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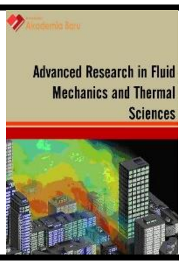

Lattice Boltzmann Simulation of Magnetohydrodynamic Free Convection in a Square Enclosure with Non-uniform Heating of the Bottom Wall

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ABSTRACT

A steady tow-dimensional Magnetohydrodynamic (MHD) free convection flow in a square cavity filled with an electrically conducting fluid which is get under a uniform magnetic field at different angles is numerically investigated using lattice Boltzmann method (LBM). The bottom wall is non-uniformly heated and vertical walls are maintained at cold temperature whereas the top wall is well isolated. The influence of pertinent parameters such as, Reyleigh number ($10^3 \leq Re \leq 10^5$), the Hartmann number ($0 \leq Ha \leq 70$) and the inclination angle of the magnetic field ($0^\circ \leq \theta \leq 180^\circ$) on the flow and heat transfer characteristics have been examined. The obtained results indicate that the heat transfer rate decreases with an increase of the Hartmann number but increases with increase of the Rayleigh number. Also, for high Rayleigh number ($Ra = 10^5$) and for the range of Hartmann number, the heat transfer and fluid flow inside the enclosure depend strongly upon the magnetic field direction.

Keywords:
MHD free convection; square cavity; lattice Boltzmann method; non-uniform heating

1. Introduction

The problems of natural convection heat transfer and flow in electrically conducting fluid in the presence of magnetic field have received considerable attention due to their wide applications in engineering and industry such as aeronautical plasma flow, crystal growth, equipment cooling, electric machinery, solar technology and nuclear reactor cooling [1-2].

The MHD free convection inside closed cavities with different boundary conditions has been studied by widely researchers using analytical, experimental and numerical methods [3-11]. Most of them extensively considered the enclosures with uniform temperature distribution on the walls, but recently, the free convection in enclosures with boundary walls having non-uniform temperatures receives considerable attention due to their use in some engineering applications. The effects of non-isothermal boundary condition of MHD flow in a square enclosure have been studied numerically by

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Oztop *et al*., [12]. They observed that the heat transfer increased with increasing amplitude of sinusoidal function and decreased with increasing Hartmann number Sivasankaran *et al.,* [13] performed a numerical study, with finite volume method, on mixed convection in a lid-driven cavity with sinusoidal temperature distribution on the side walls and a moving adiabatic top wall. The results show that the amplitude ratio increases the heat transfer rate. Sivasankaran *et al.,* [14] examined Hydro-magnetic combined convection in a lid-driven cavity with sinusoidal boundary conditions on both sidewalls. They revealed that the flow behaviour and heat transfer rate inside the cavity are strongly affected by the presence of the magnetic field. Bhuvaneswari *et al.,* [15] investigated magneto-convection in a square enclosure with sinusoidal boundary temperature distributions on both vertical walls using the finite volume method. They reported that the heat transfer rate is increased first and then decreased by increasing the phase deviation. Hossain and Abdul Alim [16] performed two dimensional MHD free convection within trapezoidal cavity with non-uniformly heated bottom wall using the finite element method. The authors found out that the average and local Nusselt number at the non-uniform heating bottom wall of the cavity depends on the dimensionless parameters and also tilts angles. Belhaj and Ben-Beya [17] considered unsteady natural convection flow in a square cavity heated from below with sinusoidal temperature distribution in the presence of uniform magnetic field. The results indicate, that for large values of Hartmann number, increasing nanoparticle volume fraction results in an increase of the normalized average Nusselt number.

Various investigations on free convection in the presence of magnetic field have been performed by researchers with applying different numerical methods. For more than one decade, the lattice Boltzmann method (LBM), based on Boltzmann equation (BE), has been demonstrated to be a very effective numerical tool for simulating fluid flow and modelling physics in fluids [18-21]. This method has also been adopted by different researchers to solve numerically the problem of the natural convection in different geometric cavities. Lattice Boltzmann Method simulation of MHD mixed convection in a lid-driven square cavity with linearly heated wall is investigated by Kefayati *et al.,* [22]. Their result showed that the heat transfer increases with increasing of Richardson number and decreases by the increment of Hartmann number for various Richardson numbers and the directions of the magnetic field. Ashorynejad *et al.,* [23] conducted a numerical study of the effect of magnetic field on the natural convection of water-Ag nanofluids in a horizontal cylindrical annulus enclosure using the Lattice Boltzmann method (LBM). According their results, the average Nusselt number and the Rayleigh number decrease with increasing Hartmann number. Mahmoudi *et al.,* [24] also used LBM to examine the effects of linear temperature distribution on the natural convection in a square enclosure filled with a nanofluid under the influence of a magnetic field. They exhibited that the magnetic field direction has effects on the flow and heat transfer rates in the cavity. Sheikholeslami *et al.,* [25] used lattice Boltzmann method to study the MHD natural convection heat transfer flow of nanofluids in a cavity heated from below in the presence of externally applied magnetic field. They have discussed the characteristics of flow and heat transfer for various values of system parameters. Bettaibi *et al.,* [26] studied the mixed convection in a differentially heated lid-driven cavity with non-uniform heating of the bottom wall using the lattice Boltzmann method. Ahrar and Djavareshkian [27] utilized the lattice Boltzmann method to investigate a nanofluid filled cavity with sinusoidal temperature boundary condition under the influence of an inclined magnetic field. They found that the influence of magnetic field direction on heat transfer was pronounced in high or moderate Rayleigh numbers. Javaherdeh and Najjarnezami [28] used the lattice Boltzmann method (LBM) to simulate the effect of magnetic field on the natural convection in a porous cavity. The sidewalls of the cavity are heated sinusoidally with a phase derivation, whereas the top and bottom walls are thermally insulated. Rahmati and Najjarnezami [29] proposed a double multi-relaxation-time lattice

Boltzmann method (2-MRT-LBM) to simulate MHD natural convection of nanofluid in a two-dimensional square cavity. The results show that for $Ra = 10^5$ and for the range of Hartmann number, the heat transfer and fluid flow depend strongly upon the direction of magnetic field.

