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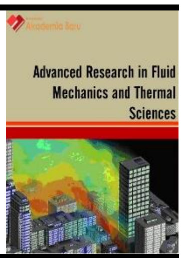

Effect of Nonlinear Temperature Profile on Thermal Convection in a Binary Fluid Saturated an Anisotropic Porous Medium

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ABSTRACT

The thermal convection in a horizontal binary fluid layer saturated an anisotropic porous medium with the effect of non-uniform temperature profile is studied analytically by linear stability analysis. The generalized eigenvalues problem subjected to various boundaries conditions are solved numerically using Galerkin method. The effects of solute Rayleigh number, Lewis number, mechanical anisotropy and thermal anisotropy parameters corresponding to the six basic temperature profiles on the stationary thermal convection are shown graphically. It is found that the effects of thermal anisotropic parameter, Lewis number and solute Rayleigh number are to reinforce the stability of the system while the effect of mechanical anisotropic parameter is to advance the onset of convection.

1. Introduction

The studies of convection driven due to density difference also known as thermal convection, Rayleigh Bénard convection or natural convection in a fluid saturated a porous medium has received many attentions of scientists and researchers due to its variety application in a real life. Bénard [1] was the first who scrutinize the thermal convection in an ordinary fluid layer. When a horizontal fluid layer is heated from below, it will cause the warmer fluid to rise up and the colder fluid to sink down due to the fact that the density of warmer fluid is less dense than the density of colder fluid. Thus, it will result in heat distribution vertically inside the system or in other words we say convection occur. The studies of convection in an isotropic porous medium has been considered by and Shivakumara *et al.,* [2] and Mokhtar *et al.,* [3]. Later, Mokhtar *et al.,* [4] studied the onset of convection in a fluid overlying an isotropic porous medium in the presence of internal heat generation.

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The studies of convection in a homogeneous isotropic porous medium may be relatively simple but it is also rather unphysical due to the assumption that it has no preferred direction. In real fields, almost all the porous mediums have an anisotropy property in a mean gradient of pressure, temperature or mass which will be consider to govern the equation for the convection problem as reported by Leslie [5]. Degan *et al.,* [6] investigated the convection problem in a fluid saturated an anisotropic porous medium. Capone *et al.,* [7] studied the effect of variable permeability on the penetrative convection in a fluid saturated an anisotropic porous medium. Capone [8] investigated the beginning of instability in a fluid saturated an anisotropic porous medium by considering both the linear and nonlinear permeability and thermal diffusivity. Sekar *et al.,* [9] have explored the threshold of both steady and unsteady convection in a ferrofluid saturated an anisotropic porous medium in the presence of magnetic field and Soret effect. Kim [10] examined the originating of thermal convection in a fluid saturated an anisotropic porous medium by relaxed energy and solved the resulting eigenvalues numerically. Bhadauria and Kiran [11] studied the effect of temperature modulation on the onset of steady and oscillatory convection in a temperature dependent viscous fluid saturated an anisotropic porous medium. Hill and Morad [12] examined the onset of thermal convection in a fluid saturated an anisotropic porous medium in order to understand the process of carbon sequestration that take place in underground saline aquifers. Aly and Ahmed [13] studied the onset of thermal and mixed convection in a non-Darcy anisotropic porous medium.

The simultaneous presence of two component liquid chemical namely a binary fluid together with the anisotropic parameters can lead to many interesting studies on the onset of convection in the system. Malashetty and Swamy [14] studied the onset of double diffusive convection in a binary fluid saturated an anisotropic porous medium in the presence of uniform temperature profile subjected to free-fee isothermal boundary. Malashetty *et al.,* [15] analyzed the onset of marginal and oscillatory double diffusive convection in the binary viscoelastic fluid saturated an anisotropic porous medium in the presence of Coriolis force. Later, Malashetty and Kollur [16] studied the threshold of diffusive convection in a couple stress fluid saturated anisotropic porous medium. Srivastava *et al.,* [17] have explored the threshold of both steady and unsteady convection in a binary fluid an anisotropic porous medium in the presence of both magnetic field and Soret effect. Bhadauria [18] have examined the arrival of steady and unsteady double diffusive convection in a binary fluid saturated an anisotropic porous medium in the presence of internal heat generation. Altawallbeh *et al.,* [19] illustrated the onset of steady and oscillatory double diffusive convection in the binary fluid saturated an anisotropic porous medium in the presence of internal heating and Soret effect. El moustaine and Cheddadi [20] studied the onset of double diffusive convection in an annular space filled with binary fluid which bounded by two very long cylinders.

In many practical studies, one considers more the nonlinear temperature gradient cases instead of uniform profile cases. A pioneering study of a convection problem in a fluid layer in the presence of nonlinear temperature profile has been considered by Nield [21]. Siddheshwar and Paranesh [22] solved numerically the onset of steady thermal convection in a micropolar fluid in the presence of non-uniform temperature gradient by Galerkin method for the case of symmetric and non-symmetric velocity boundaries together with adiabatic or isothermal temperature boundaries. Char and Chen [23] attempted to study the effect of non-uniform temperature profile on the threshold of oscillatory mode Bénard -Marangoni convection with the presence of electric and magnetic field. Degan and Vasseur [24] studied the thermal convection in a fluid-saturated-anisotropic porous medium by considering the effect of nonlinear temperature profile. Idris *et al.,* [25] examined the threshold of Bénard Marangoni convection in a micropolar fluid in the presence of non-uniform temperature profile. Mokhtar *et al.,* [26] examined the linear stability analysis on the threshold of stationary thermal instability in an isotropic porous medium by considering the effect of non-uniform temperature profile and magnetic field and they solved the eigenvalue obtained numerically using Galerkin method for free-free and rigid-free together with adiabatic temperature boundary. Idris and Hashim [27] investigated the effect of cubic temperature gradient on the onset of Bénard Marangoni convection in a ferrofluid with the presence of feedback control. Shivakumara *et al.,* [28] studied the effect of nonlinear temperature profile on the onset of thermal convection in a couple stress fluid saturated an isotropic porous medium.

The aim of the present paper is to study the threshold of steady thermal convection in a binary fluid saturated an anisotropic porous medium. As far as our concern, there are no studies have been done to investigate the effect of nonlinear temperature profile in a binary fluid saturated an anisotropic porous medium. Therefore, we further the studies by considering nonlinear temperature profile subjected to various boundaries conditions. We applied the linear stability analysis and the resulting eigenvalue is solved using single-term Galerkin method.

