Nor Azwadi Che Sidik [47] noted the applications of nanofluids and various minichannel configurations for heat transfer improvement. Nanofluids have heat transfer applications as a microelectronic fuel cells, electronic cooling, domestic refrigerator chillers, solar water heating chillers, heat pipes, lubrication, oil

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recovery, detergency and processes of soil remediation,. Husam Abdulrasool Hasan *et al.,* [49] discussed the heat transfer enhancement using nanofluids for Cooling a Central Processing Unit (CPU) System. Due to their higher thermal conductivity nanofluids are used for liquid cooling computer processors.

Nanofluids also have Bio-medical application such as drug delivery, hyperthermia, and magnetic cell separation. Some nanofluids are used in cancer imaging in a cancer therapy. Since magnetic nanoparticles are more adhesive to tumour cells without damaging nearby healthy tissues, nanofluids allow doctor to deliver high local doses of drugs or radiation to the patient. Nano-cryosurgery is one of the recent Bio-medical applications, effective treatment to kill cancer cell without any side effects with lower cost and safe recovery to the patient. Jing Liu and Zhong-Shan Deng [10] and Chandran [24] discussed the surgical procedure of cancer treatment for minimizing the complication of surgery and kill the tumours within the target region. Imaging technology is one of the advantages in cryosurgery along with the treatment of skin cancer, brain, breast, liver, lung, prostate tumour, glaucoma etc.

A fluid which obeys the Newton law of viscosity are called Newtonian fluid, on the other hand in non-Newtonian fluid the relationship between stress and rate of strain are not linear. Some of the non-Newtonian fluids are fruit juices, printer ink, polymers, suspension of starch and sand in water, blood, and gypsum pastes. Because of biological, engineering and industrial applications non-Newtonian nanofluid flow and heat transfer over stretching sheets has considerable interest. Crane [2] investigated the concept of fluid flow caused by the stretching sheet. Qasim [29] observed the effect of heat and mass transfer in Jeffery fluid over stretching sheet. Sakiadis [1] first presented the concept of boundary layer flow over a moving surface. Many researchers are extended the idea of boundary layer flow over a stretching sheet. Bachok *et al.,* [11], Olanerwaju*et al.,* [17], Hamed [13], Makinde and Aziz [14] etc. Aminreza Noghrehabadi and Amin Samimi [18] observed that Brownian motion effect and thermophorosis effect which improves boundary layer flow of nanofluid over a moving surface. Sugunamma *et al.,* [34] analyzed the flow and heat transfer characteristic of a nanofluid in a porous medium bounded by a moving vertical semi-infinite permeable flat plate.

MHD boundary layer flow of heat and mass transfer have many physical and engineering applications such as plasma physics, packed-bed catalytic reactors, thermal insulation, geothermal reservoirs etc**.** Rami [6] studied MHD flow in presence of thermal radiation Anuar Ishak [15] studied the effect of radiation on MHD boundary layer flow of a viscous fluid. Khan *et al.,* [19] studied the MHD heat and mass transfer nanofluid flow on a moving surface.

Viscosity of a fluid is the measure of its resistance to gradual deformation by shear stress. Elasticity is the ability of a body to resist a distorting influence and to get back its shape and size when force is removed. Viscoelasticity is the property of a material during the deformation it exhibits both viscous and elastic characteristic. Fluids like soap solution, polymers which have elastic properties as well as fluid properties. Such fluids are called viscoelastic fluids. Rathy [3] illustrated viscoelastic model by a spring and dashpot assembly. He explained that restoring force is directly proportional to the extension of the spring. The more we stretch a spring the harder it pushes back. The elastic properties of a material can be represented by a spring and the viscous properties by a dashpot. Maxwell model is the combination of spring and the dashpot such that the same force acts on a spring as well as the dashpot. This model can be applied to fluids by replacing force by stress tensor and rate of extension by strain tensor, we get Maxwell fluid. The study of this fluid is more interesting and informative.

Mustafa *et al.,* [37] analyzed the steady flow of visco-elastic Maxwell nanofluid induced by an exponentially stretching sheet subject to convective heating. Muhammad Awais *et al.,* [38] analyzed the comparison of analytic and numeric solutions of heat generation/absorption effects in a boundary layer flow of Maxwell nanofluid. Ramzan *et al.,* [39] studied the effect of Soret/Dufour and mixed convective flow of Maxwell nanofluid past a porous vertical stretched surface. Sidra Aman *et al.,* [40] analyzed heat transfer enhancement in free convection flow of Carbon nanotubes Maxwell nanofluids with four different types of molecular liquids. Hloniphile Sithole *et al.,* [41] analyzed an unsteady MHD Maxwell nanofluid flow over a shrinking sheet with convective boundary conditions using spectral local linearization method. Ramana Reddy *et al.,* [42] studied the effect of cross diffusion on non-Newtonian fluids flow past a stretching sheet with non-uniform heat source/sink Asim Aziz *et al.,* [43] analyzed the heat transfer capabilities and the energy generation of non-Newtonian Maxwell nanofluid in presence of slip and convective boundary condition. Alireza Rahbari *et al.,* [44] have done the comparison of semi-analytical and numerical solution of heat transfer and MHD flow of non-Newtonian Maxwell fluid through a parallel plate channel. Saraswathi *et al.,* [45] studied the effects of heat source/sink and chemical reaction on MHD Maxwell nanofluid over a convectively stretching sheet using Homotopy Analysis Method.In this paper we applied semi analytical method HAM to Maxwell nanofluid flow over a linearly stretching sheet under the influence of external magnetic field and non-uniform heat source/sink. Hamzeh Taha Alkasasbeh [48] solved the nonlinear system of partial differential equations with boundary conditions of the Micropolar Casson fluid behaviour on steady MHD natural convective flow about a solid sphere by the Keller-box method. Saidu Bello AbuBakar *et al.,* [46] discussed the numerical Prediction of laminar nanofluid flow in Rectangular Micro channel.