In general, in our best knowledge, MHD free convection of electrically conducting fluid in a square enclosure with non-uniform heating of the bottom wall using Lattice Boltzmann method (LBM) have not been investigated in any paper previously. Thus, the main aim of the present study is to examine the effects of the Rayleigh number, Hartmann number and the inclination angle of the magnetic field on the fluid flow and heat transfer inside the cavity. The results are presented in terms of streamlines and isotherms inside the enclosure, local and average Nusselt number along the bottom and right walls.

2. Problem Description and Mathematical Formulation

The geometry of the present study is shown in Figure 1. It is a tow-dimensional square cavity with length L filled with a viscous, incompressible and electrically conducting fluid. The cavity is heated from the bottom wall with nonuniformly distributed temperature such that $T=(T_h-T_c)\ \sin(\pi x=L)+T_c$, the top wall is insulated and the left and right vertical walls are maintained at constant cold temperature (T_c). The thermo-physical properties of the fluid are considered to be constant except the density variation is approximated by the standard Boussinesq model. A uniform magnetic field with a constant magnitude B is applied at an angle (θ) with respect to the horizontal plane. It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field. Further, the viscous heating and compression work are neglected.

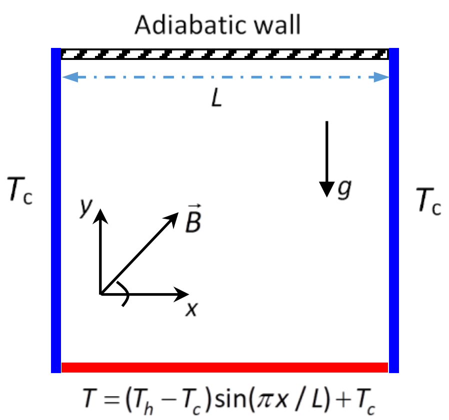


Fig. 1. Geometry of the present study

Under the above-noted simplified assumptions, the governing equations for MHD natural convection flow using conservation of mass, momentum and energy can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+F_x \tag{2}$$

$$u\frac{\partial \upsilon}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}\right)+F_y \quad (3)$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

where F_x, F_y are are the total body forces at x and y directions, respectively, and are defined as follows

$$F_x=\frac{Ha^2\nu}{L^2}\left(\upsilon\sin\theta\cos\theta-u\sin^2\theta\right) \quad (5)$$

$$F_y=g\beta\left(T-T_c\right)+\frac{Ha^2\nu}{L^2}\left(u\sin\theta\cos\theta-\upsilon\cos^2\theta\right) \quad (6)$$

In the above equations $Ha=LB\sqrt{\sigma/\rho\nu}$ is the Hartmann number as σ is electrical conductivity, B is the magnitude of the magnetic field, L is the length of the cavity and θ is the direction of the magnetic field. The governing equations are subject to the following boundary conditions

$$\text{bottomwall:}\ u=\upsilon=0,\ T=(T_h-T_c)\sin\left(\frac{\pi x}{L}\right)+T_c \quad (7)$$

$$\text{topwall:}\ u=\upsilon=0,\quad \left.\frac{\partial T}{\partial y}\right|_{y=L}=0 \quad (8)$$

left wall: $u = v = 0,\ T(0,y) = T_c$ (9)

right wall: $u = v = 0,\ T(L,y) = T_c$ (10)

3. Simulation of MHD with Lattice Boltzmann Method (LBM)

3.1 Brief Introduction to LBM

In thermal lattice Boltzmann method, for incompressible flows problems, two different distribution functions f and g are used for solving flow and temperature fields, respectively. For the flow field

$$f_i(x+e_i\Delta t,t+\Delta t)-f_i(x,t)=-\frac{1}{\tau_f}\left[f_i(x,t)-f_i^{eq}(x,t)\right]+\Delta tF_i \quad (11)$$

For the temperature field

$$g_i(x+e_i\Delta t,t+\Delta t)-g_i(x,t)=-\frac{1}{\tau_g}\left[g_i(x,t)-g_i^{eq}(x,t)\right] \quad (12)$$

where t and Δt denote time and its interval, respectively, e_i is the discrete lattice velocity in direction i and F is the external force in direction of lattice velocity.

In the simulation of MHD natural convection, the external force term F_i in the Eq. (11) is given by

$$F_i = F_{ix} + F_{iy} \tag{13}$$

$$F_{ix} = 3\omega_i \left[\frac{Ha^2 \nu}{M^2} \left(\upsilon \sin\theta \cos\theta - u \sin^2\theta \right) \right] \tag{14}$$

$$F_{iy} = 3\omega_i \left[g\beta\left(T - T_c\right) + \frac{Ha^2 \nu}{M^2} \left(u \sin\theta \cos\theta - \upsilon \cos^2\theta \right) \right] \tag{15}$$

The relaxation time for the flow field (τ_f) and temperature field (τ_g) are related to kinematic viscosity and thermal diffusivity of fluid, and these are expressed as follows

$$\tau_f = 3\nu + 0.5; \quad \tau_g = 3\alpha + 0.5 \tag{16}$$

The kinematic viscosity (ν) and the thermal diffusivity (α) are then related to the relaxation time by

$$\nu = \left[\tau_f - \frac{1}{2} \right] e_s^2 \Delta t; \quad \alpha = \left[\tau_g - \frac{1}{2} \right] e_s^2 \Delta t \tag{17}$$

where e_s the speed of sound which is equals to $e_s = e/\sqrt{3}$.

In the present model, local equilibrium distribution function for both flow f_i^{eq} and temperature fields g_i^{eq}, in Eq. (11) and (12) are given by

$$f_i^{eq}(x,t) = \omega_i \rho \left[1 + \frac{e_i.u}{e_s^2} + \frac{1}{2}\frac{(e_i.u)^2}{e_s^4} - \frac{1}{2}\frac{u.u}{e_s^2} \right] \tag{18}$$

$$g_i^{eq}(x,t) = \omega_i T \left[1 + \frac{e_i.u}{e_s^2} \right] \tag{19}$$

u and ρ are the macroscopic velocity and density, respectively.