2. Methodology

We consider an infinite horizontal binary fluid layer saturated anisotropic porous medium of thickness d heated from below with the gravity force, $\mathbf{g}=(0,0,-g)$ acting vertically downward on it. The fluid are subjected to uniform adverse vertical temperature profile, $\Delta T = T_l - T_u$, where $T_l > T_u$, and concentration profile, $\Delta S = S_l - S_u$, where $S_l > S_u$, are maintained between the plane. The porous medium is assumed to have a vertical anisotropy and a horizontal isotropy property in terms of mechanical and thermal parameters. Following Altawallbeh *et al.,* [19], the governing equations based on the Boussinesq approximation are

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\rho_0}{\phi}\frac{\partial \mathbf{u}}{\partial t} + \nabla p + \frac{\mu}{\mathbf{K}} \cdot \mathbf{u} - \rho \mathbf{g} = 0, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa_T \left(\nabla^2 T\right), \tag{3}$$

$$\phi \frac{\partial S}{\partial t} + (\mathbf{u} \cdot \nabla) S = \kappa_s \left(\nabla^2 S\right), \tag{4}$$

where, $\mathbf{u}=(u,v,w)$ is the velocity vector, p is the pressure, ϕ is the porosity, $\mathbf{K} = K_x(\mathbf{ii}+\mathbf{jj}) + K_z(\mathbf{kk})$ is the is the permeability tensor, μ is the dynamic viscosity, γ is the ratio of heat capacity, T is the temperature, $\kappa_T = \kappa_{T_x}(\mathbf{ii}+\mathbf{jj}) + \kappa_{T_z}(\mathbf{kk})$ is the thermal diffusivity tensor , S is the solute concentration and κ_s is the solute diffusivity.

The density of the fluid is assumed to vary linearly with temperature and solute concentration

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \beta(S - S_0)\right], \tag{5}$$

where, ρ_0 and T_0 is the reference density and temperature respectively, α and β is the coefficient of thermal and solute expansion respectively.

We assumed that the basic state of the fluid to be motionless which takes the form

$$\mathbf{u}_b=(0,0,0), p=p_b(z), T=T_b(z), \frac{-d}{\Delta T}\frac{dT_b}{dz}=f(z), S=S_b(z), \rho=\rho_b(z), \tag{6}$$

where, $f(z)$ is a non-dimensional temperature gradient which satisfy the following condition

$$\int_0^1 f(z)dz=1. \tag{7}$$

Substitute Eq. (6) into Eq. (1) – (4) to obtain

$$\frac{dp_b}{dz}=-\rho_b g,\ \frac{d^2T_b}{dz^2}=0,\ \frac{d^2S_b}{dz^2}=0,\ \rho_b=\rho_0\left[1-\alpha\left(T_b-T_0\right)+\beta\left(S_b-S_0\right)\right]. \tag{8}$$

where, subscript b indicate the basic state. The solution of the conduction state of the fluid takes the form

$$T_b=\frac{-\Delta T}{d}z+T_l,\ S_b=\frac{-\Delta S}{d}z+S_l. \tag{9}$$

Subjected to an infinitesimal perturbation, the quiescent state of the fluid are given by

$$\mathbf{u}=\mathbf{u}_b+\mathbf{u}',\ T=T_b+T',\ S=S_b+S',\ p=p_b+p',\ \rho=\rho_b+\rho', \tag{10}$$

which the primes represent the infinitesimal perturbation quantities. Substituting Eq. (10) into Eq. (1) – (5) together with the basic state solution and yields

$$\nabla\cdot\mathbf{u}'=0, \tag{11}$$

$$\frac{\rho_0}{\phi}\frac{\partial\mathbf{u}'}{\partial t}+\nabla p'+\frac{\mu}{\mathbf{K}}\cdot\mathbf{u}'+\left(\alpha T'-\beta S'\right)\mathbf{g}=0, \tag{12}$$

$$\gamma\frac{\partial T'}{\partial t}+\left(\mathbf{u}'\cdot\nabla\right)T'-w'\frac{\Delta T}{d}f(z)=\kappa_{T_x}\left(\nabla_h^2T'\right)+\kappa_{T_z}\frac{\partial^2T'}{\partial z^2}, \tag{13}$$

$$\phi\frac{\partial S'}{\partial t}+\left(\mathbf{u}'\cdot\nabla\right)S'+w'\frac{dS_b}{dz}=\kappa_s\left(\nabla^2S'\right), \tag{14}$$

$$\rho'=\rho_0\left[-\alpha T'+\beta S'\right], \tag{15}$$

where, $\nabla_h^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

Eliminate the pressure term by applying the curl identity on Eq. (12) and using the following transformation

$$(x',y',z')=\left(x^*d,y^*d,z^*d\right),t=\frac{\gamma d^2t^*}{\kappa_{T_z}},p'=\frac{\mu\kappa_{T_z}p^*}{K_z},(u',v',w')=\left(\frac{\kappa_{T_z}u^*}{d},\frac{\kappa_{T_z}v^*}{d},\frac{\kappa_{T_z}w^*}{d}\right),$$
$$T'=\left(\Delta T\right)T^*,S'=\left(\Delta S\right)S^*, \tag{16}$$

to get the non-dimensional form of Eq. (11) – (15) as

$$\left[\frac{Da}{\gamma\phi\Pr}\frac{\partial}{\partial t^*}\nabla^2+\nabla_h^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^{*2}}\right]w^*-Ra\nabla_h^2T^*+Ra_s\nabla_h^2S^*=0, \tag{17}$$

$$\left[\frac{\partial}{\partial t^*}-\eta\nabla_h^2-\frac{\partial^2}{\partial z^{*2}}+\mathbf{u}^*\cdot\nabla\right]T^*-f(z)w^*=0, \tag{18}$$

$$\left[\varepsilon_n\frac{\partial}{\partial t^*}-\frac{1}{Le}\nabla^2+\mathbf{u}^*\cdot\nabla\right]S^*-w^*=0, \tag{19}$$

where, $Da=\frac{K_z}{d^2}$ is the Darcy number, $Pr=\frac{\nu}{\kappa_{T_z}}$ is the Prandtl number, $\mathrm{Ra}=\frac{\alpha \mathrm{g}\Delta \mathrm{TdK}_z}{\nu\kappa_{T_z}}$ is the thermal Rayleigh number, $Ra_s=\frac{\beta g\Delta SdK_z}{\nu\kappa_{T_z}}$ is the solute Rayleigh number, $\nu=\frac{\mu}{\rho_0}$ is the kinematic viscosity, $Le=\frac{\kappa_{T_z}}{\kappa_s}$ is the Lewis number, $\xi=\frac{K_x}{K_z}$ and $\eta=\frac{\kappa_{T_x}}{\kappa_{T_z}}$ is the mechanical and thermal anisotropy parameter respectively, $\varepsilon_n=\frac{\phi}{\gamma}$ is the normalized porosity.

