

## Analysis of Parameters Affecting Thrust and Base Pressure in Suddenly Expanded Flow from Nozzle

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### ABSTRACT

In the case of flow from the nozzle, the net thrust is the vital and critical consideration. The net thrust force is the difference of the thrust developed by the jet from the nozzle and the base drag. The pressure in the base area of a duct with the sudden expansion usually is lower than the ambient atmospheric pressure, which is responsible for creating a very high value of the base drag. It is essential to design an enlarged duct and selection of the suitable values of the flow parameters for efficient utilization of the fuel by producing the high thrust and low base drag. In this research paper, various combinations of parameters are analyzed using CFD analysis and by considering the Design of Experiments & ANOVA. The parameters considered for this research work are Nozzle Pressure Ratio (NPR), Length to Diameter Ratio (L/D), Mach number (M), Inlet temperature of air (T), and the Area Ratio (AR). Based on the results obtained during the present study, it is found that the base pressure and thrust created by the flow are dependent and functions of the following parameters, and they are the NPR, the Mach number, the area ratio of the duct (AR), and the L/D. The inlet temperature of the air does not have any significant effect on the base pressure as well as on the Thrust.

#### Keywords:

Base pressure; base drag; CDF; mach number; thrust

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## 1. Introduction

In high-speed projectiles, the flow separation at the blunt base takes place in several aerospace applications, such as rockets, missiles, space shuttle, bombs, and hence it has been the subject of many investigations for several years. The internal and external suddenly expanded flow characteristics are studied, and they are found to be the same [1]. It is observed that the high-speed aerodynamic flow introduces many undesirable flow characteristics, such as like flow separation, very unsteady pressure fluctuations, thrust loss due to flow entrainment, and so forth [2-8]. These flow-induced influences will undermine the performance of the aerospace vehicle. Efforts have been made

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to develop more efficient aerodynamic vehicles. Attempts have been made study such flow field and tried to correlate. Similar attempts were made to understand and to minimize these undesirable effects of the low base pressure at the base in transonic flow aerodynamics. This research work was undertaken with the intention of both developing a better understanding of the base flow and investigating the influence of the flow and geometrical parameters on thrust and base pressure values.

In the suddenly expanded flow field, the thrust and the base pressure occurrence depends on several parameters. Based on literature and to initiate the study, the parameters considered for the CFD analysis are the NPR, M, AR, and the inlet temperature of the air [9-17]. The shape of the human-powered submarine is studied and concluded that the fineness ration for various shapes submarine should be in the range of 6 to 7 [19]. Based on literature and as a first trial, the length to diameter ratio of an enlarged duct is kept constant at 5 [16, 20]. In the literature, no one has worked on the effectiveness of the parameters on base pressure and the thrust. This research paper includes the main effect and interaction effects of the various parameters on base pressure and the thrust.

## 2. Parameters Considered for the Analysis

The three levels for all the parameters are considered for the analysis. Table 1 shows the parameters and their levels considered for the study. The geometry of the nozzle and its assembly with the enlarged duct is shown in Figure 1. The nozzles are designed for M = 1.5, 2.0, and 2.5. The dimensions of the nozzles for Mach number 1.5, 2.0, and 2.5 are shown in Table 2.

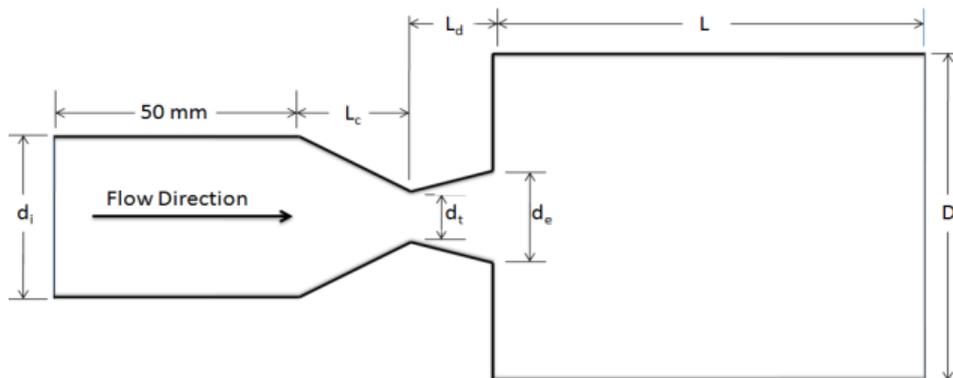
**Table 1**  
Parameters and Levels for L/D = 5

Parameter	Levels	Values
Mach number (M)	3	1.5, 2.0 and 2.5
Nozzle pressure ratio (NPR)	3	2, 5 and 8
Area ratio (AR)	3	2, 5 and 8
The inlet temperature of the air in K (T)	3	300, 1300 and 2300
Length to diameter ratio of the enlarged duct (L/D)	1	5

The NPR is the ratio of the pressure at the inlet of the nozzle to the exit of the nozzle. The nozzle pressure ratio is the most useful parameter to develop thrust. When the flow is subsonic or sonic, the main jet/flow exiting from the converging nozzle is always correctly expanded. However, in the case of supersonic flow, when the flow is coming out from the converging-diverging nozzle, there can be three conditions. The main jet may be correctly expanded, over expanded, or under-expanded.