The D2Q9 lattice model is applied to the present study. According to this model, the weighting factors ω_i and the discrete lattice velocity e_i are defined as follows

$$\omega_i = \begin{cases} 4/9 & i = 0 \\ 1/9 & i = 1,2,3,4 \\ 1/36 & i = 5,6,7,8 \end{cases} \tag{20}$$

$$e_i = \begin{cases} (0,0) & i=0 \\ \left(\cos\left[\frac{(i-1)}{2}\pi\right], \sin\left[\frac{(i-1)}{2}\pi\right]\right) & i=1,2,3,4 \\ \sqrt{2}\left(\cos\left[\frac{(i-1)}{2}\pi + \frac{\pi}{4}\right], \sin\left[\frac{(i-1)}{2}\pi + \frac{\pi}{4}\right]\right) & i=5,6,7,8 \end{cases} \tag{21}$$

Finally, macroscopic quantities can be calculated in terms of these variables, with the following formula

$$\text{Flow density: } \rho(x,t) = \sum_i f_i(x,t) \tag{22}$$

$$\text{Momentum: } \rho u(x,t) = \sum_i f_i(x,t) e_i \tag{23}$$

$$\text{Temperature: } T = \sum_i g_i(x,t) \tag{24}$$

3.2 Boundary Conditions
3.2.1 Boundary conditions for flow field

Bounce-back boundary conditions were applied on all solid boundaries, which mean that incoming boundary populations are equal to out-going populations after the collision.

$$f_{6,n} = f_{8,n}, \quad f_{7,n} = f_{5,n}, \quad f_{3,n} = f_{1,n} \tag{25}$$

3.2.2 Boundary conditions for temperature field

The adiabatic boundary condition is used on the north boundaries. For the north boundary, the following conditions are imposed:

$$g_{7,n} = g_{5,n}, \quad g_{8,n} = g_{6,n}, \quad g_{4,n} = g_{2,n} \tag{26}$$

Temperature at the west, east and bottom walls are known. In the bottom wall $T = (T_h - T_c)$ sin $(\pi x/L) + T_c$. Since we are using D2Q9, the unknowns are g_2, g_5, g_6 which are evaluated as

$$\begin{aligned} g_2 &= T_n(y)(\omega_2 + \omega_4) - g_4 \\ g_5 &= T_n(y)(\omega_5 + \omega_7) - g_7 \\ g_6 &= T_n(y)(\omega_6 + \omega_8) - g_8 \end{aligned} \tag{27}$$

3.3 Nusselt Number

Nusselt number, *Nu*, is one of the most important dimensionless parameters in describing the convective heat transport. The local Nusselt number at the bottom wall (Nu_b) and at the right wall (Nu_r) are calculated as

$$Nu_b = -\frac{L}{T_h - T_c}\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{28}$$

$$Nu_r = -\frac{L}{T_h - T_c}\left.\frac{\partial T}{\partial y}\right|_{x=L} \tag{29}$$

The average Nusselt numbers at the bottom and right walls are

$$\overline{Nu_b} = \frac{1}{L}\int_0^L Nu_b dx \tag{30}$$

$$\overline{Nu_r} = \frac{1}{L}\int_0^L Nu_r dy \tag{31}$$

4. Grid Testing and Code Validation

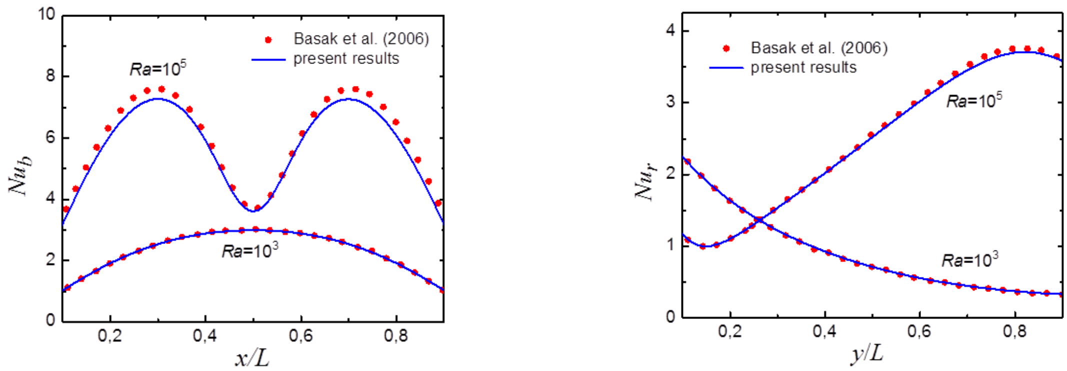
In the present study, a grid testing procedure was conducted to guarantee a grid-independent solution of numerical code. Four different mesh combinations were explored for the case of $Ra = 10^5$, $Ha = 0$ and $Pr = 0.7$. The present code was tested by calculating the average Nusselt number on the bottom and right walls and the results are presented in Table 1. It was found that a grid size of (100 × 100) ensures a grid independent solution.

Table 1
Grid independence test for Pr=0.7, Ha=0 and Ra=10^5

Grid size	Nu_b (Bottom wall)	Nu_r (Right wall)
60×60	4.79286718	2.54953647
80×80	4.86477757	2.54351401
100×100	4.90215445	2.53793979
120×120	4.90514324	2.53363348

To check the validity of the numerical simulation, a comparison is performed between the results obtained by the present numerical method with the results of Basak *et al.,* [30], for $Pr = 0.7$, $Ha = 0$, $Ra = 10^3$ and $Ra = 10^5$.

Figure 2 shows the effects of Rayleigh number (Ra) on the local Nusselt numbers at the bottom and right walls (Nu_b, Nu_r) which are compared with the results of Basak *et al.,* the comparison shows an excellent agreement between the present calculations and the results of Basak *et al.,* [30].



(a) (b)

Fig. 2. Comparison of local Nusselt number on the bottom wall (a) and right wall (b) between the present result and numerical results by Basak *et al.,* [30] (Pr = 0.7, Ha = 0)

5. Results and Discussion

A present numerical study has been carried out to investigate the MHD natural convection heat transfer within square cavity with non-uniformly heated bottom. The results obtained for various values of Hartmann number (Ha = 0, 10, 30 and 50), Rayleigh number (Ra = 10^3, 10^4 and 10^5), magnetic field direction (θ = 0° to 180°) and constant Prandtl number (Pr = 0.7).

5.1 Effect of Hartmann and Rayleigh Numbers

Streamlines and isotherms of flow for different values of Hartmann number (Ha = 0 − 50) and Rayleigh number (Ra = 10^3 − 10^5) and for horizontal magnetic field (θ = 0°) are shown in Figure 3 and 4.