We applied the linear stability analysis to eliminate the nonlinear term on Eq. (17) – (19). The vertical velocity, temperature and concentration are assumed to be periodic waves and hence we seek the solution in the form of normal mode expansion as

$$\left(w,T,S\right)=\left(W\left(z\right),\Theta\left(z\right),\Phi\left(z\right)\right)\exp\left[i\left(lx+my\right)+\sigma t\right], \tag{20}$$

where, l and m are horizontal wave number and σ is the growth rate parameter, which generally a complex quantity. Substituting Eq. (20) into the linearized version of Eq. (17) – (19), we obtained

$$\left[\frac{\sigma Da}{\gamma\phi\Pr}\left(D^2-a^2\right)+\left(\frac{D^2}{\xi}-a^2\right)\right]W+a^2Ra\Theta-a^2Ra_s\Phi=0, \tag{21}$$

$$\left[\sigma-\left(D^2-\eta a^2\right)\right]\Theta-f(z)W=0, \tag{22}$$

$$\left[\sigma\varepsilon_n-\frac{1}{Le}\left(D^2-a^2\right)\right]\Phi-W=0, \tag{23}$$

where, $D=d/dz$ and $a^2 = l^2+m^2$. Equations 21 - 23 are to be solved subjected to the following boundary condition:

(a) Rigid-rigid isothermal

$$W = DW = \Theta = \Phi = 0 \text{ at } z = 0,1. \tag{24}$$

(b) Rigid-free isothermal

$$\begin{aligned} &W = DW = \Theta = \Phi = 0 \text{ at } z = 0. \\ &W = D^2W = \Theta = D\Phi = 0 \text{ at } z = 1. \end{aligned} \tag{25}$$

(c) Lower rigid isothermal and upper free adiabatic

$$\begin{aligned} &W = DW = \Theta = \Phi = 0 \text{ at } z = 0, \\ &W = D^2W = D\Theta = D\Phi = 0 \text{ at } z = 1. \end{aligned} \tag{26}$$

(d) Lower rigid isothermal and upper rigid adiabatic

$$\begin{aligned} &W = DW = \Theta = \Phi = 0 \text{ at } z = 0, \\ &W = DW = D\Theta = D\Phi = 0 \text{ at } z = 1. \end{aligned} \tag{27}$$

A uniform and five non-uniform basic temperature gradient models are chosen as shown in Table 1.

Table 1
Six models of basic state temperature profiles

Model	Basic state temperature profile		$f(z)$
1	Linear	(Ra_{c1})	$f1 = 1$
2	Inverted parabola	(Ra_{c2})	$f2 = 2(1-z)$
3	Cubic 1	(Ra_{c3})	$f3 = 3(z-1)^2$
4	Cubic 2	(Ra_{c4})	$f4 = 0.6 + 1.02(z-1)^2$
5	Heating from below	(Ra_{c5})	$f5 = \begin{cases} \varepsilon^{-1}, & 0 \le z < \varepsilon \\ 0, & \varepsilon < z \le 1 \end{cases}$
6	Cooling from above	(Ra_{c6})	$f6 = \begin{cases} 0, & 0 \le z < 1-\varepsilon \\ \varepsilon^{-1}, & 1-\varepsilon < z \le 1 \end{cases}$

We used a single term Galerkin technique to find the eigenvalues of the Eq. (21) – (23) subject to boundary conditions 24 - 27. The basis functions of the variables are given by

$$W = \sum_{n=1}^{N} A_n W_n, \Theta = \sum_{n=1}^{N} B_n \Theta_n, \Phi = \sum_{n=1}^{N} C_n \Phi_n, \tag{28}$$

where, A_n, B_n and C_n are constants and W_n, Θ_n and Φ_n are the trial functions that satisfies the boundary condition (24) – (27). Multiplying Eq. (21) by W_m, Eq. (22) by Θ_m, Eq. (23) by Φ_m and integrate by parts the resulting equation with respect to z from 0 to 1, we obtained the system of homogeneous algebraic equations in the form

$$A_n C_{mn} + B_n D_{mn} + C_n E_{mn} = 0,$$
$$A_n F_{mn} + B_n G_{mn} + C_n H_{mn} = 0,$$
$$A_n I_{mn} + B_n J_{mn} + C_n K_{mn} = 0, \tag{29}$$

where, the coefficient C_{mn} - K_{mn} required the inner products of basic functions

$$C_{mn} = -\frac{\sigma}{\Pr}\left\langle (DW)^2 \right\rangle - \frac{\sigma a^2}{\Pr}\left\langle W^2 \right\rangle - \frac{1}{\xi}\left\langle (DW)^2 \right\rangle - a^2\left\langle W^2 \right\rangle,$$

$$D_{mn} = a^2 Ra \left\langle \Theta \cdot W \right\rangle,$$

$$E_{mn} = -a^2 Ra_s \left\langle \Phi \cdot W \right\rangle,$$

$$F_{mn} = -\left\langle f(z) W \cdot \Theta \right\rangle,$$

$$G_{mn} = \sigma\left\langle \Theta^2 \right\rangle + \left\langle (D\Theta)^2 \right\rangle + \eta a^2 \left\langle \Theta^2 \right\rangle,$$

$$H_{mn} = 0,$$

$$I_{mn} = -\left\langle W \cdot \Phi \right\rangle,$$

$$J_{mn} = 0,$$

$$K_{mn} = \sigma\left\langle \Phi^2 \right\rangle + \frac{1}{Le}\left\langle (D\Phi)^2 \right\rangle + \frac{a^2}{Le}\left\langle \Phi^2 \right\rangle. \tag{30}$$

where, $\langle \ldots \rangle$ represent the integration with respect to z from 0 to 1. We set $\sigma = 0$ to study the onset of stationary mode convection.

Equation 29 can be asserted in the form

$$LX = 0, \tag{31}$$

where, L is the determinant of $M \times M$ matrices and X is the eigenvector. Equation 30 is solved using a MAPLE software to obtain the critical Rayleigh number, Ra_c with respect to various boundary conditions for the system.

3. Results and Discussions

In this paper, we used the linear stability analysis to investigate analytically the effects of non-uniform temperature profile on the threshold of steady thermal convection in a binary fluid saturated an anisotropic porous medium. We have performed the results numerically by single-term Galerkin method with respect to boundary conditions. The critical value of thermal depth which depend on the parameters of the problem, ε for model 5 and 6 are shown in Table 2. The function of Rayleigh number, Ra are express in terms of various parameters such as solute Rayleigh number, Lewis number, mechanical anisotropy parameter, thermal anisotropy parameter and $f(z)$. The effects of various parameter on the critical Rayleigh number, Ra_c for different function of $f(z)$ are depicted in Figure 1(a) - 4(d).