The enlarged duct diameter is calculated according to the area ratio. The area ratio is the ratio of the duct cross-sectional area to the exit area of the nozzle. The nozzle exit diameter is kept constant at 10 mm for all the cases. The reason for keeping the nozzle exit diameter as 10 mm since, in the literature, some papers are available to enable us to compare our results. The duct diameter (D) for area ratio 2, 5, and 8 are 14.14 mm, 22.36 mm, and 28.28 mm, respectively. The length of an enlarged duct (L/D) is calculated according to the area ratio by keeping the length to diameter ratio fixed at 5 since the flow will remain attached to the duct wall. The length of the duct for area ratio 2, 5, and 8 are 70.7 mm, 111.8 mm, and 141.4 mm, respectively.

The temperature during combustion of the fuel in missile/rocket is very high and needs to be considered as a parameter to observe the effect of it on the base pressure and the thrust. The three levels of temperature considered for the study are 300 K, 1300 K, and 2300 K.



**Fig. 1.** Nozzle with circular duct

### 3. CFD Analysis of Effect of Area Ratio Mach Number and Nozzle Pressure Ratio

The CFD analysis is done for all the possible combinations of the parameters considering full factorial design. The CFD analysis involves the pre-processing, analysis and post-processing. The CFD analysis is done on a workstation computer of 32 GB RAM and an octa-core processor. All the cases have run until the solution gets converged.

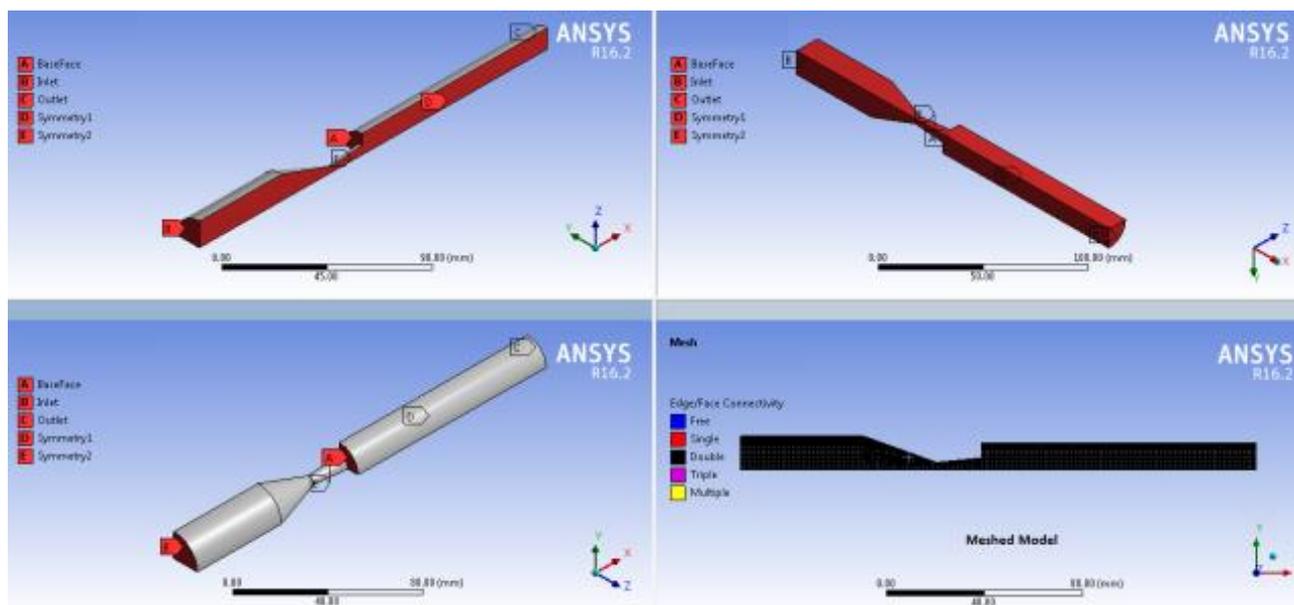
#### 3.1 Modeling, Meshing, and CFD Analysis

Academic licensed ANSYS Workbench is used for creating and meshing the geometries. ANSYS Fluent software is used for the CFD analysis. The various geometries are modeled using dimensions given in Table 2.