It can be observed that streamlines and isotherms are generally symmetrical about the vertical centre line of the bottom wall due to the symmetry of the problem geometry and boundary conditions. For all Hartmann and Rayleigh number, two circulation cells are formed in different rotation direction. The strength of these cells decreases as the Hartmann number increases and increases as the Rayleigh number increases.

The application of horizontal magnetic field has the tendency to slow down the motion of the fluid in the cavity at all Rayleigh number. As a result, the centre of circulation cells gets pushed towards lower horizontal wall of the cavity when Hartmann number gets increased. The isotherms are also affected by variations in the Rayleigh and Hartmann numbers. For low Rayleigh number, conduction dominant heat transfer compared to the convection mechanism. However, with increase in the Rayleigh number, the effect of natural convection increases and the isotherms are condensed next to the side walls. From the isotherms lines, we notice that the natural convection is reduced with increasing in Hartman number.

Distributions of the local Nusselt number at the right wall (Nu_r) and at the bottom wall (Nu_b) are shown in Figure 5(a) and (b), respectively. It is seen from Figure 5(a) the local Nusselt number on the right wall decreases with distance at the side or cold wall for Ra = 10^3 only. However, at Ra = 10^4 and 10^5, decreasing and increasing trends are observed for local Nusselt number. It can be seen from the Figure 5(b) that the local Nusselt number at the bottom wall increases from zero at both the edges of the bottom wall towards the centre with its maximum value at the centre for Ra = 10^3. For $Ra \geq 10^4$ Nu_b oscillates along the bottom wall, this behaviour is related to the presence of two symmetric cells rotating in opposite directions.

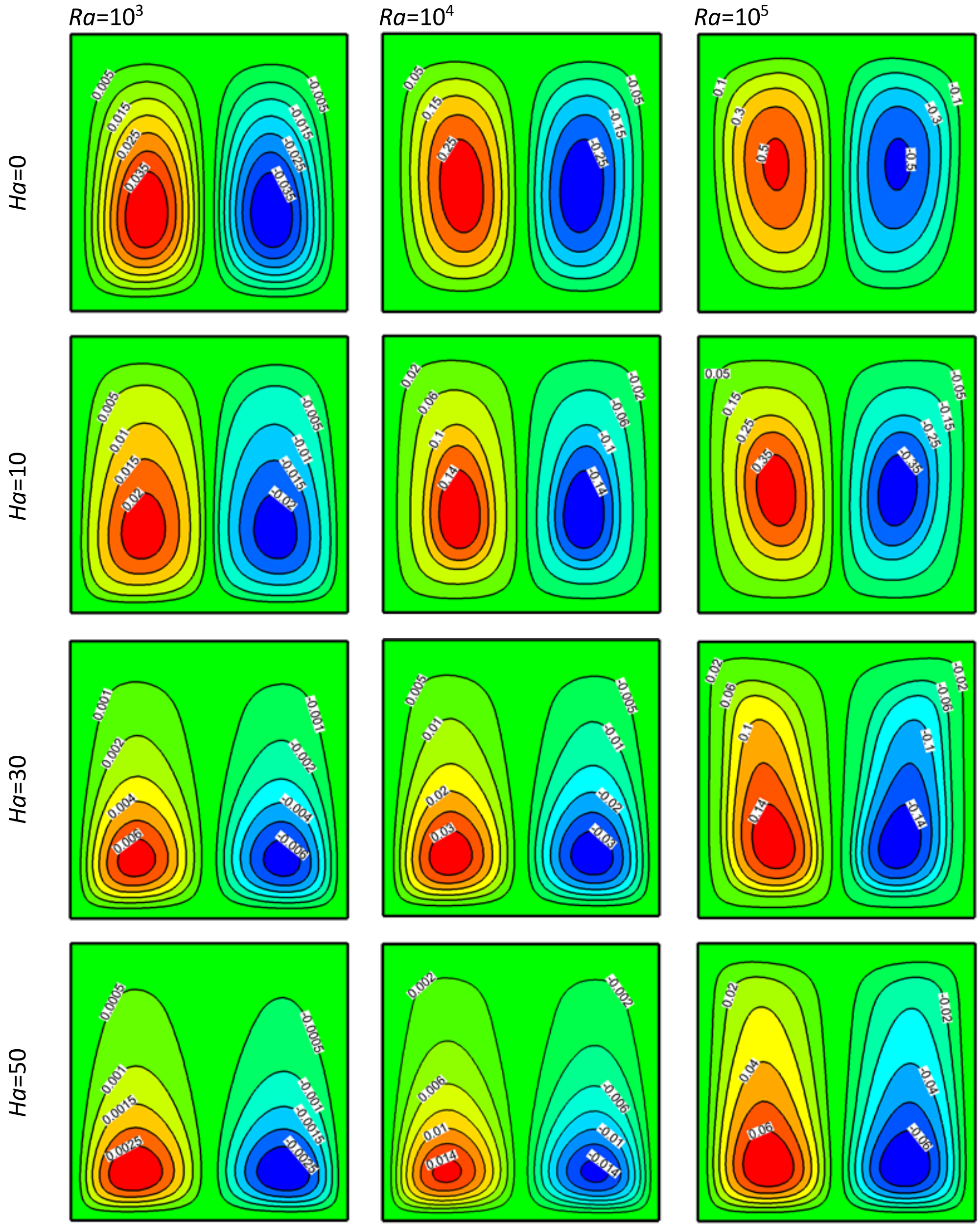


Fig. 3. Streamlines for different Hartmann and Rayleigh numbers and for $\vartheta = 0°$