Table 2
Critical thermal depth, ε_c for $Ra_s = 10$, $Le = 5$, $\xi = 0.5$ and $\eta = 0.3$. (a) rigid-rigid isothermal, (b) rigid-free isothermal, (c) rigid isothermal-free adiabatic and (d) rigid isothermal-rigid adiabatic

Boundary profiles	(a)	(b)	(c)	(d)
heating from below	0.71	0.77	0.87	0.78
cooling from above	0.71	0.66	0.55	0.64

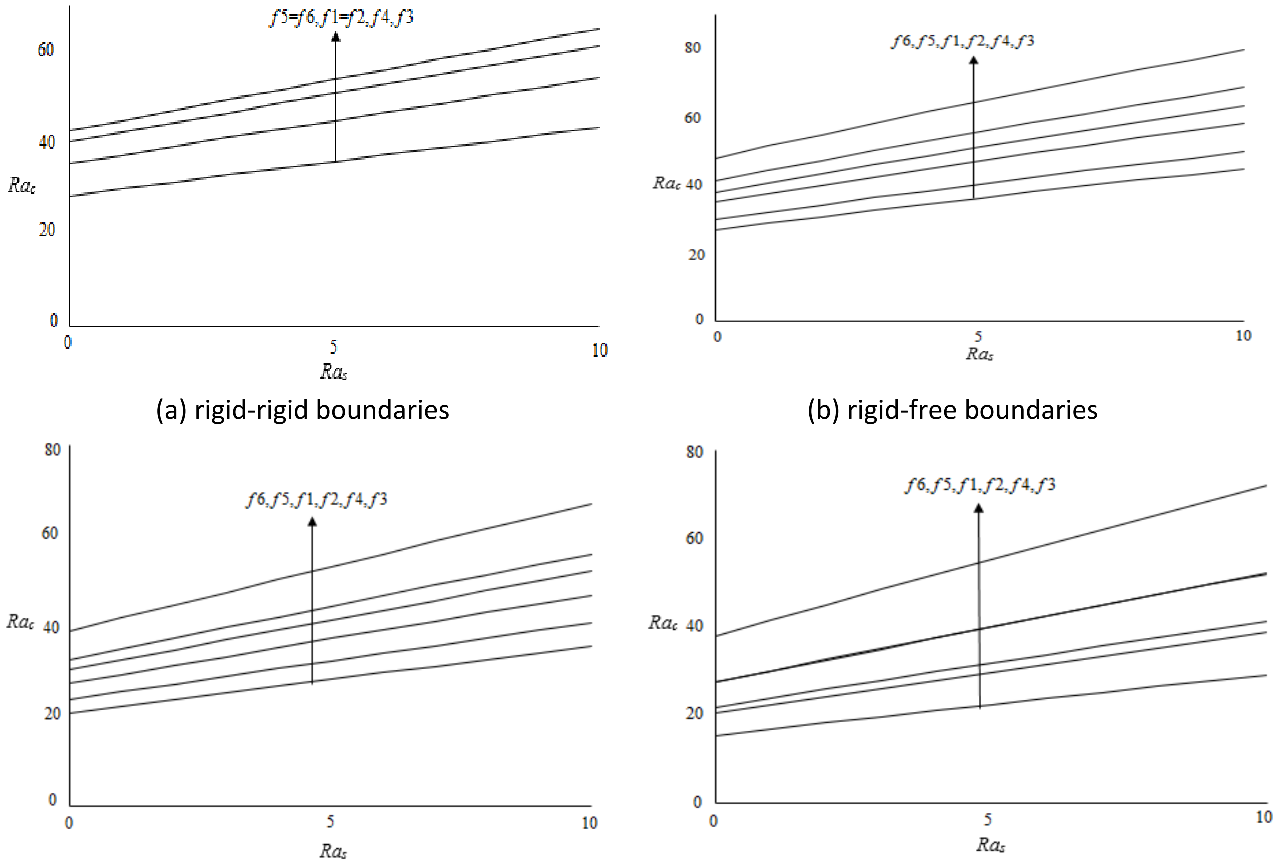
Table 3
Comparison table of Ra_c for boundary profile (a) rigid-rigid isothermal, (b) rigid-free isothermal and (c) rigid isothermal-free adiabatic

Boundary conditions	Siddheshwar and Pranesh [22]	Present
(a)	$\mathrm{Ra}_{c6}=\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}=\mathrm{Ra}_{c2}$	$\mathrm{Ra}_{c6}=\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}=\mathrm{Ra}_{c2}$
(b)	$\mathrm{Ra}_{c6}<\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}<\mathrm{Ra}_{c2}$	$\mathrm{Ra}_{c6}<\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}<\mathrm{Ra}_{c2}$
(c)	$\mathrm{Ra}_{c6}<\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}<\mathrm{Ra}_{c2}$	$\mathrm{Ra}_{c6}<\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}<\mathrm{Ra}_{c2}$

For the validity of the present studies, we have done the comparison studies between the present work and the result obtained by Siddheshwar and Pranesh [22] as shown in Table 3. Our results for the boundary conditions (a), (b) and (c) has shown a similarity with the result obtained by Siddheshwar and Pranesh [22]. In this paper, we also study the case of rigid-rigid boundaries with the lower part is isothermal and upper part is adiabatic and find that $\mathrm{Ra}_{c6}<\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}<\mathrm{Ra}_{c2}<\mathrm{Ra}_{c4}<\mathrm{Ra}_{c3}$. This is because when the plate is cooled from above (model 6) and heated from below, it caused the temperature difference between upper and lower plate to become greater. Since the rate of heat flow is directly proportional to the temperature differences, thus increasing the temperature difference will also increase the rate of heat flow in the fluid. Hence, convection set up earlier for model 6. It is observed that for all boundaries type, convection is delay for model 3.

Figure 1(a), 1(b), 1(c) and 1(d) indicates that increasing the solute Rayleigh number, Ra_s with the fixed value of other parameters will increase the Ra_c for all the six models of non-uniform temperature profile with respect to rigid-rigid isothermal, rigid-free isothermal, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic boundary conditions respectively. This can be explained by the fact that an increase in Ra_s corresponds to an increase in concentration difference between the lower and upper plate. Since the density of the fluid depend linearly on temperature and concentration, increase the concentration difference will lead to increase in the density of fluid. Thus, the movement of warmer fluid upward is resisted by the high concentration of the fluid and hence convection is delay. From Figure 1(a), it can clearly be seen that $\mathrm{Ra}_{c5}=\mathrm{Ra}_{c6}$ is the most destabilize and Ra_{c3} is the most stabilize, while in Figure 1(b), 1(c) and 1(d) Ra_{c6} is the most destabilize and Ra_{c3} is the most stabilize as we increase the value of Ra_s. The value of Rac6 with respect to the upper rigid and upper free adiabatic boundaries are smaller than the value of Ra_{c6} for the upper rigid and upper free isothermal boundaries for various value of Ra_s respectively. This is because the uses of adiabatic plate can prevent the heat loss to the surrounding during convection as reported by Reynolds *et al.,* [29] and thus enhance the onset of convection.