**Table 2**  
 Nozzle parameters for various Mach numbers

Parameter	Mach number		
	1.5	2.0	2.5
Inlet diameter in mm ( $d_i$ )	19.94	25.90	28.00
Throat diameter in mm ( $d_t$ )	9.22	7.70	6.16
Exit diameter in mm ( $d_e$ )	10	10	10
Convergent length in mm ( $L_c$ )	20	25	30
Divergent length in mm ( $L_d$ )	14.88	13.16	18.28
Convergent angle in degree ( $\theta_c$ )	150	200	200
Divergent angle in degree ( $\theta_d$ )	1.50	50	60

Figure 2 shows the geometry and meshed model for  $M = 2.5$  and  $AR = 5$ . As the geometry is axisymmetric about the axis, the quarter portion of the geometry is considered for the CFD analysis. The two faces of the quarter portion of geometry are named as symmetry1 and symmetry2, as shown in Figure 2. The symmetry1 and symmetry2 are defined as symmetry in ANSYS Fluent during CFD analysis. The face at the base of the duct is named as the base face, and it is defined as a wall in Fluent during analysis. The base face is used to calculate an average base pressure during post-processing.  $k-\epsilon$  turbulence model is used for the analysis.



**Fig. 2.** Geometry and meshed model for  $M = 2.5$ ,  $AR = 5$  and  $L/D = 5$

The boundary condition defined at the inlet in Ansys is the pressure at the inlet, and the outlet is defined as the pressure at the outlet. The inlet pressure is calculated as per nozzle pressure ratio, and the outlet pressure is set to zero gauge pressure [9-17]. The NPR considered for CFD simulations are 2.0, 5.0, and 8.0. The density-based steady-state solver is used in CFD analysis [9-17].

The mesh element count should be optimum to complete the CFD analysis within a reasonable time and accuracy. The element count can reduce using hexahedral mesh elements instead of tetrahedral elements. The entire hexahedral mesh is generated by dividing the geometry into some sections, and each section has meshed separately with the hexahedral meshing scheme, as shown in Figure 2.

The CFD analysis is based on the fundamental governing equations of fluid dynamics – the continuity, the momentum, and the energy equations. They are the mathematical equations of three fundamental physical principles upon which all of fluid dynamics is based.

- i. Mass is conserved (the Continuity equation)
- ii. Newton's second law (Momentum equation)
- iii. Energy is conserved (Energy equation)

The equations considered for a fluid flow analysis are the continuity equation, the momentum equation, and the energy equations are as shown in Eq. (1)-(6). Under the dynamic conditions when the medium is moving, the characteristic features for incompressible and compressible flow situations are:

The volume flow rate is even at each cross-section of a stream tube for incompressible flow.

$$\dot{Q} = AV \tag{1}$$

The mass flow rate is constant at any cross-section of a stream tube for compressible flow.

$$\dot{m} = \rho AV \tag{2}$$

where,

A is cross-sectional area of stream tube

V and  $\rho$  are velocity and density of the fluid, respectively, at that cross-section

For three dimensional compressible flow

$$\text{div}(\rho V) = 0$$

i.e.

$$\frac{\partial(\rho U_x)}{\partial x} + \frac{\partial(\rho U_y)}{\partial y} + \frac{\partial(\rho U_z)}{\partial z} = 0 \quad (3)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (4)$$

The momentum equation and energy equations are given in Eq. (5)-(6).

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i} \tau_{ij} \quad (5)$$

$$\frac{\partial}{\partial t} \left[ \rho \left( e + \frac{V^2}{2} \right) \right] + \frac{\partial}{\partial x_j} \left[ \rho u_j \left( e + \frac{V^2}{2} \right) + P + q_j - u_i \tau_{ij} \right] = 0 \quad (6)$$

where,

$u$  is instantaneous velocity

$V$  is velocity modulus

$\rho$  is gas density

$P$  is gas pressure

$q_j$  is heat flux and

$\tau_{ij}$  is viscous stress tensor

### 3.2 Factorial Design for Four Factors with Three Levels

The full factorial design is used to make all the possible combinations of the factors/parameters. In each trial or replicate of analysis, all the possible combinations of the levels of the factors are investigated. The main effect and interaction effects of each factor are studied.

The CFD analysis was carried out for a total of 81 different combinations of factors and their levels. The responses in the form of thrust and total base pressure are recorded for all the cases. All the values of the base pressure are gauge pressure, and it is converted into absolute pressure by adding ambient atmospheric pressure. Then the absolute pressure is divided by atmospheric pressure to convert into dimensionless base pressure. The dimensionless base pressure is calculated for all the cases and plotted.

#### 4. The Main Effects and Interaction Effects on Base Pressure

The objective is to assist in the practical interpretation of the results obtained from CFD analysis; the main effects of factors on base pressure are plotted using Minitab software. In Figure 3 to 6, the results present the effects of Mach number (M), the area ratio (AR), the nozzle pressure ratio (NPR), and the inlet temperature of the air (T) on the dimensionless base pressure. These plots show that there are marginal response averages at the levels of these factors.