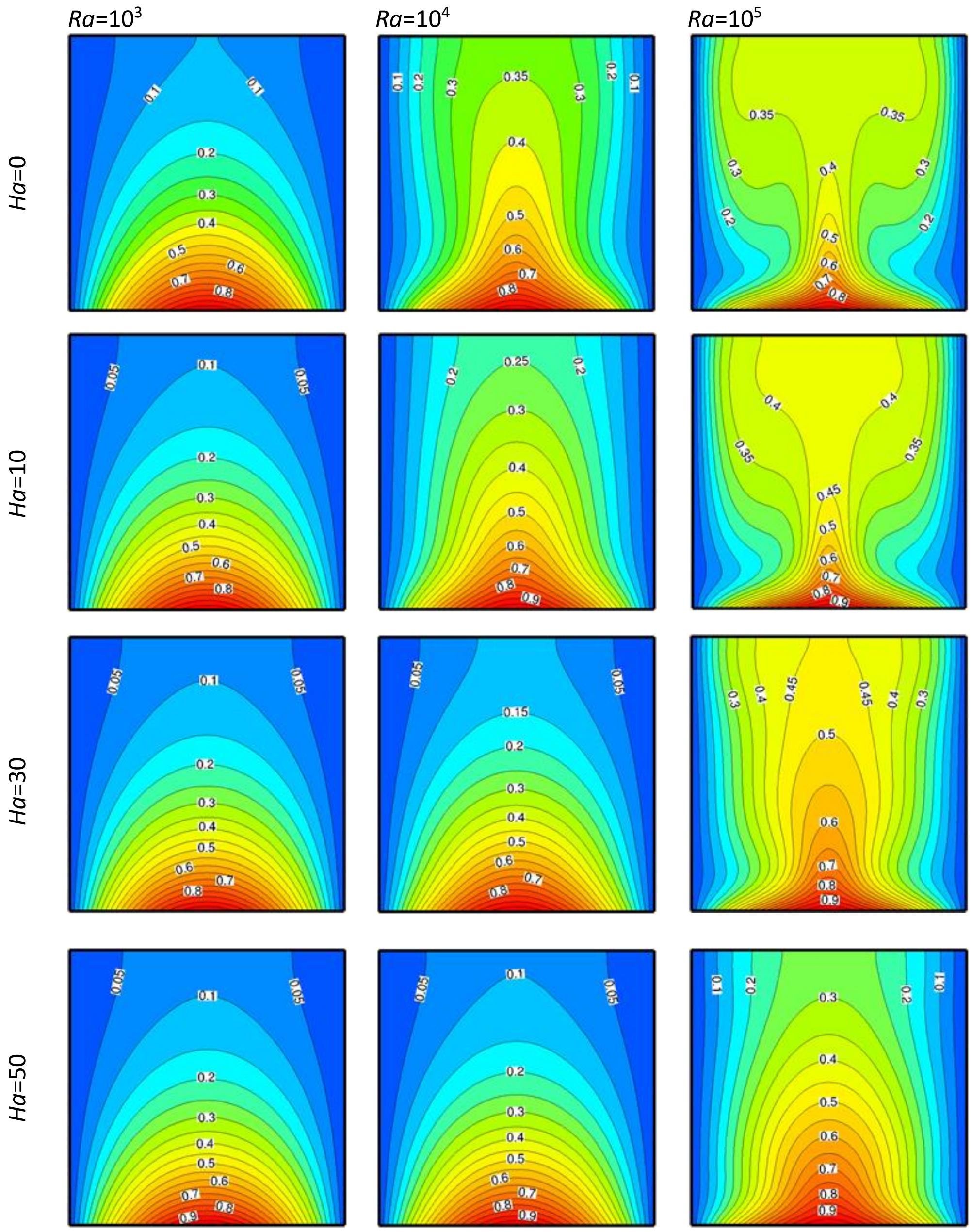


Fig. 4. Isotherms for different Hartmann and Rayleigh numbers and for $\vartheta = 0°$

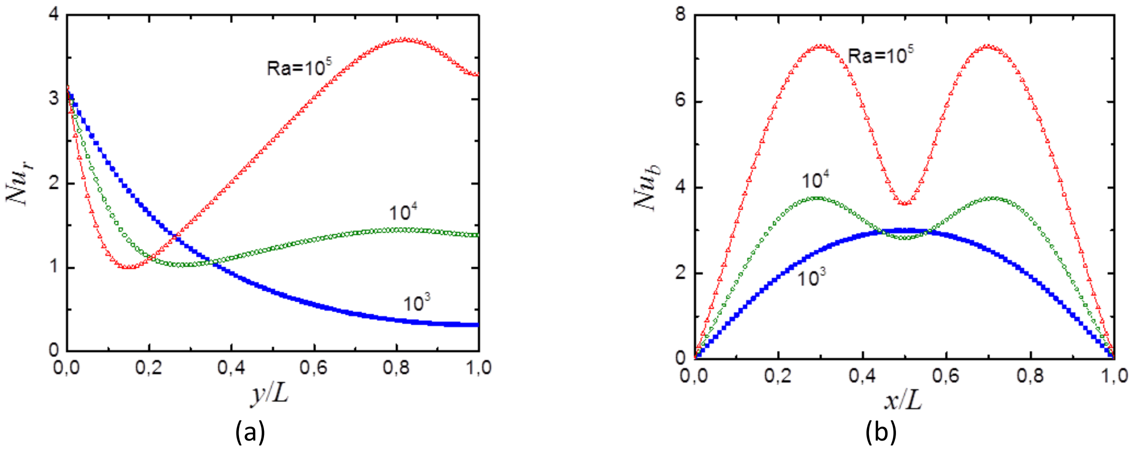


Fig. 5. Local Nusselt number distributions on the right wall (a) and bottom wall (b) for Ha = 0 and $\vartheta = 0°$ for different Rayleigh number

Figure 6(a) and (b) shows the distribution of the local Nusselt number at the right wall and at the bottom wall for various values of Hartmann number, for $Ra = 10^5$ and $\theta = 0°$. It is observed that the local Nusselt number on the right wall for $Ha = 50$ decreases monotonically whereas for $Ha = 30$ Nu_r decreases and then increases. However, for $Ha = 10$ local Nusselt number decreases, attains a minimum and then increases to attain its maximum value and again decreases.