Figure 1(d) shows that the difference between the values of Ra_{c2} and Ra_{c4} are small enough $(\mathrm{Ra}_{c2} - \mathrm{Ra}_{c4} < 1)$.



(a) rigid-rigid boundaries

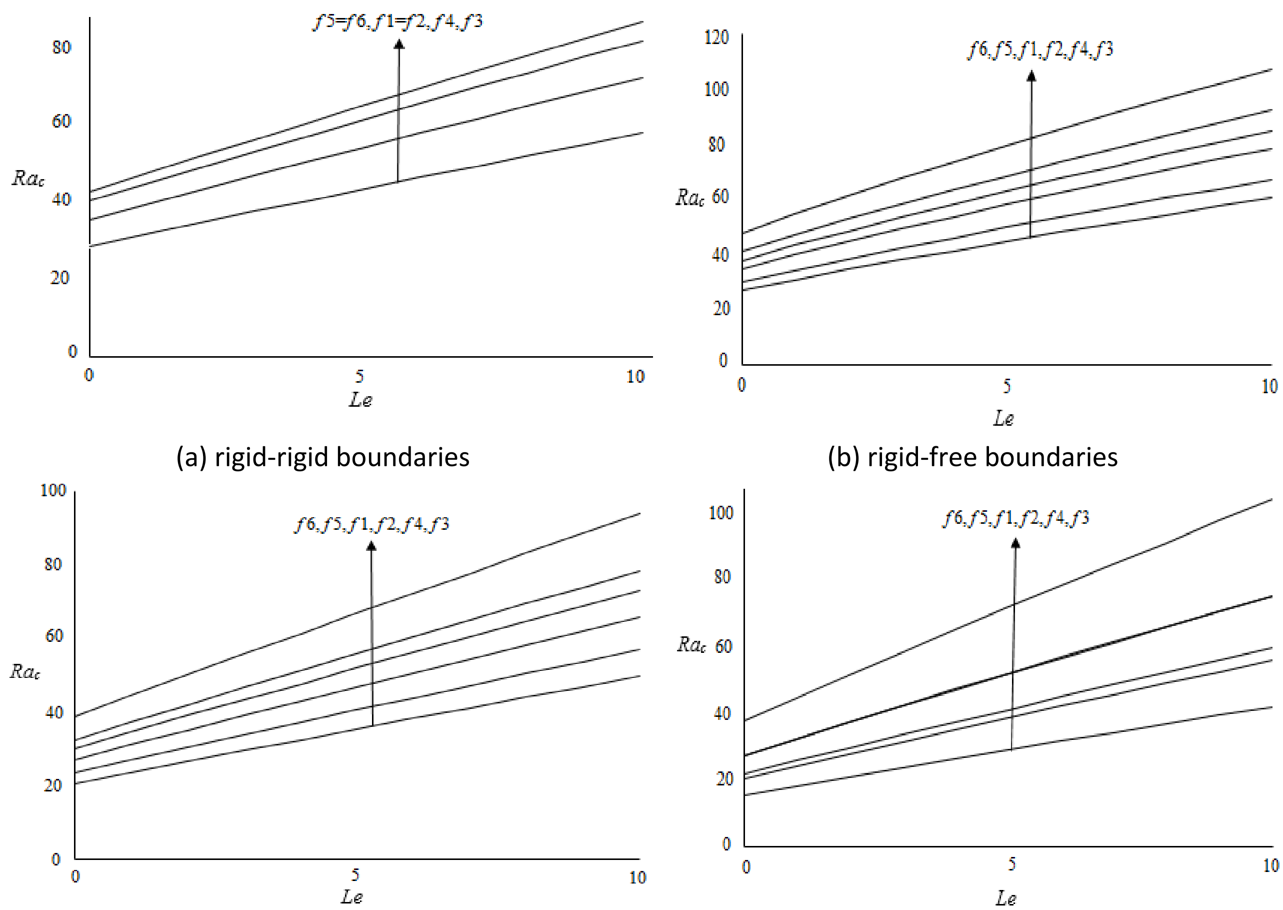
(b) rigid-free boundaries

(c) lower rigid isothermal-upper rigid adiabatic boundaries

(d) lower rigid isothermal-upper free adiabatic boundaries

Fig. 1. Variation of Ra_c with Ra_s for different $f(z)$ with respect to various boundaries conditions for $Le = 5$, $\xi = 0.5$, and $\eta = 0.3$

The effect of Lewis number, Le for the fixed value of other parameters and various models of non-uniform temperature profile on the Ra_c with respect to rigid-rigid isothermal, rigid-free isothermal, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic boundary conditions are presented in Figure 2(a), 2(b), 2(c) and 2(d). It show that the value of Ra_c increases as Le increases, hence it delay the onset of stationary convection for all the non-uniform temperature profile models. These results complement with the results obtained by Malashetty and Kollur [16] for the case of linear temperature gradient which they conclude that Le acted as stabilizer for the steady mode convection and acted as destabilizer for the unsteady mode convection in a coupled stress fluid saturated an anisotropic porous medium.