2. Basic Equations

Consider the 2D flow of non-Newtonian Maxwell nanofluid past a linearly stretching surface. u is the velocity in the direction of x-axis parallel to the flow surface, v is the velocity in the direction of y-axis is orthogonal to it as in the physical model. It is assumed that wall stretches with velocity given by $U_w(x)$ = ax where $a > 0$ is the stretching rate. A magnetic field B is applied oblique to the sheet. In this analysis we ignored induced magnetic field, Joule heating and electric fields. The impact of cross diffusion, frictional heat and irregular heat are retained. Let T and C are temperature of the fluid and nanoparticle concentration of the fluid respectively. Let T_0andC_0 are the reference temperature of fluid and nanoparticle concentration where as $T_\infty$and C_∞ taken to be ambient temperature of fluid and nanoparticle concentration. The governing equations of the flow are taken as discussed by Ramana Reddy *et al.,* [45], Hayet *et al.,* [9], Abel *et al.,* [21], and Raju *et al.,* [38]. By taking $\gamma \to \infty$where γ is the Casson nanofluid parameter and the relaxation time$\delta \neq 0$. The geometry of flow is as shown in Figure 1, continuity, momentum, energy and nanoparticle concentration equation with the effect of cross diffusion on MHD non-Newtonian fluid flow past a stretching sheet with non uniform heat source/sink are given by

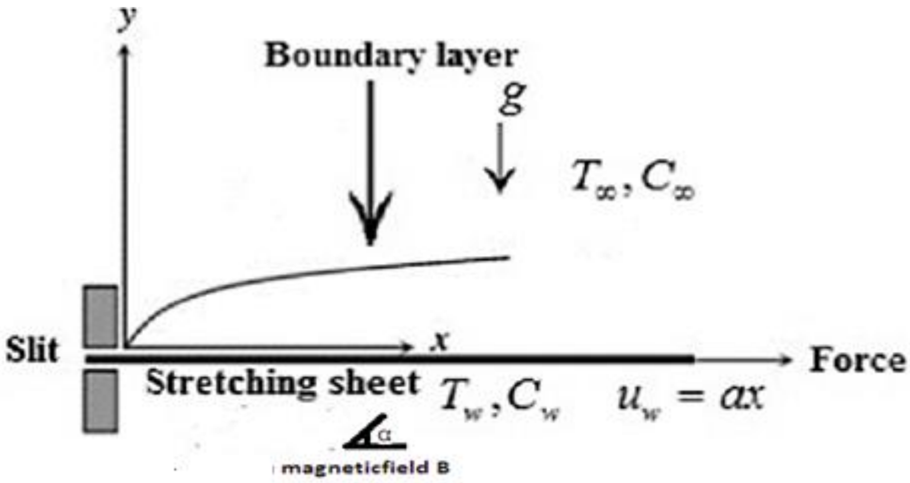

Fig.1. Physical model and co-ordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \rho\delta\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) - \sigma B^2 Sin^2\alpha\left(u + \delta v\frac{\partial u}{\partial y}\right) + g\beta_T\rho(T - T_\infty) + g\beta_c\,\rho(C - C_\infty) \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \rho\frac{D_T}{C_s}\frac{\partial^2 C}{\partial y^2} + Q^* \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$v$is the kinematic viscosity coefficient, $k$is the thermal conductivity, D_B the Brownian diffusion coefficient, $D_T$the Thermophoresis diffusion coefficient, $B$is the transverse magnetic field strength of the base fluid, α is the inclined angle of the magnetic field, g is the gravitational force, σis theelectrical conductivity of the base fluid, $\tau$is the ratio of the nanoparticle heat capacity and the base fluid heat capacity, N_b is the Brownian motion parameter, N_t is the Thermophoresis parameter, C_f is theSkin-friction coefficient,$C_s$the Concentration susceptibility, a is the Stretching rate, λ is the thermal Buoyancy parameter, N is the Buoyancy ratio parameter, Ec is the Eckert number, D is the Dufour number, M is the magnetic field parameter, $\beta_1$is Deborah number, β_T and β_c are Thermal and concentration expansion co-efficient, s1 and s2 are thermal and concentration stratification parameters.

The associated boundary conditions are taken as

$$v = 0, U = U_W(x) = ax,\; T = T_W = T_0 + bx, C = C_W = C_0 + cx \; at \; y = 0 \tag{5}$$

$$U \to 0,\; T \to T_\infty = T_0 + px, C \to C_\infty = C_0 + qx \; as \; y \to \infty \tag{6}$$

where a, b, c, p, q are constantsIn Eq. (3) Q^* denote the non-uniform heat source or sink

$$Q^* = \frac{kU_W(x)}{xv}\left[A^*(T_W - T_0)\frac{\partial f}{\partial \eta} + (T - T_\infty)B^*\right] \tag{7}$$

A^* and B^* are the non-uniform heat source/sink parameters, Here Q^* is the heat sourcing if $A^* > 0, B^* > 0$ and Q^* is the heat sink if $A^* < 0, B^* < 0$.

To reduce the governing equations into a system of ordinary differential equations, we introduce the following Similarity transformations,

$$\Psi = (av)^{\frac{1}{2}} xf(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_0} \tag{8}$$

$$\varphi(\eta) = \frac{C-C_\infty}{C_w-C_0}, \qquad \eta = (a/v)^{1/2} y \tag{9}$$

where $f(\eta)$,$\theta(\eta)$and $\varphi(\eta)$ are the dimensionless stream function, temperature, and nanoparticle concentration respectively and η is the similarity variable.The stream function Ψ is defined as

$$u = \frac{\partial \Psi}{\partial y} \text{and } v = -\frac{\partial \Psi}{\partial x} \tag{10}$$