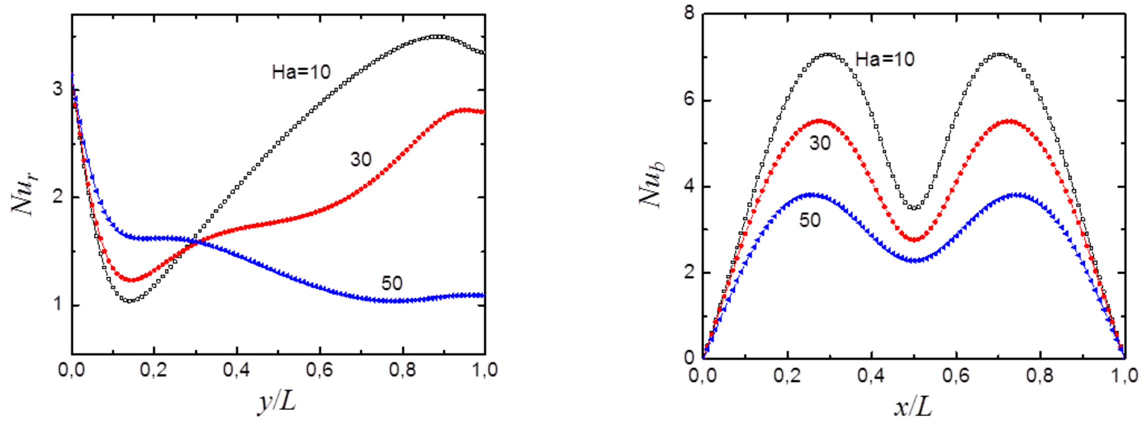


Fig. 6. Local Nusselt number distributions on the right wall (a) and bottom wall (b) for $Ra = 10^5$ and $\vartheta = 0°$ for different Hartmann number

Figure 7(a) and (b) presents the variation average Nusselt number on the right wall (Nu_r) and bottom wall (Nu_b) with Hartmann number for different Rayleigh number for $\theta = 0°$. It is observed that the heat transfer rate increases with increasing Rayleigh number but it decreases when the Hartmann number increases.

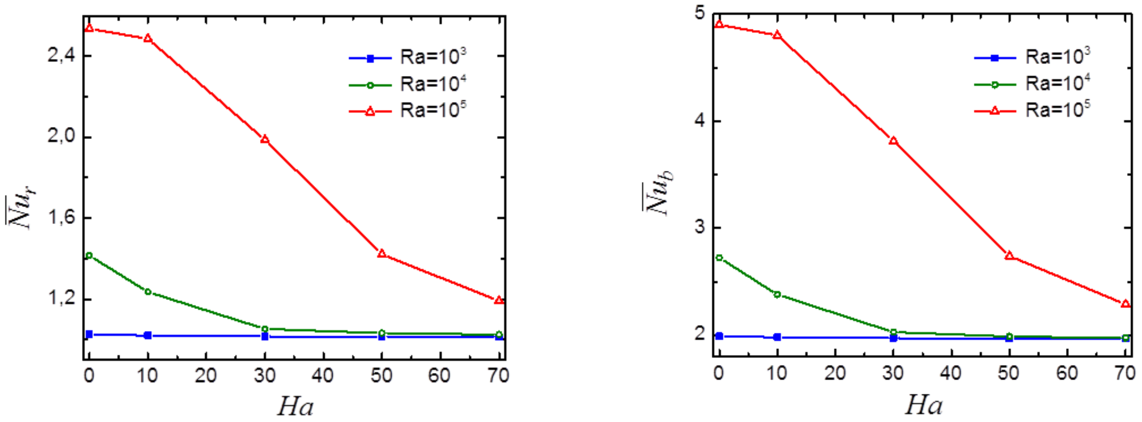


Fig. 7. Variation of the average Nusselt number on the right wall (a) and bottom wall (b) with Hartmann number for different Rayleigh number for $\theta = 0°$

5.2 Effect of the Magnetic Field Direction

The change of the magnetic field direction causes the modification of the Lorentz force direction relative to the gradient temperature which controls the heat transfer rate [24].

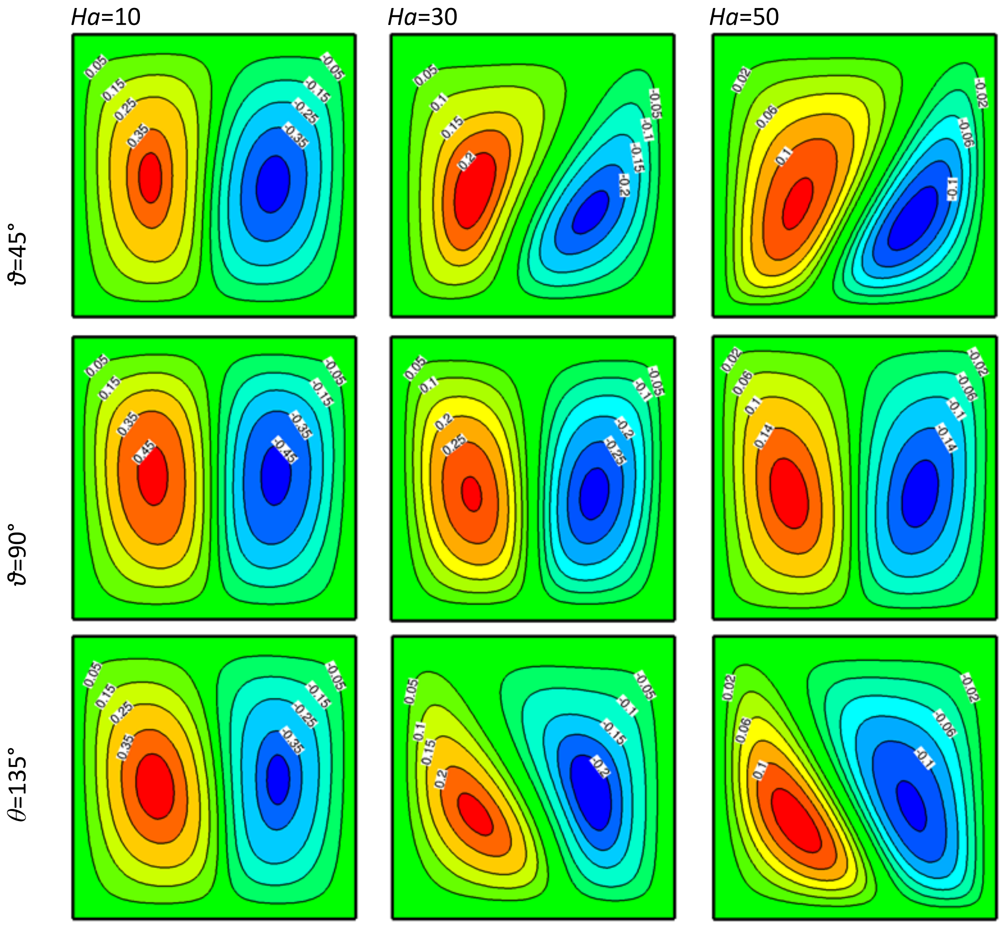


Fig. 8. Streamlines for different direction of the magnetic field and Hartmann number for $Ra = 10^5$

Figure 8 and 9 presents the effect of the direction of the magnetic field on the streamlines and isotherms of flow for three different Hartmann numbers, as Rayleigh number is fixed at $Ra = 10^5$. For all Hartmann number two circulation cells are formed inside the cavity. The symmetry of these cells is broken for $\theta = 45°$ and 135°.