(a) rigid-rigid boundaries

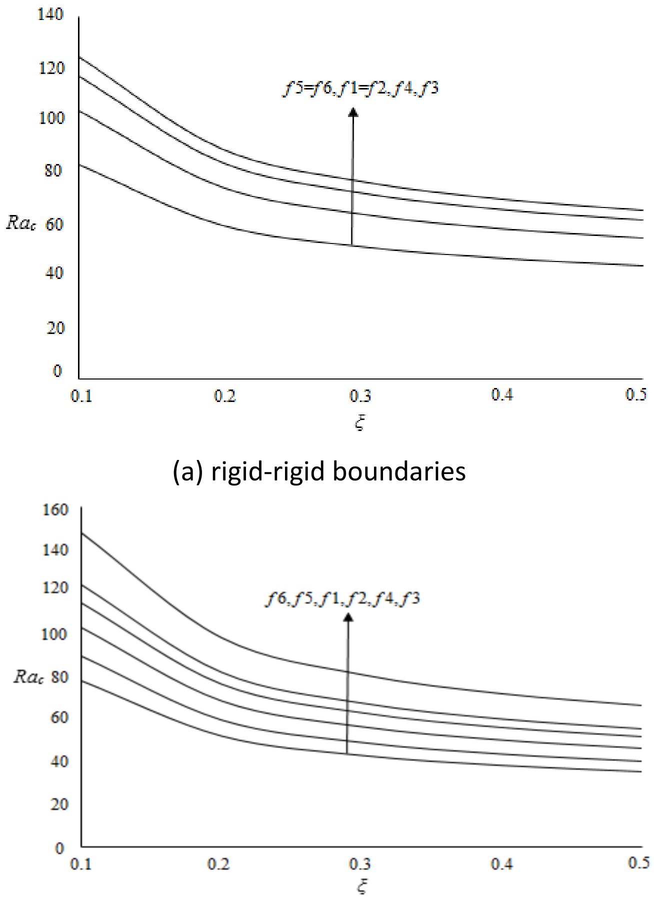
(b) rigid-free boundaries

(c) lower rigid isothermal-upper rigid adiabatic boundaries

(d) lower rigid isothermal-upper free adiabatic boundaries

Fig. 2. Variation of Ra_c with Le for different $f(z)$ with respect to various boundaries conditions for $Ra_s = 10$, $\xi = 0.5$, and $\eta = 0.3$

Figure 3(a), 3(b), 3(c) and 3(d) are the effects of mechanical anisotropic parameter, ξ on the onset of stationary convection for various non-uniform basic temperature gradients where the system are bounded by rigid-rigid isothermal, rigid-free isothermal, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic plate respectively. On increasing the value of ξ, we found that the value of Ra_c decrease for all the six models of non-uniform temperature gradient. The value of ξ is directly proportional to the horizontal permeability, K_x of the porous and inversely proportional to the vertical permeability, K_z of the porous medium. This is due to the facts that increasing the value of K_x will cause the size of the cell to become larger while decreasing the value of K_z will result in larger temperature difference between the lower and upper plate as reported by Degan *et al.,* [6]. As a result, these will trigger the heat flow vertically upward through the porous medium and thus enhance the onset of convection.



(a) rigid-rigid boundaries

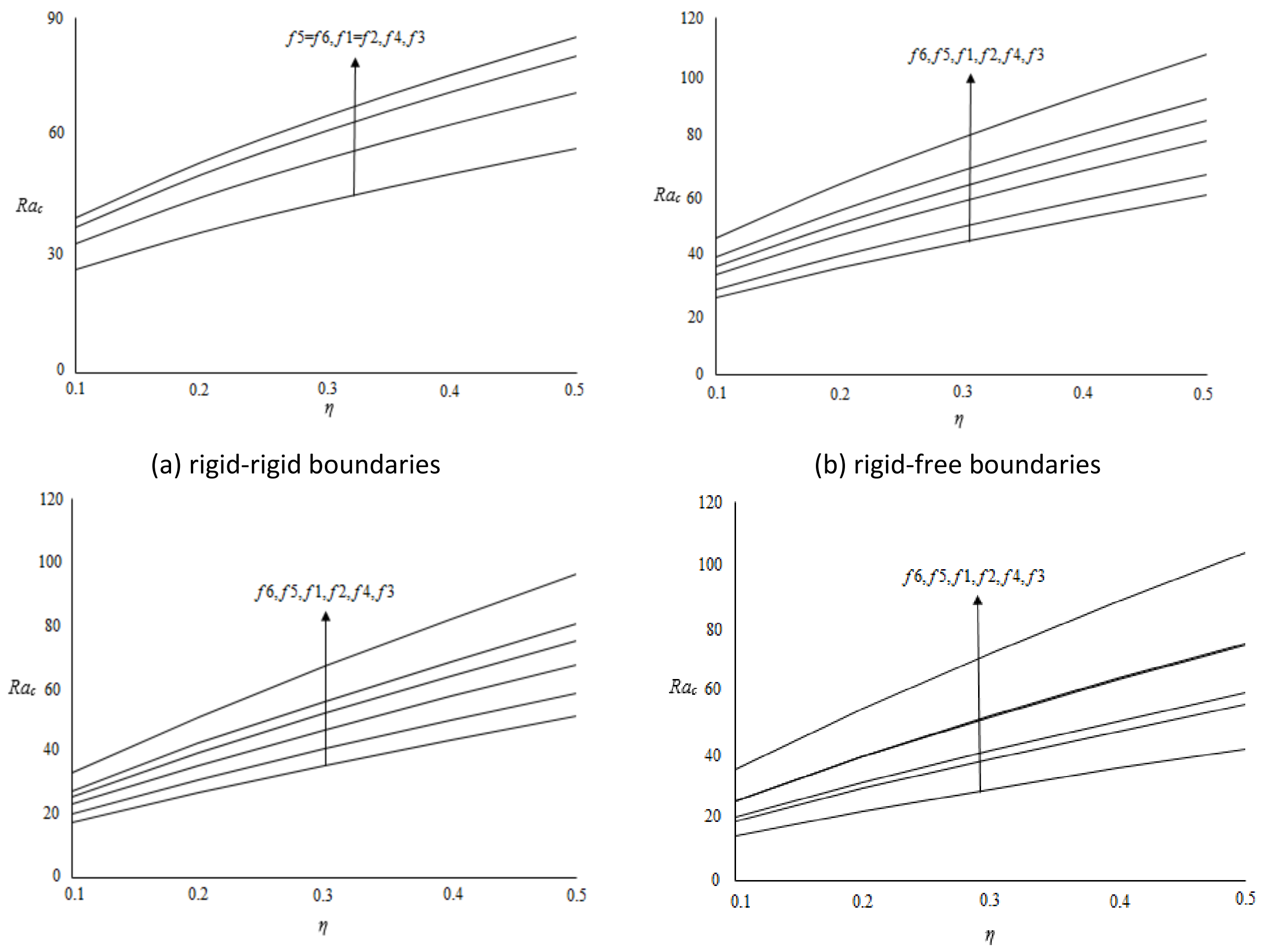
(b) rigid-free boundaries

(c) lower rigid isothermal-upper rigid adiabatic boundaries

(d) lower rigid isothermal-upper free adiabatic boundaries

Fig. 3. Variation of $\mathrm{Ra_c}$ with ξ for different $f(z)$ with respect to various boundaries conditions for $Ra_s = 10$, $Le = 5$ and $\eta = 0.3$

The critical Rayleigh number Ra_c with thermal anisotropic parameter, η for different models of non-uniform temperature profile with respect to rigid-rigid isothermal, rigid-free isothermal, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic plate are presented in Figure 4(a), 4(b), 4(c) and 4(d) respectively. It is observed that with increasing the value of η the value of Ra_c also increase. Increasing η is corresponds to decrease in vertical thermal diffusivity κ_{T_z} which will slow down the heat flow vertically through it. This is due to the fact that the substances with low thermal diffusivity will conducts heat slowly relative to its volumetric heat capacity. Therefore, the effect of η is to inhibit the threshold of stationary convection.