Using similarity transformation and associated boundary conditions, the continuity equation is identically satisfied. Momentum equation, energy equation and nanoparticle concentration reduces to ODE. The governing coupled non linear equations for this problem is

$$f''' + \beta_1(2f'f'' - f^2f''') + (1 + M\beta_1 \mathrm{Sin}^2(\alpha))(ff'') - M\mathrm{Sin}^2(\alpha)(f') - (f')^2 + \lambda\theta + \lambda N\varphi = 0, \tag{11}$$

$$\theta'' + \Pr f\theta' + \Pr Ec \left(f''\right)^2 + \Pr D\varphi'' + A^* f' + B^*\theta = 0 \tag{12}$$

$$\varphi'' + Le\, f\varphi' + \frac{N_t}{N_b}\theta'' = 0 \tag{13}$$

$$f(0) = 0, f'(0) = 1, f' \to 0 \text{ as } \eta \to \infty \tag{14}$$

$$\theta(0) = 1 - s1, \theta \to 0 \text{ as } \eta \to \infty \tag{15}$$

$$\varphi(0) = 1 - s2, \varphi \to 0 \; as \; \eta \to \infty \tag{16}$$

Here $f'$is the dimensionless fluid velocity, θ is the dimensionless temperature and φ is the nanoparticle concentration respectively. Pr the Prandtl number, Le Lewis number, q_w is the heat flux, q_m Mass flux, Re_x is the Local Renolds number where

$$\Pr = \frac{\mu C_p}{k}, \quad Le = \frac{\nu}{D_B}, \beta_1 = \delta a \tag{17}$$

$$M = \frac{\sigma B^2}{\rho a}, \quad \lambda = \frac{g\beta_T(T_w-T_0)}{au_w}, \quad Ec = \frac{x\sigma^2}{C_p(T_w-T_0)} \tag{18}$$

$$N_b = \frac{(\rho c)_p D_B (C_w-C_0)}{\nu(\rho c)_f}, \quad N_t = \frac{(\rho c)_p D_T(T_w-T_0)}{\nu(\rho c)_f T_\infty}, \quad N = \frac{\beta_C(C_w-C_0)}{\beta_T(T_w-T_0)} \tag{19}$$

$$C_f = \frac{\tau_w}{\rho u^2}, \quad D = \frac{D_T(C_w-C_0)}{C_s C_p \nu(T_w-T_0)} \tag{20}$$

$$\tau_w = \mu(\frac{\partial u}{\partial y})_{y=0},\ q_w = -k(\frac{\partial T}{\partial y})_{y=0},\ q_m = -\mathrm{D_B}(\frac{\partial C}{\partial y})_{y=0} \tag{21}$$

local Renolds number

$$\mathrm{Re_x} = \frac{a^2x^2}{b} \tag{22}$$

using similarity variables in $\mathrm{C_f}$,we get dimensionless form as

$$C_f = (\mathrm{R}e_{\mathrm{x}})^{\frac{1}{2}}\mathrm{f}''(0) \tag{23}$$

Limiting case: Ramana Reddy *et al.,* [45] explain that the comparative study of two different fluid flows by taking $\gamma \to \infty$and the relaxation time $\delta \neq 0$ for Maxwell fluid whereas for Casson fluid γ is very small and the relaxation time $\delta = 0$. We have extended the study for Maxwell nanofluid by taking $\gamma \to \infty$and the relaxation time $\delta \neq 0$ which exactly matches with the previous result.

3. Methodology

In this paper we solve governing equations by using homotopy analysis method [26] which gives convergence for chosen linear operator.The governing coupled non linear equations for this problem are written as

$$N[\mathrm{f}(\eta)] = \mathrm{f}''' + \beta_1(2\mathrm{f}'\mathrm{f}'' - \mathrm{f}^2\mathrm{f}''') + (1 + \mathrm{M}\ \beta_1\mathrm{Sin}^2(\alpha))(\mathrm{ff}'') - \mathrm{MSin}^2(\alpha)(\mathrm{f}') - (\mathrm{f}')^2 + \lambda\theta + \lambda\mathrm{N}\varphi \tag{24}$$

$$N[\theta(\eta)] = \theta'' + \Pr \mathrm{f}\,\theta' + \Pr \mathrm{Ec}\,(\mathrm{f}'')^2 + \Pr D\varphi'' + \mathrm{A}^*\mathrm{f}' + \mathrm{B}^*\theta \tag{25}$$

$$N[\varphi(\eta)] = \varphi'' + \mathrm{Le}\,\mathrm{f}\,\varphi' + \frac{N_t}{N_b}\theta'' \tag{26}$$

by selecting an auxiliary linear operator for the equation (24), (25), (26) respectively as

$$\mathrm{L_f} = \frac{\partial^3}{\partial\eta^3} + \frac{\partial^2}{\partial\eta^2},\ \mathrm{L_\theta} = \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta},\ \mathrm{L_\varphi} = \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta} \tag{27}$$

consider$\mathrm{L_f}[\mathrm{f}] = 0,\ \mathrm{L_\theta}[\theta] = 0, \mathrm{L_\varphi}[\varphi] = 0$ and using boundary conditions (14), (15), (16) for f, θ, φ we get the initial approximations are $\mathrm{f_0}(\eta)$, $\theta_0(\eta)$, $\varphi_0(\eta)$ as

$$\mathrm{f_0}(\eta) = 1 - \mathrm{e}^{-\eta} \tag{28}$$

$$\theta_0(\eta) = (1 - \mathrm{s}1)\mathrm{e}^{-\eta} \tag{29}$$

$$\varphi_0(\eta) = (1 - \mathrm{s}2)\mathrm{e}^{-\eta} \tag{30}$$

Homotopy equations for (24), (25), (26) are constructed as below

$$(1-p)L_f[F(\eta,p)-f_0(\eta)] = \left\{\frac{\partial^3 F}{\partial\eta^3}+\beta_1\left(2\frac{\partial F}{\partial\eta}\frac{\partial^2 F}{\partial\eta^2}-F^2\frac{\partial^3 F}{\partial\eta^3}\right)+\left(1+M\beta_1 Sin^2(\alpha)\right)F\frac{\partial^2 F}{\partial\eta^2}-MSin^2(\alpha)\frac{\partial F}{\partial\eta}-\left(\frac{\partial F}{\partial\eta}\right)^2+\lambda G+\lambda NE\right\} \quad (31)$$

$$(1-p)L_\theta[G(\eta,p)-\theta_0(\eta)] = hp\left\{\frac{\partial^2 G}{\partial\eta^2}+PrF\frac{\partial G}{\partial\eta}+PrEc\left(\frac{\partial^2 F}{\partial\eta^2}\right)^2+PrD\frac{\partial^2 E}{\partial\eta^2}+A^*\frac{\partial F}{\partial\eta}+B^*G\right\} \quad (32)$$

$$(1-p)L_\varphi[E(\eta,p)-\varphi_0(\eta)] = hp\left\{\frac{\partial^2 E}{\partial\eta^2}+LeF\frac{\partial E}{\partial\eta}+\frac{N_t}{N_b}\frac{\partial^2 G}{\partial\eta^2}\right\} \quad (33)$$