Figure 3 is plotted by considering the mean value of the base pressure for all the cases at the values of Mach numbers as 1.5, 2.0, and 2.5. From the figure, it is found that the Mach number has a positive effect on base pressure. The base pressure increases with the increment in the inertia level. When the Mach number increases, the inertia level of the primary jet/flow increases, and the main jet/flow becomes under-expanded. As the flow becomes under-expanded, the reattachment length is increased. With the increase in reattachment length, the main flow from the nozzle may not be reattached to the enlarged duct, and then it may be exhausted in the atmosphere for short duct length. When the flow is not reattached to the enlarged duct, the base area will directly come in contact with the atmospheric pressure and hence will result in large values of the base pressure.

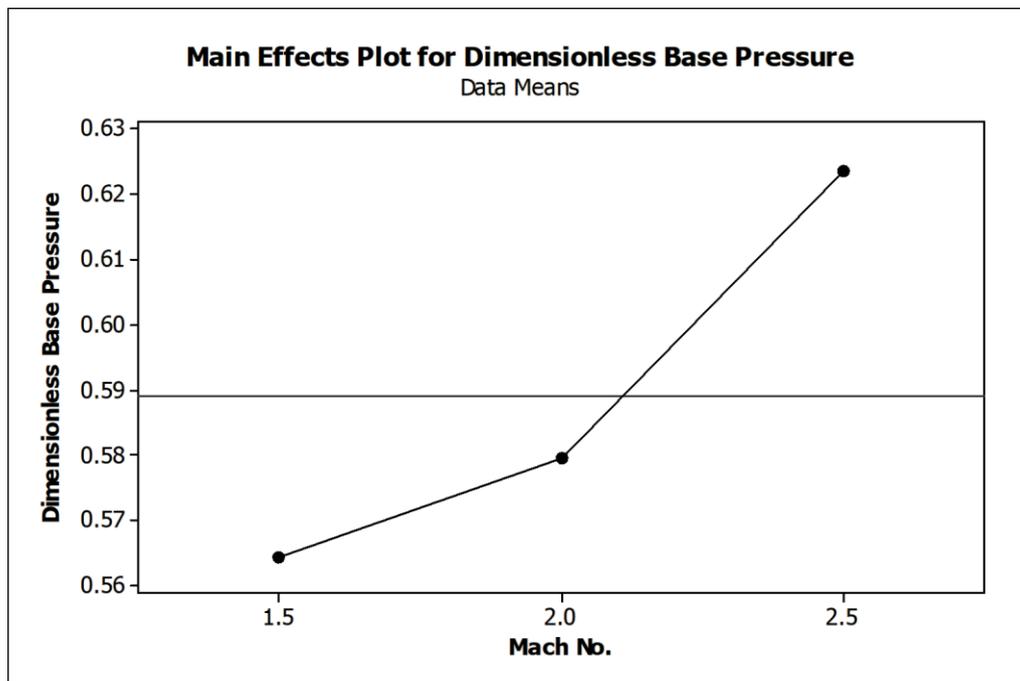


Fig. 3. The main effect plot of Mach number for dimensionless base pressure

Figure 4 is plotted by considering the mean value of the base pressure for all the cases at the values of area ratios. From the figure, it is seen that the base pressure increases with the increase in area ratio. With the increment in duct diameter, the reattachment length increases, and hence, the base pressure will increase since the strength of the vortex is the same.