(a) rigid-rigid boundaries

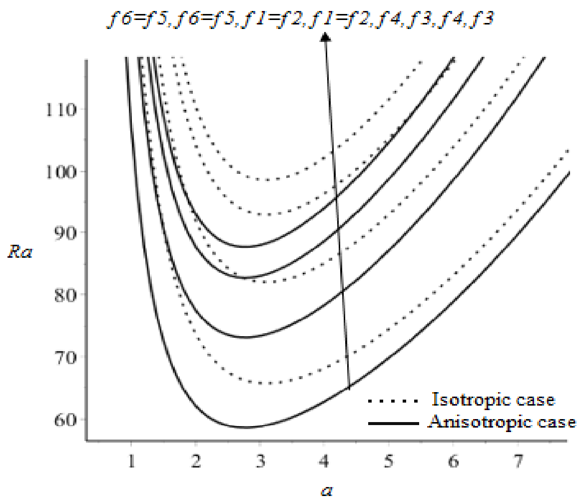
(b) rigid-free boundaries

(c) lower rigid isothermal-upper rigid adiabatic boundaries

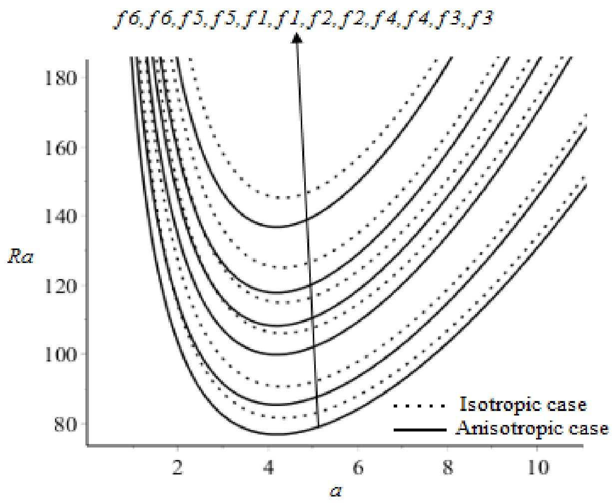
(d) lower rigid isothermal-upper free adiabatic boundaries

Fig. 4. Variation of Ra_c with η for different $f(z)$ with respect to various boundaries conditions for $Ra_s = 10$, $Le = 5$, and $\xi = 0.5$

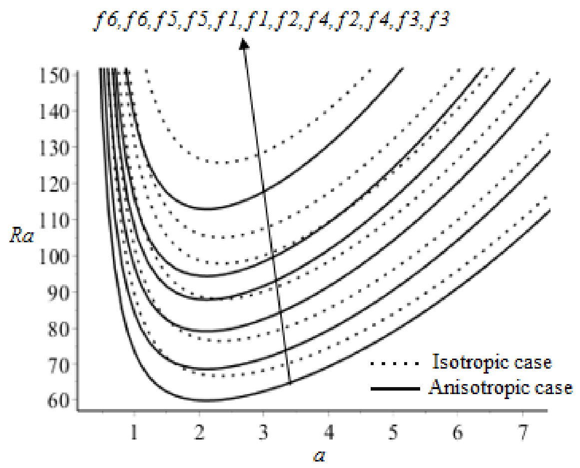
Figure 5(a), 5(b), 5(c) and 5(d) show the influence of $f(z)$ on the neutral stability curves for isotropic $(\xi = 1)$ and anisotropic $(\xi = 1.5)$ cases with respect to rigid-rigid isothermal, rigid-free isothermal, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic plate respectively. The stationary thermal convection is more advance for the anisotropic case when $\xi > 1$ as compared to the isotropic case $(\xi = \eta = 1)$ in each $f(z)$ for all boundaries conditions types. In Figure 5(a), we found that both isotropic and anisotropic neutral curves for $f4$ and $f3$ are lying above the isotropic curve of uniform temperature profile $(f1)$ while both isotropic and anisotropic neutral curve for $f5$ and $f6$ are lying below the isotropic curve of $f1$. The neutral curves for $f6 = f5$ and $f1 = f2$ in both anisotropic and isotropic cases with respect to the symmetric boundary condition(rigid-rigid) while none of the curve are similar for every $f(z)$ with respect to non-symmetric cases indicates that the convection are also affected by the boundaries conditions type. Ra_{c5} and Ra_{c6} for anisotropic cases $(\xi > 1)$ have the more destabilizing effect on the system. The neutral curves for $f4$, $f3$ and $f2$ are lying above the isotropic neutral curve of uniform temperature profile $(f1)$ while both isotropic and anisotropic neutral curve for $f6$ and $f5$ are lying below the isotropic curve of $f1$ as shown in Figs. 5(b), 5(c) and 5(d). Therefore, convection set up earlier for the combination of $f6 = f5$ and anisotropic case $(\xi > 1)$ while set up last for the combination of $f3$ and isotropic case.



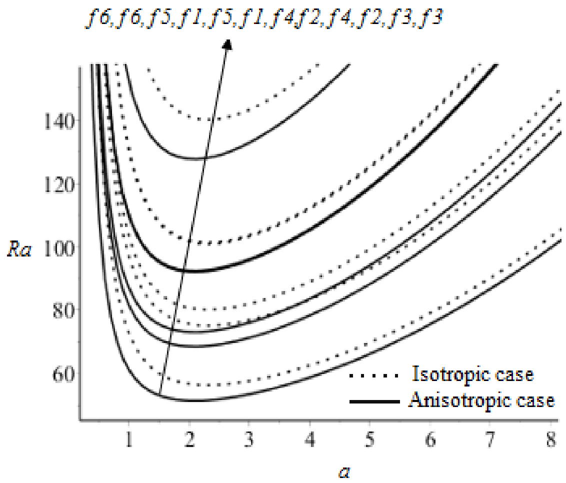
(a) rigid-rigid boundaries



(b) rigid-free boundaries



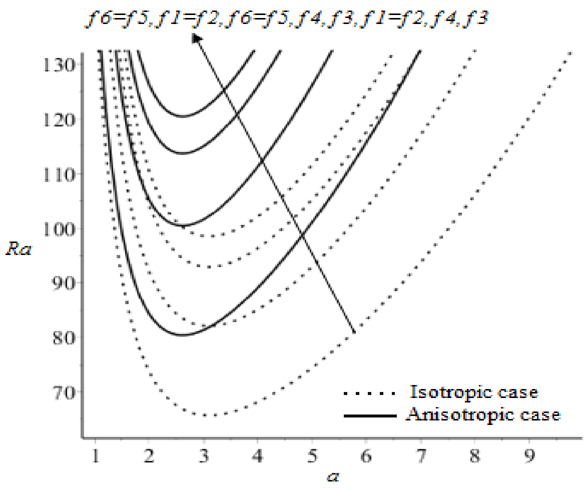
(c) lower rigid isothermal-upper rigid adiabatic boundaries