For p=0 and p=1 we have

$$F(\eta,0)=f_0(\eta),\quad F(\eta,1)=f(\eta) \quad (34)$$

$$G(\eta,0)=\theta_0(\eta),\quad G(\eta,1)=\theta(\eta) \quad (35)$$

$$E(\eta,0)=\varphi_0(\eta),\quad E(\eta,1)=\varphi(\eta) \quad (36)$$

thus as pincreases from 0 to 1, the solution $f_0(\eta)$,$\theta_0(\eta)$, $\varphi_0(\eta)$ varies from the initial guess to the exact solution $f(\eta)$, $\theta(\eta)$, $\varphi(\eta)$.Boundary conditions are,

$$F(0,p)=0, F_\eta(0,p)=1,\ F_\eta(\infty,p)=0 \quad (37)$$

$$G(0,p)=1-s1, G(\infty,p)=0 \quad (38)$$

$$E(0,p)=1-s2, E(\infty,p)=0 \quad (39)$$

Applying Maclaurin's series expansion to $F(\eta,p)$, $G(\eta,p)$ and $E(\eta,p)$ and using (28)-(30) we get

$$F(\eta,p)=\phi_0(\eta)+\textstyle\sum_{k=1}^{\infty}\phi_k(\eta)p^k \quad (40)$$

$$G(\eta,p)=\Psi_0(\eta)+\textstyle\sum_{k=1}^{\infty}\Psi_k(\eta)p^k \quad (41)$$

$$E(\eta,p)=\xi_0(\eta)+\textstyle\sum_{k=1}^{\infty}\xi_k(\eta)p^k \quad (42)$$

The convergence region of the above series depends upon the auxiliary linear operator L, and the non-zero auxiliary parameter h which are to be selected such that solution converges at p = 1 and hence we get

$$f(\eta)=\phi_0(\eta)+\textstyle\sum_{k=1}^{\infty}\phi_m(\eta) \quad (43)$$

$$\theta(\eta)=\Psi_0(\eta)+\textstyle\sum_{k=1}^{\infty}\Psi_m(\eta) \quad (44)$$

$$\varphi(\eta)=\xi_0(\eta)+\textstyle\sum_{k=1}^{\infty}\xi_m(\eta) \quad (45)$$

Differentiating equation (31), (32) and (33) m times about the embedding parameter p, using Leibnitz theorem and setting p = 0, dividing by m! We get the following equations.

$$L[\phi_m - \chi_m \phi_{m-1}] = hR_m(\eta) \tag{46}$$

$$L[\Psi_m - \chi_m \Psi_{m-1}] = hS_m(\eta) \tag{47}$$

$$L[\xi_m - \chi_m \xi_{m-1}] = hT_m(\eta) \tag{48}$$

where $\chi_m = \begin{cases} 0 & \text{when } m \le 1 \\ 1 & \text{when } m > 1 \end{cases}$ and

$$R_m(\eta) = \phi'''_{m-1}(\eta) + \beta_1 \sum_{k=0}^{m-1} \phi'_{m-1-k}(\eta)\phi''_k(\eta) - \beta_1 \sum_{k=0}^{m-1} \phi'''_{m-1-k}(\eta) \sum_{r=0}^{k} \phi_{k-r}(\eta)\phi_r(\eta) - M\,\mathrm{Sin}^2(\alpha)\phi'_{m-1}(\eta) + \sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\phi''_k(\eta) + M\beta_1 \mathrm{Sin}^2(\alpha) \sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\phi''_k(\eta) - \sum_{k=0}^{m-1} \phi'_{m-1-k}(\eta)\phi'_k(\eta) + \lambda\Psi_{m-1} + \lambda N\xi_{m-1} \tag{49}$$

$$S_m(\eta) = \Psi''_{m-1}(\eta) + Pr\sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\Psi'_k(\eta) + PrEc\sum_{k=0}^{m-1} \phi''_{m-1-k}(\eta)\phi''_k(\eta) + PrD\,\xi''_{m-1}(\eta) + A^*\phi'_{m-1}(\eta) + B^*\Psi_{m-1} \tag{50}$$

$$T_m(\eta) = \xi''_{m-1}(\eta) + Le\sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\xi'_k(\eta) + \frac{N_t}{N_b}\Psi''_{m-1}(\eta) \tag{51}$$

along with boundary conditions

$$\phi_m(0) = 0,\ \phi'_m(0) = 0,\ \phi'_m(\infty) = 0 \tag{52}$$

$$\Psi_m(0) = 0, \Psi_m(\infty) = 0 \tag{53}$$

$$\xi_m(0) = 0, \xi_m(\infty) = 0 \tag{54}$$

we solve these non-linear equations given by (48), (49) and (50) for ϕ_m,Ψ_m, ξ_m by MATHEMATICA. Using these coefficients in (45), (46) and (47) we get the solution of the given equations. Data of above solutions are analyzed through graphs for different characteristic parameters.