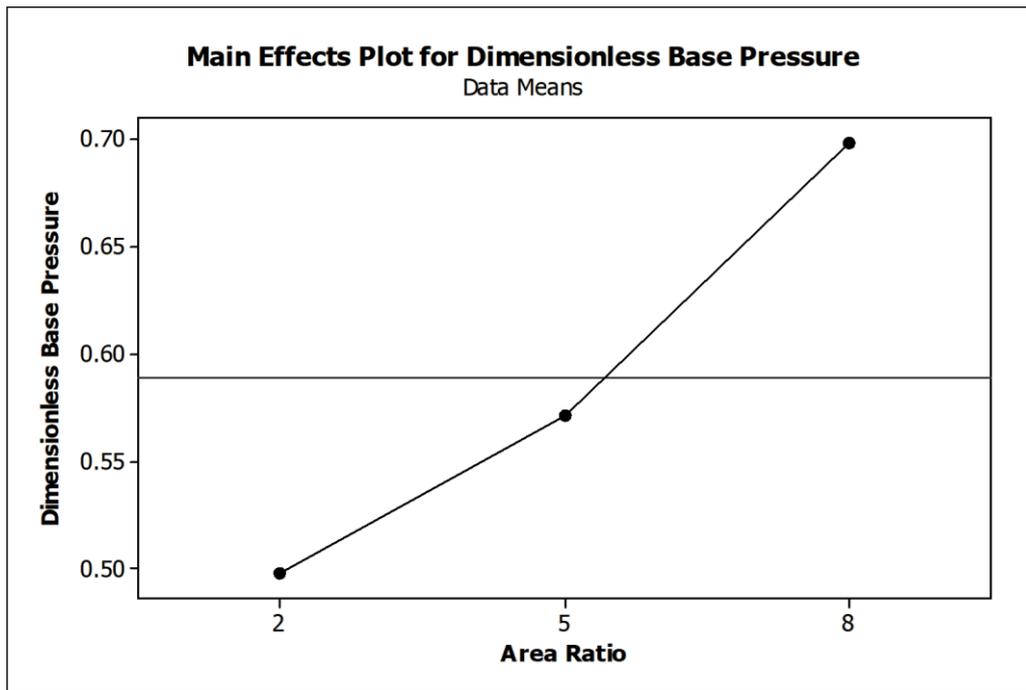


Fig. 4. The main effect plot of area ratio for dimensionless base pressure

Figure 5 is plotted by considering the mean value of the base pressure for all the cases at the values of nozzle pressure ratios (NPR) as 2, 5, and 8. From the figure, it is observed that with the increment in the nozzle pressure ratio (NPR), the base pressure reduces significantly. At the lower nozzle pressure ratio, the main jet from the nozzle is over-expanded.

With the increase in NPR, the main jet/flow becomes correctly expanded and then under-expanded. For under-expanded flow, the flow gets reattached to the enlarged duct, and the base area is separated from the atmosphere. When the base area is completely separated from the atmosphere, the base pressure is reduced.

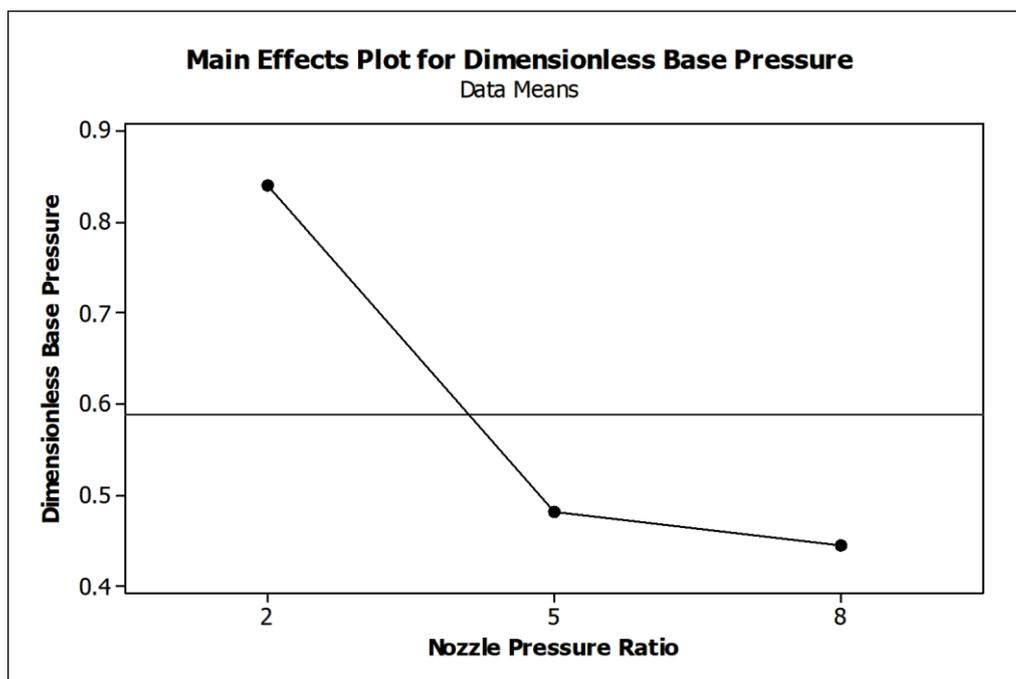


Fig. 5. The main effect plot of nozzle pressure ratio for dimensionless base pressure

Figure 6 is plotted by considering the mean value of the base pressure for all the cases at inlet temperatures of the air as 300 K, 1300 K, and 2300 K using Minitab software. From the figure, it is seen that the inlet temperature of the air has a negligible effect on the base pressure. The variation in the base pressure over the entire range of temperature from 300 K to 2300 K is 0.002, which is insignificant.