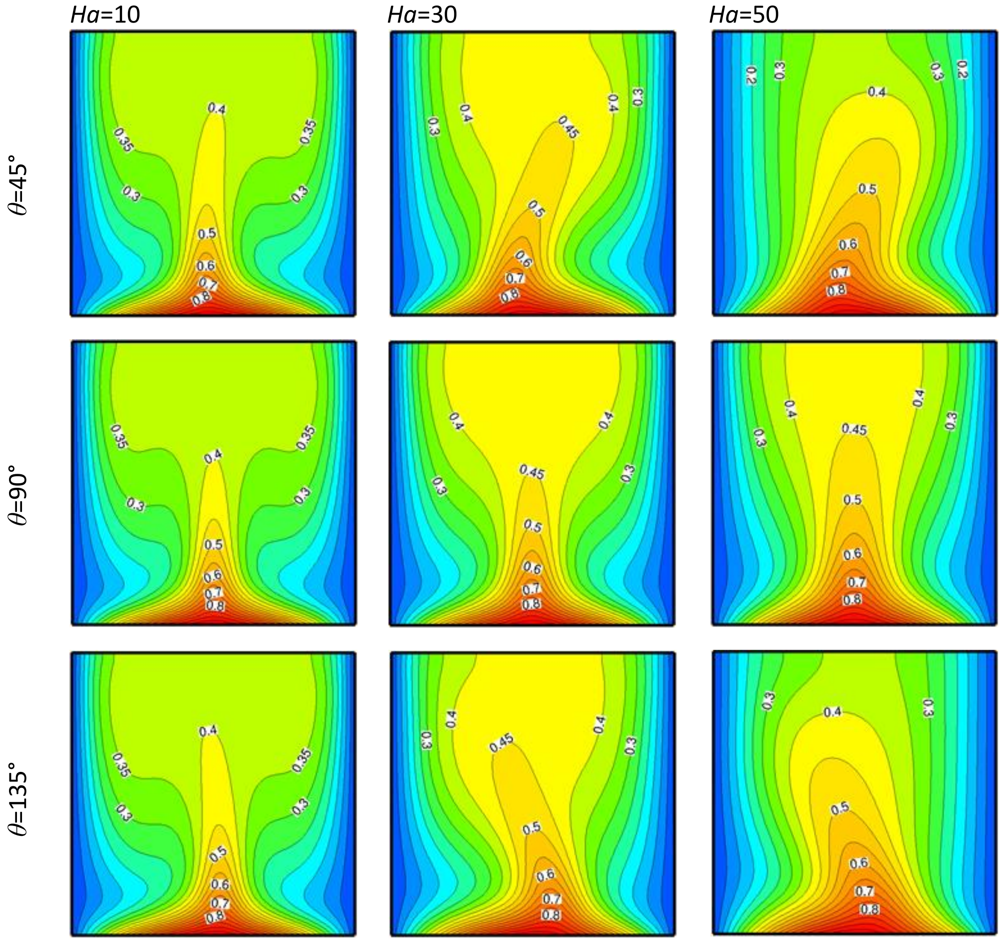


Fig. 9. Isotherms for different direction of the magnetic field and Hartmann number for $Ra = 10^5$

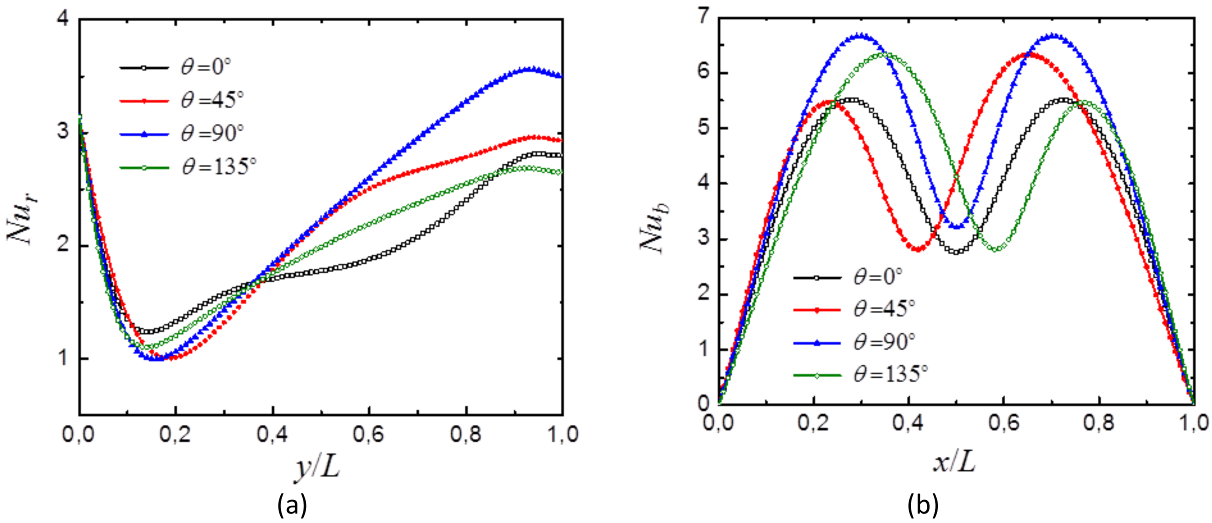


Fig. 10. Local Nusselt number distributions on the right wall (a) and bottom wall (b) for $Ra = 10^5$ and $Ha = 30$ for different θ

Figure 10(a) and (b) illustrates the distributions of the local Nusselt number on the right and bottom walls for different direction of magnetic field for Ra = 10^5 and Ha = 30. It may be noted that, the local Nusselt numbers on the right wall initially decreases and later increases with distance for all values of ϑ. As can be seen from Figure 10(b), the local Nusselt number on the bottom wall (Nu_b) oscillates with the direction.