4. Result and Analysis

The solutions for the (51)-(53) system of equation with corresponding boundary conditions are obtained by homotopy analysis method. With the help of Mathematica equations are solved and discussed through graphs. The variations of velocity, temperature and nanoparticle fraction are discussed for different values of N, Ec, M, Pr, D, N_b, N_t, Le, A^*, B^*, α, λ. In Figure 2-4 we have observed that the increase in the values of α suppress the fluid velocity where as enhance the temperature of the fluid and nanoparticle fraction. In Figure 5-7, shows that velocity of the fluid increases as buoyancy ratio parameter N increase, but higher the value of N suppresses the fluid temperature and nanoparticle concentration. In Figure 8-10 as fluid moves fast, higher the value of Eckeret number Ec, fluid velocity and temperature of the fluid increases due to the frictional heat but we notice that the reduction in the nanoparticle concentration as Eckret number Ec increases. In Figure 11-13 we notice that the A^* enhances the thermal boundary layer thickness and it acts as an agent to generate heat. Hence magnification in velocity and temperature distributions are observed but fluid concentration decreases by increasing in the value of A^*.In Figure 14 Brownian motion parameter N_b increases the nanoparticle concentration. In Figure 15 higher the Dufour values temperature of the fluid increases. In Figure 16 higher the value of thermophrosis parameter N_t lowers the nanoparticle concentration. In Figure 17 we observed nanoparticle fraction decreases as Lewis number increases.