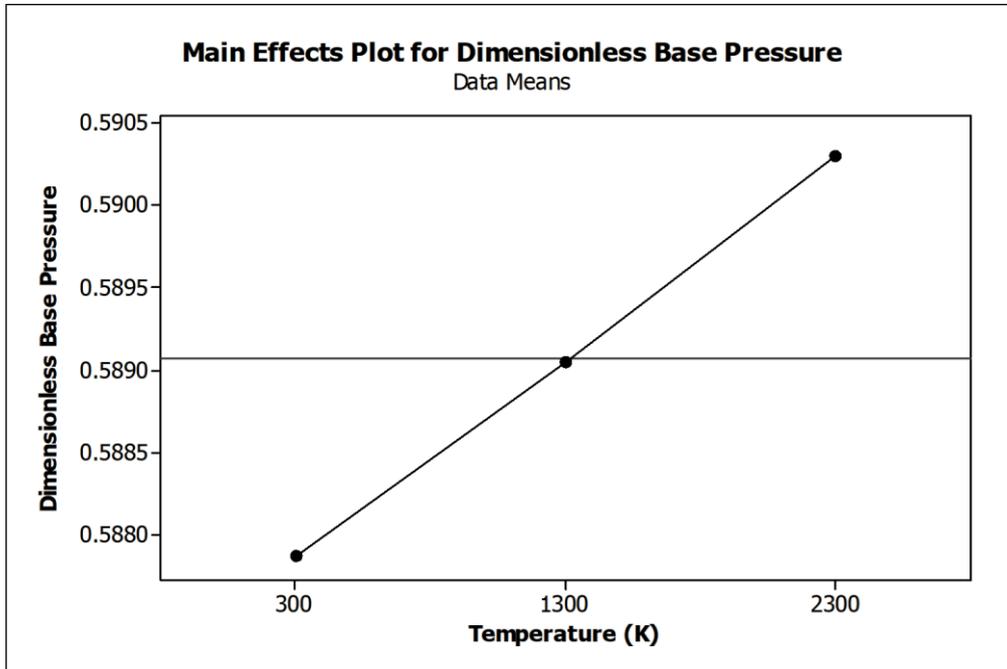


Fig. 6. The main effect plot of temperature for dimensionless base pressure

Figure 7 shows the interaction plots between all the factors, the M, AR, the NPR, and the inlet temperature of the air. Based on the plots, it is observed that the factors M, AP, and NPR have significant interaction with each other. The interaction of the inlet temperature of air with any other factor is relatively small, as shown by the similar shape of the curves in Figure 7.

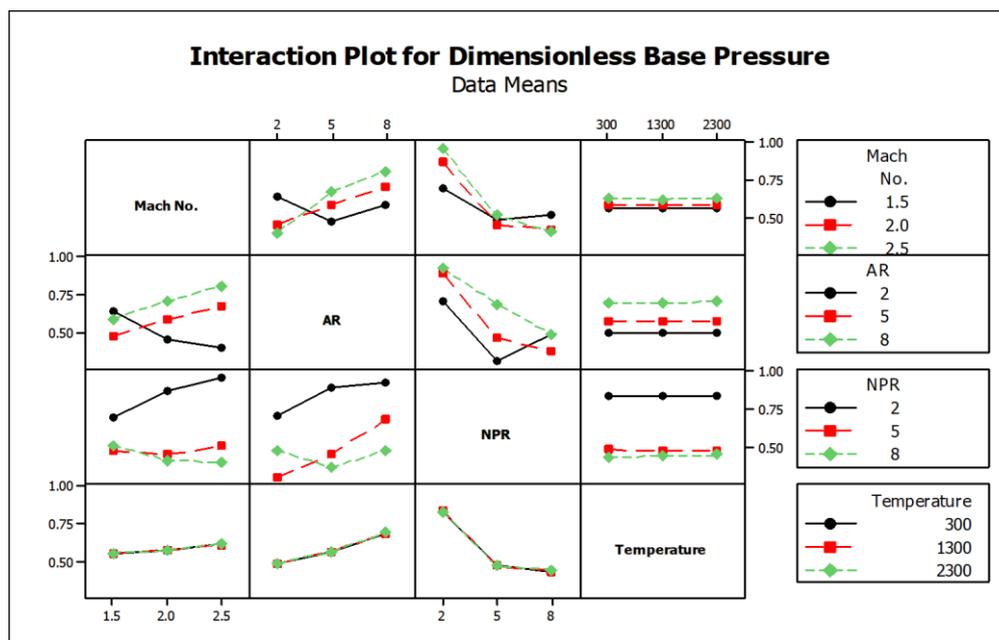


Fig. 7. The interaction effect plots for dimensionless base pressure at  $L/D = 5$

## 5. The Main Effects and Interaction Effects on Thrust

From the results of CFD analysis, the main effects of all the factors on the thrust are plotted using Minitab software. Figure 8 to 11 show the main effect plots for M, AR, NPR, and the air inlet temperature on the thrust generated by the jet from the CD nozzle.