Figure 11(a) and (b) shows the variation of the average Nusselt numbers on the right wall (Figure 11(a)) and bottom wall (Figure 11(b)) as a function of the direction of magnetic field for different Hartmann numbers for Ra = 10^5. It is observed that as Hartmann number increases, the heat transfer rate decreases for all values of ϑ. Also, for all values of Hartmann numbers, the heat transfer rate reaches its maximum value for ϑ = 90° on bottom wall of the cavity. The figure also shows that the distribution of the average Nusselt number of the vertical right wall (Nu_r) is not symmetrical, contrary to Nu_b.

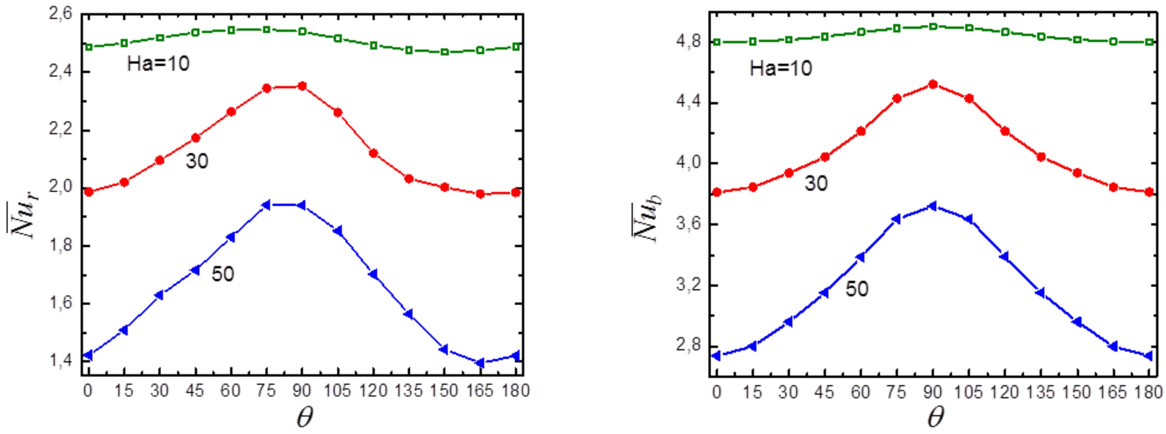


Fig. 11. Variation of the average Nusselt number on the right wall (a) and bottom wall (b) with θ for different Hartmann number for Ra = 10^5

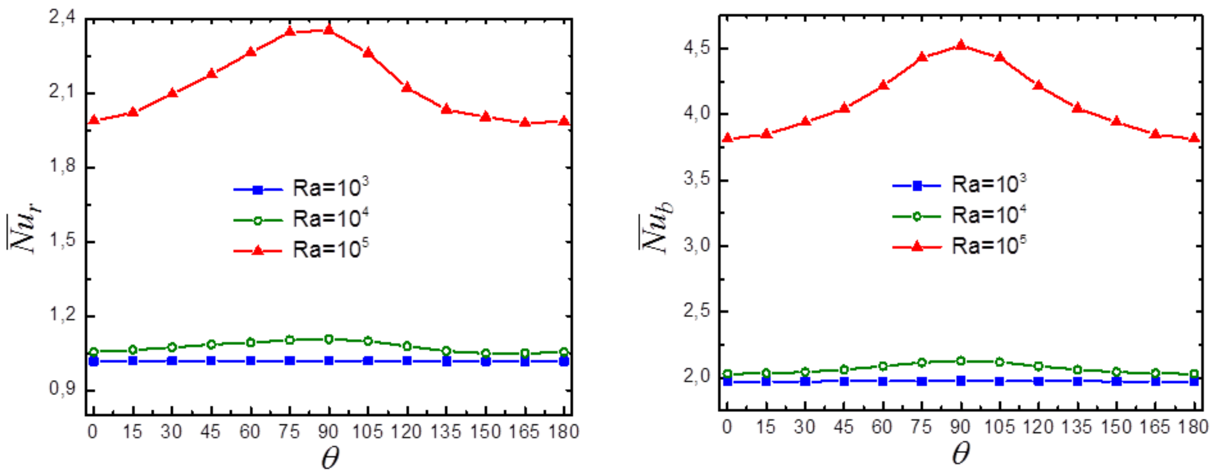


Fig. 12. Variation of the average Nusselt number on the right wall (a) and bottom wall (b) with θ for different Rayleigh number for Ha = 30

Variations in the average Nusselt number on the right and bottom walls as function of the direction of magnetic field for different Rayleigh numbers and for Ha = 30 are presented in Figure 12. For low values of Rayleigh number ($Ra \leq 10^4$), the values of Nu_r and Nub are almost same due to dominant conduction mode of heat transfer, the direction of magnetic field does not affect the heat transfer in the enclosure. For *Ra* = 10^5, the average Nusselt number on the right and bottom walls increases when ϑ increases in the range of 0° to 90° and decreases for 90° < ϑ < 180°, and reaches its maximum value for the angle ϑ = 90°.

6. Conclusions

In the present paper, MHD free convection in a tow-dimensional square cavity with sinusoidal temperature distribution on the bottom wall has been investigated with Lattice Boltzmann method. Effects of different parameters such as Rayleigh number, Hartmann number and the direction of the magnetic field have been considered. With respect to the present results the following conclusions are drawn

i. A good agreement, valid with previous numerical investigations demonstrates that the Lattice Boltzmann Method is an appropriate method for different applicable problems.
ii. Heat transfer rate is augmented by the growth of Rayleigh number.
iii. Heat transfer rate and fluid flow inside the cavity declines with the increment of Hartmann number for various Rayleigh numbers and for all magnetic field direction.
iv. For *Ra* = 10^5 and for 10 ≤ *Ha* ≤ 50, the heat transfer mechanism depends on the magnetic field angle. For the bottom wall, the average Nusselt number is maximum for θ = 90°. For the right wall Nu_r oscillates with θ.
v. The direction of magnetic field does not affect the heat transfer in the enclosure for low values of Rayleigh number.
vi. For *Ra* = 10^5 and *Ha* = 30, with increasing the magnetic field angle from 0° to 90° the average Nusselt number on the right and bottom walls increases and with increasing the magnetic field angle from 90° to 180° the average Nusselt numbers decreases.

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