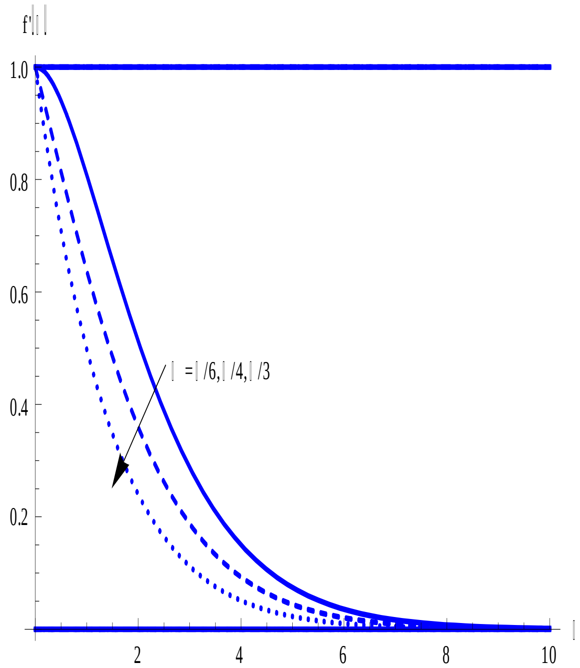

Fig. 2. Velocity profiles for different values of α

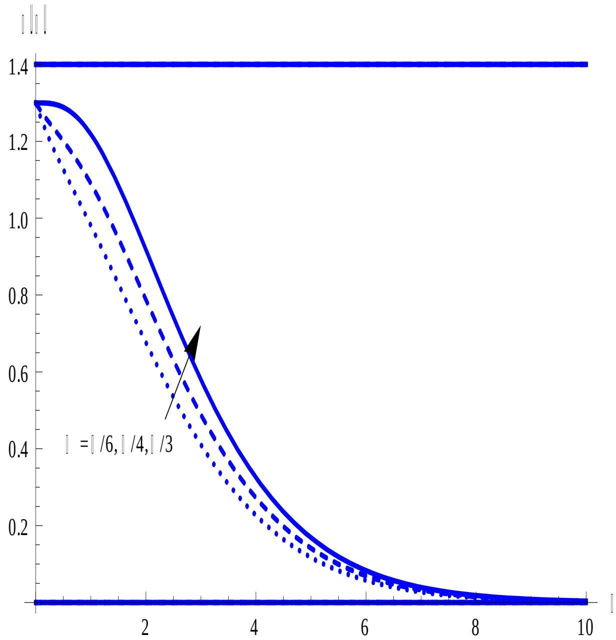

Fig. 3. Temperature distribution for different values of α

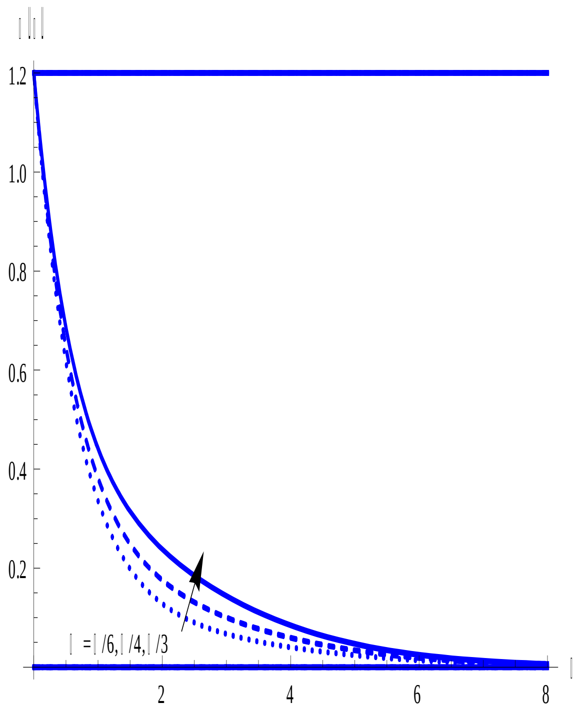

Fig. 4. Nanoparticle fraction for different values of α

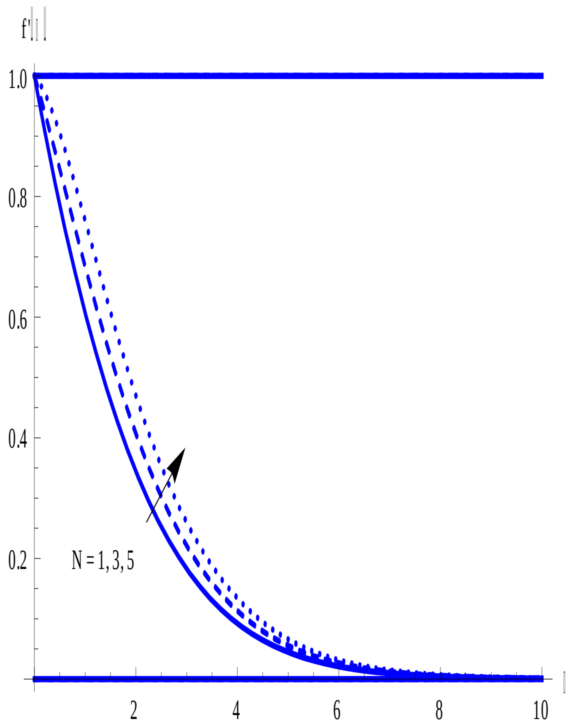

Fig. 5. Velocity profiles for different values of N

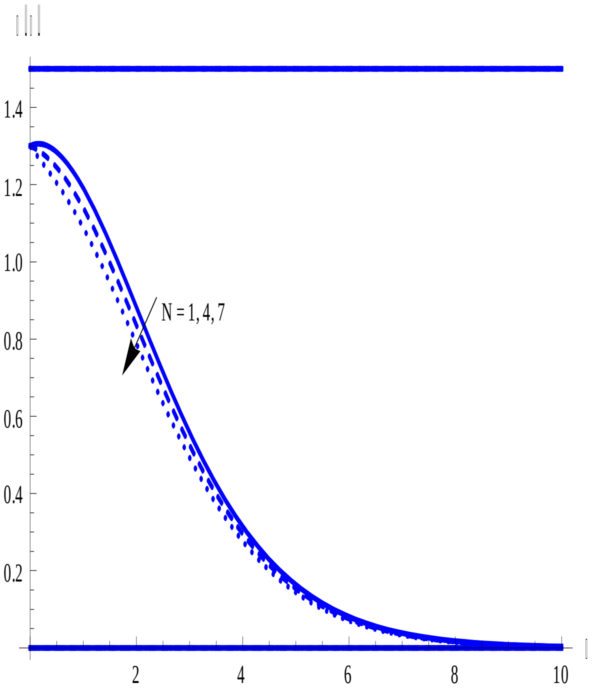

Fig. 6. Temperature distribution for different values of N

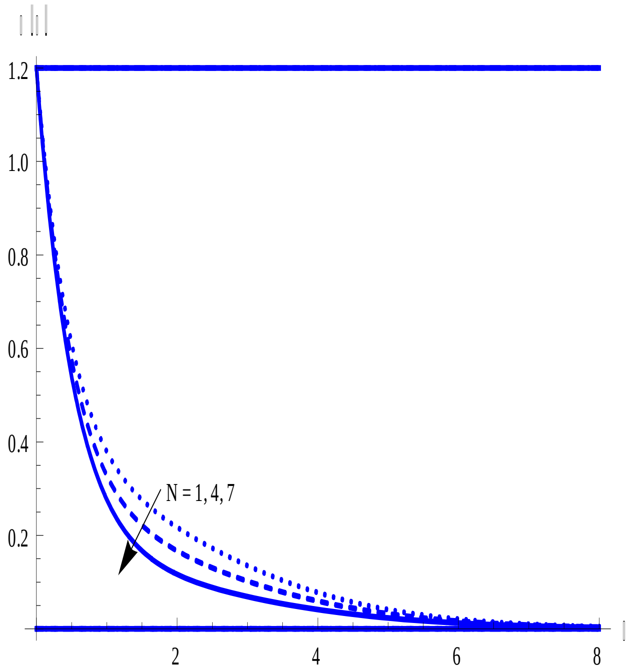

Fig. 7. Nanoparticle fraction for different values of N

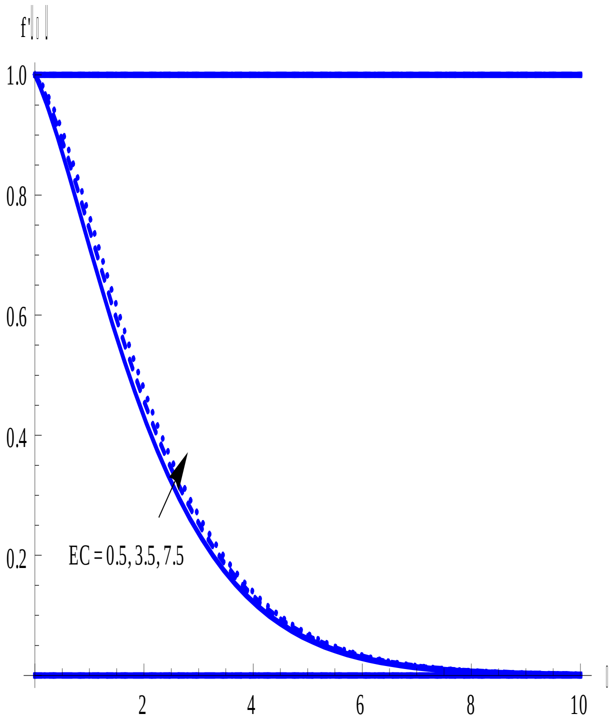

Fig. 8. Velocity profiles for different values of Ec