The thrust produced by the jet exiting from the CD nozzle depends on the mass flow rate of the main jet, the velocity of the main jet, exit area of the nozzle and the pressure at the exit of the nozzle. The thrust force from the C-D nozzle can be calculated using Eq. (7).

$$Thrust(T) = \dot{m}_j v_j + A_e(p_e - p_a) \quad (7)$$

For subsonic or sonic Mach numbers, the flow is always correctly expanded. For supersonic Mach numbers, the flow may be over expanded, correctly expanded, or under-expanded. Depending on the pressure at the exit of the nozzle, the three conditions of expansion are as follows

- i. when  $p_e = p_a$ ; then the main jet is correctly expanded,
- ii. when  $p_e < p_a$ ; then the nozzle is over-expanded, and
- iii. when  $p_e > p_a$ , then the main jet is under-expanded.

Figure 8 is plotted by considering the mean value of the thrust for all the cases at various values of Mach numbers, namely 1.5, 2.0, and 2.5. From Figure 8, it is observed that the thrust force increases with increase in Mach number. When the Mach number increases, the inertia level of the primary jet/flow increases, and the main jet/flow becomes under-expanded. If the flow becomes under-expanded, the velocity of the main jet increases to remarkably high value, and hence the thrust is the maximum.

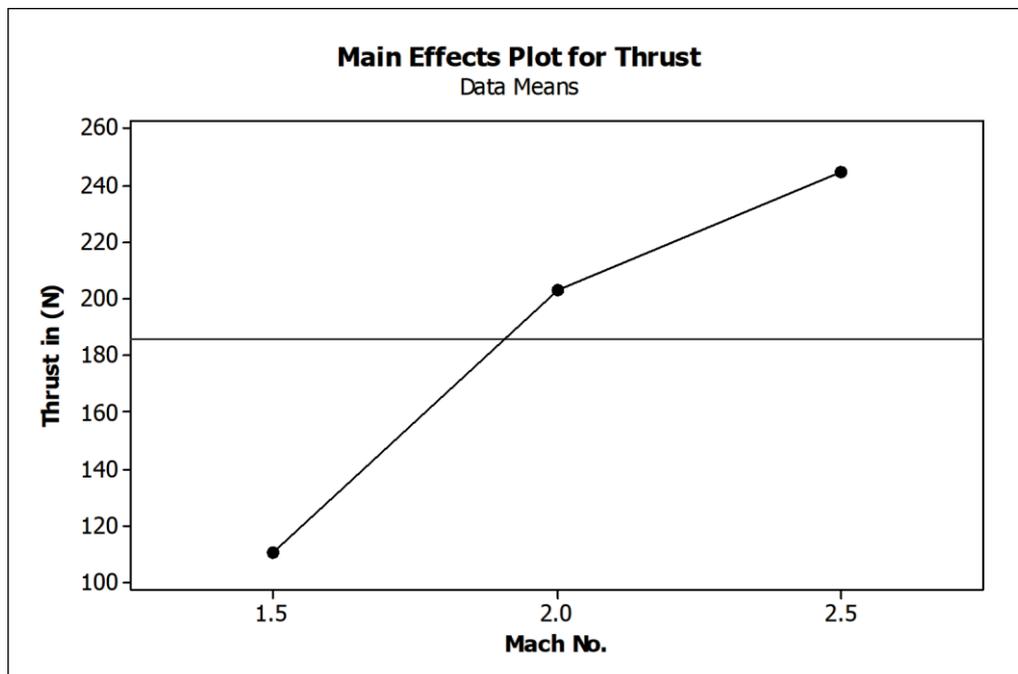


Fig. 8. The main effect plot of Mach number for thrust

Figure 9 is plotted by considering the mean value of the thrust for all the cases for the values of the area ratios, namely 2, 5, and 8. From the figure, it is observed that the thrust increases rapidly

with the increase in the area ratio from 2 to 5, but when the area ratio increases from 5 to 8, the increase in thrust is marginal. As the area ratio increases, the duct diameter increases, and hence the relaxation/relief to the flow exiting from the nozzle increases. Since the more relief is available to the flow after the nozzle exit, the main jet completely under-expanded at higher NPR, and hence thrust increases. It can be concluded that the area ratio should be increased with the increment in the level of expansion (i.e., NPR) to maximize the thrust.