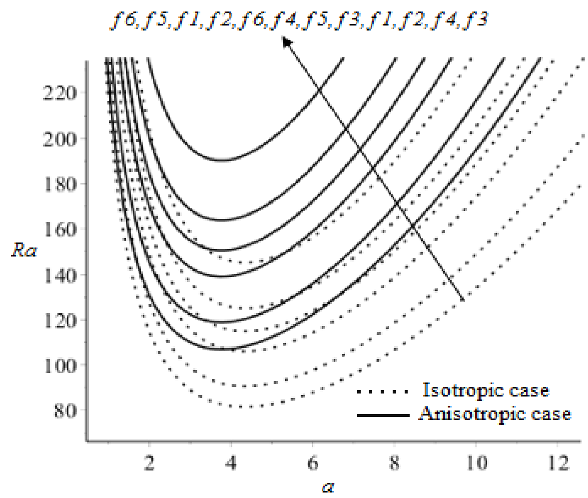
(d) lower rigid isothermal-upper free adiabatic boundaries

Fig. 5. Plot of Ra versus a for different $f(z)$ in isotropic $(\xi = 1)$ and anisotropic $(\xi = 1.5)$ porous medium with respect to various boundaries conditions for $Ra_s = 10$, $Le = 5$ and $\eta = 0.3$

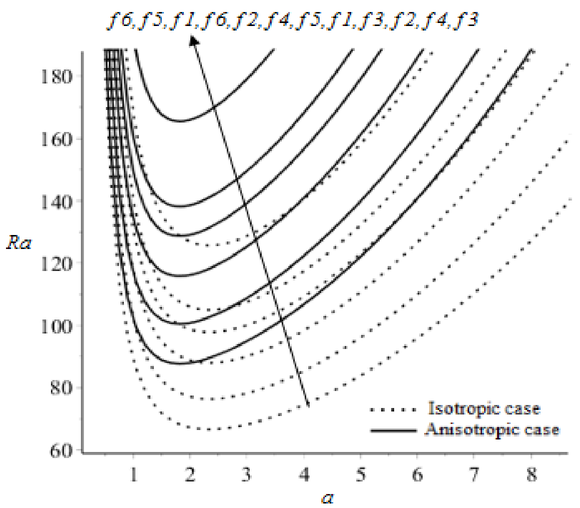
The behavior of neutral curve for isotropic $(\eta = 1)$ and anisotropic $(\eta = 1.5)$ cases with respect to rigid-rigid isothermal, rigid-free isothermal, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic plate are revealed in Figure 6(a), 6(b), 6(c) and 6(d) respectively. The system is more stable in the case of anisotropic porous when $\eta > 1$ as compared to isotropic case $(\xi = \eta = 1)$ for all $f(z)$ with respect to all boundaries types. In Figure 6(a), the onset of stationary convection in an isotropic porous medium in the presence of uniform temperature profile is similar with $f2$. From Figs. 6(b), 6(c) and 6(d), we find that both anisotropic and isotropic neutral curves for $f6$ and isotropic neutral curves for $f5$ are lying below the isotropic neutral curve for uniform temperature profile $(f1)$ while other neutral curves lying above it. Thus, convection is more advance for the combination of $f6$ and isotropic case while for the combination of $f3$ and anisotropic case $\eta > 1$, convection is delayed.



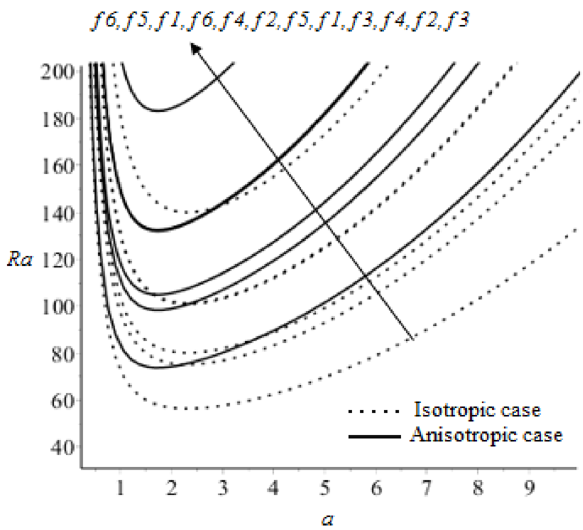
(a) rigid-rigid boundaries



(b) rigid-free boundaries



(c) lower rigid isothermal-upper rigid adiabatic boundaries



(d) lower rigid isothermal-upper free adiabatic boundaries

Fig. 6. Plot of Ra versus a for different $f(z)$ in isotropic $(\eta = 1)$ and anisotropic $(\eta = 1.5)$ porous medium with respect to various boundaries conditions for $Ra_s = 10$, $Le = 5$ and $\xi = 0.5$

4. Conclusions

The stationary thermal convection in a binary fluid saturated an anisotropic porous medium in the presence of non-uniform temperature profiles is investigated analytically using linear stability analysis. The resulting eigenvalues obtained from the governing equations are solved numerically by Galerkin method. The function of Rayleigh number is obtained with the purpose of investigating the effect of solute Rayleigh number, Lewis number, mechanical anisotropy and thermal anisotropy parameter with different models of basic temperature gradient on the onset of thermal convection.

The effect of solute Rayleigh number can make the system become more stable and choosing $Le>1$ can give a positive impact on the onset of stationary convection, since it acts as stabilizer on the system. The effect of mechanical anisotropy parameter can advance the threshold of stationary thermal convection while the presence of thermal anisotropy parameter can slow down the threshold of stationary convection in the system. We can conclude that for the case of rigid-rigid boundaries with both upper and lower plate are isothermal, $\mathrm{Ra}_{c6}=\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}=\mathrm{Ra}_{c2}<\mathrm{Ra}_{c4}<\mathrm{Ra}_{c3}$. For the case of rigid-free isothermal boundaries, lower rigid isothermal-upper rigid adiabatic and lower rigid isothermal-upper free adiabatic, $\mathrm{Ra}_{c6}<\mathrm{Ra}_{c5}<\mathrm{Ra}_{c1}<\mathrm{Ra}_{c2}<\mathrm{Ra}_{c4}<\mathrm{Ra}_{c3}$.

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