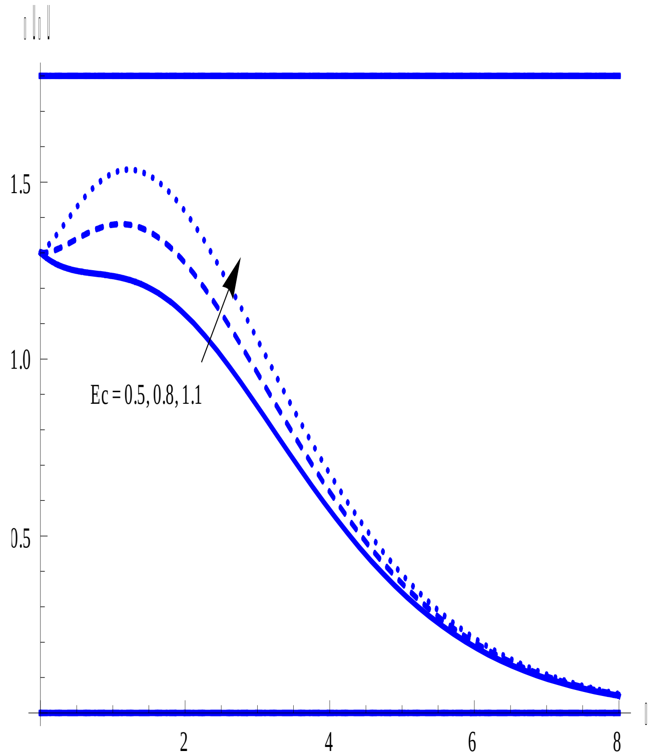

Fig. 9. Temperature distribution for different values of Ec

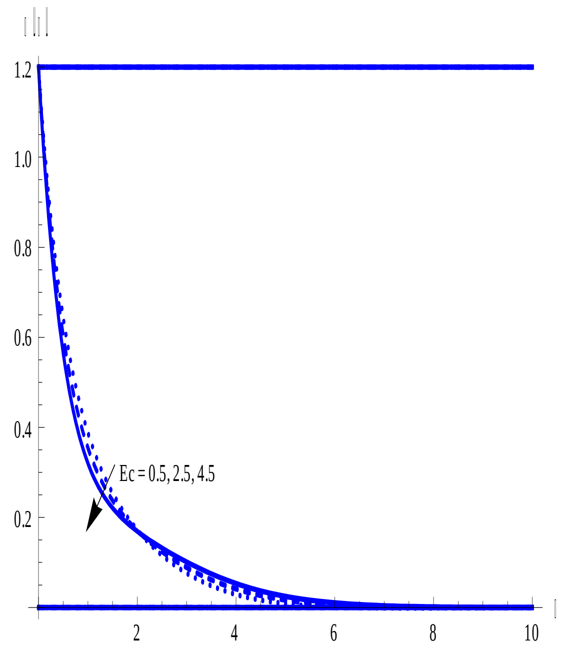

Fig. 10. Nanoparticle fraction for different values of Ec

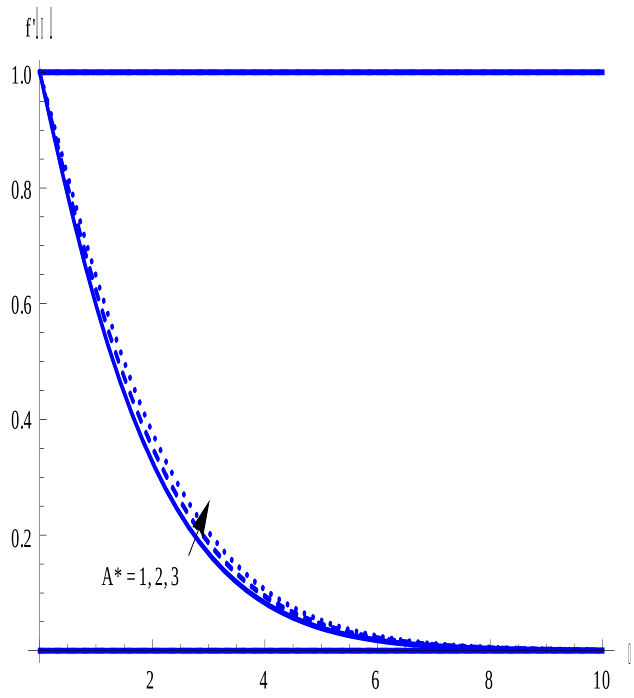

Fig. 11. Velocity profiles for different values of A*

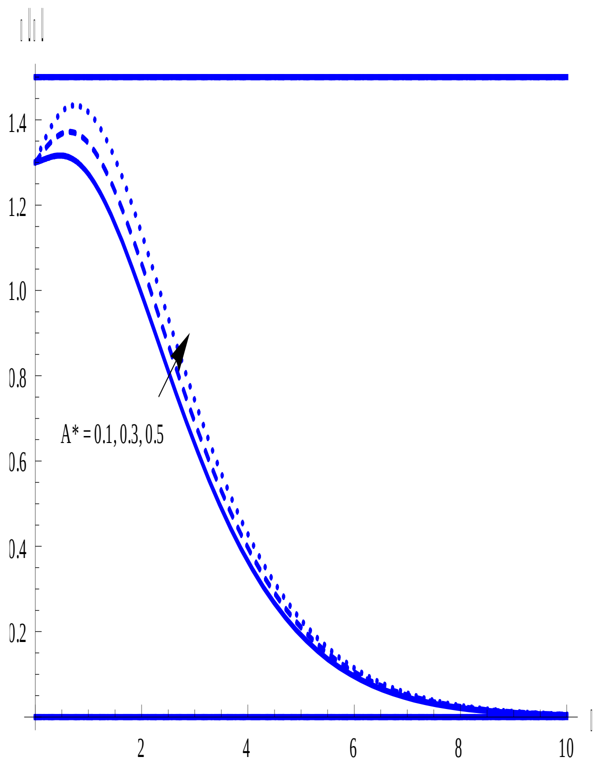

Fig. 12. Temperature distribution for different values of A*

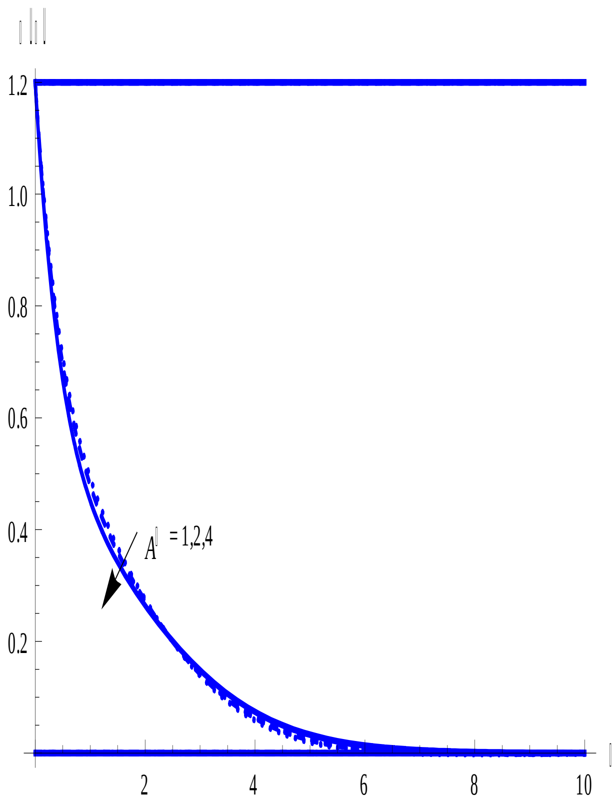

Fig. 13. Nanoparticle fraction for different values of A*

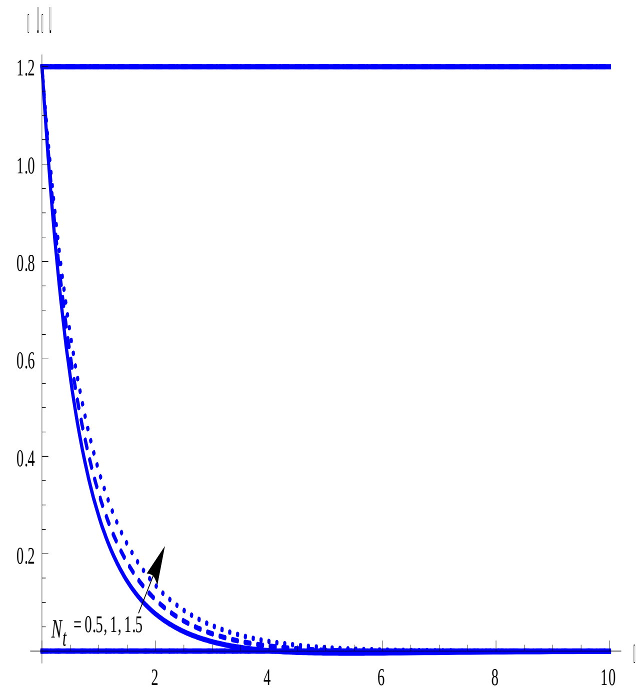

Fig. 14. Nanoparticle fraction for different values of N_t

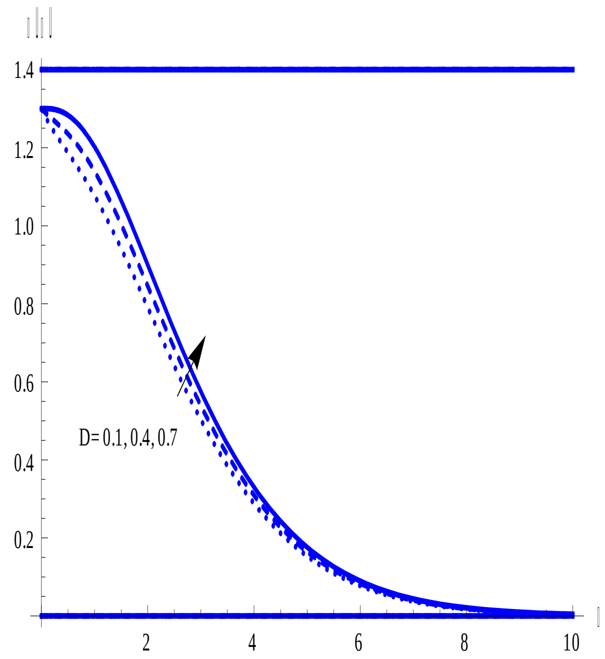

Fig. 15. Temperature distribution for different values of D

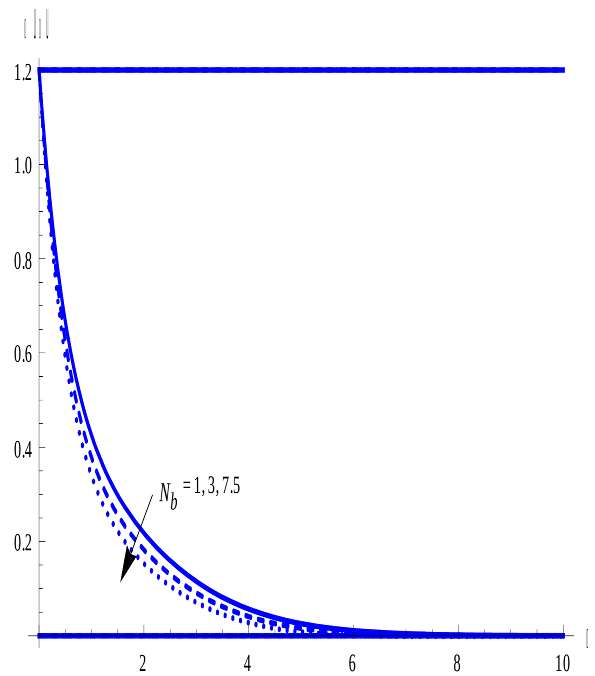

Fig. 16. Nanoparticle fraction for different values of N_b

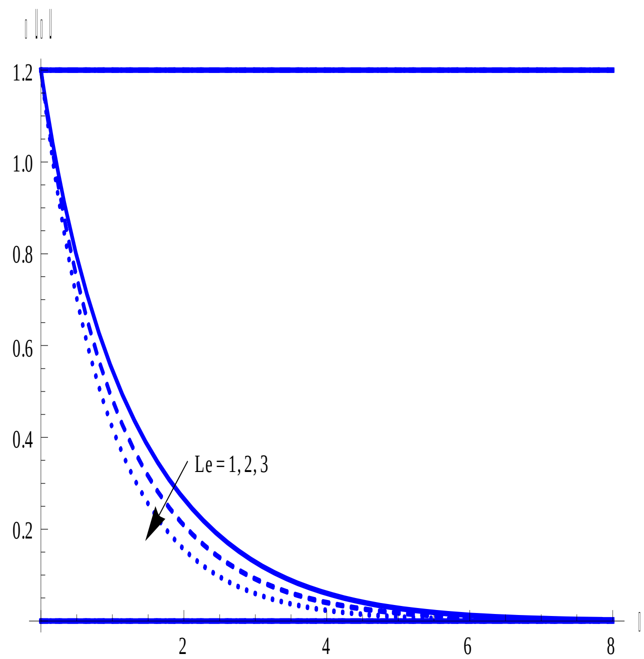

Fig. 17. Nanoparticle fraction for different values of Le

5. Conclusions

In this study we analysed that the homotopy analysis method works well for non-linear differential equations. We have proved that homotopy analysis method solution exactly matches with numerical result obtained by J. V. Ramana Reddy *et al.,* [42] in which Runge-Kutta- Fehelberg integration scheme is applied directly to ODE. We have checked with R-K Merson method also.

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