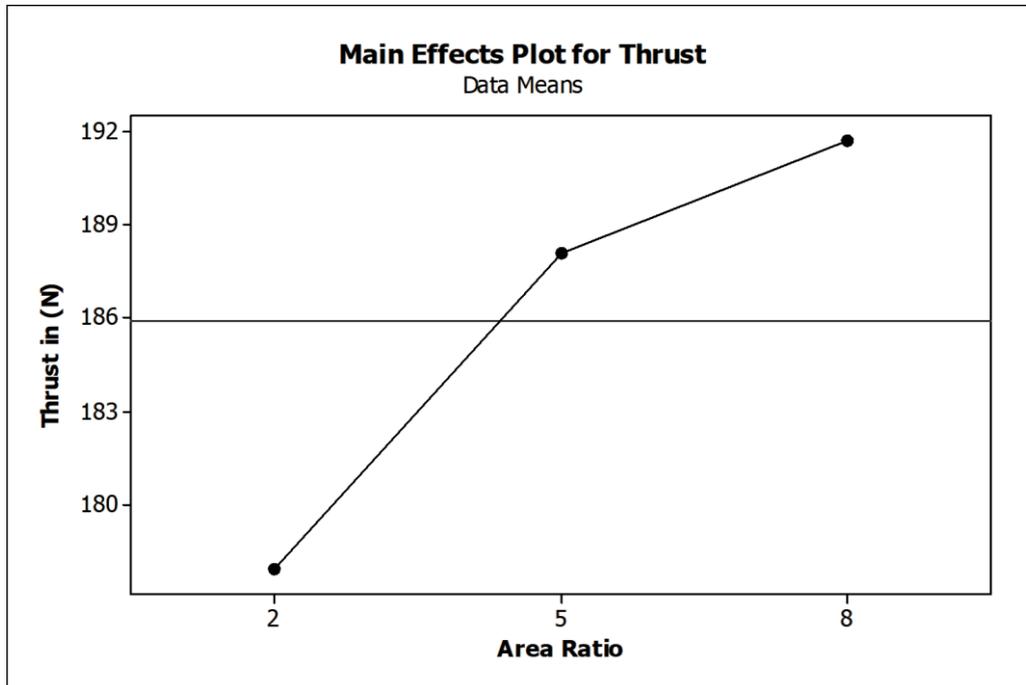


Fig. 9. The main effect plot of area ratio for thrust

Figure 10 is plotted by considering the mean value of the thrust for all the cases at the values of nozzle pressure ratios as 2, 5, and 8. From Figure 10, it is observed that with the increase in the level of expansion (i.e., NPR), the thrust increases significantly. At the lower nozzle pressure ratio, the flow is over-expanded.

Figure 11 is plotted by considering the mean value of the thrust for all the cases at the values of the inlet temperature of the air as 300 K, 1300 K, and 2300 K. It is noted that the thrust is not significantly affected by the inlet temperature of the air. The variations in the thrust force created by the jet flow from the flow accelerating devices (i. e., CD nozzle), over the entire range of temperature from 300 K to 2300 K is 0.12 N, which is not significant.

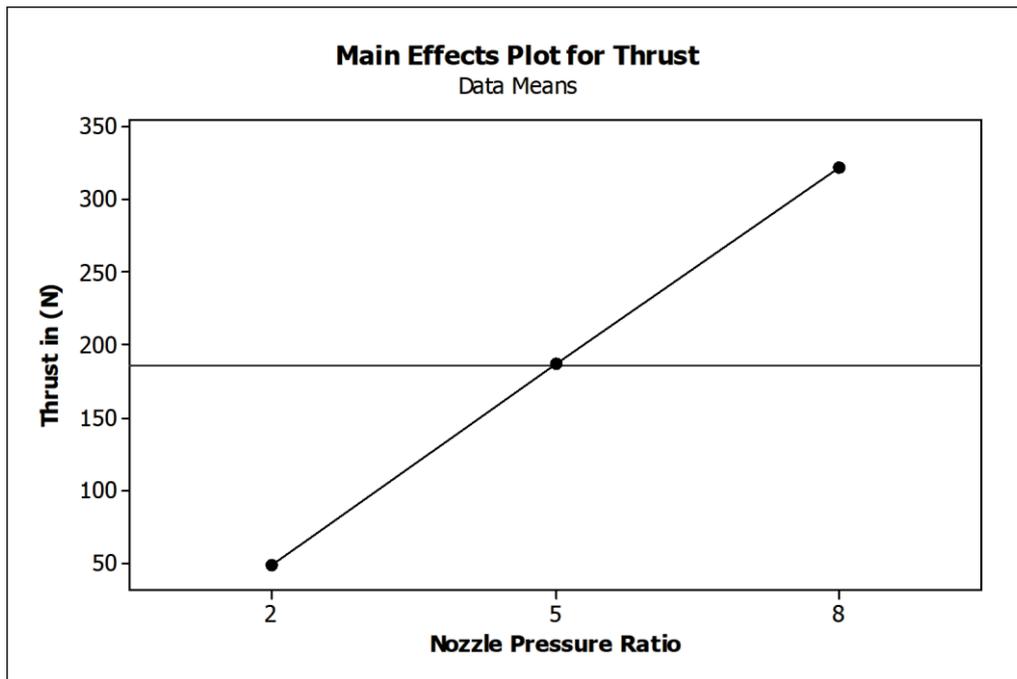


Fig. 10. The main effect plot of nozzle pressure ratio for thrust

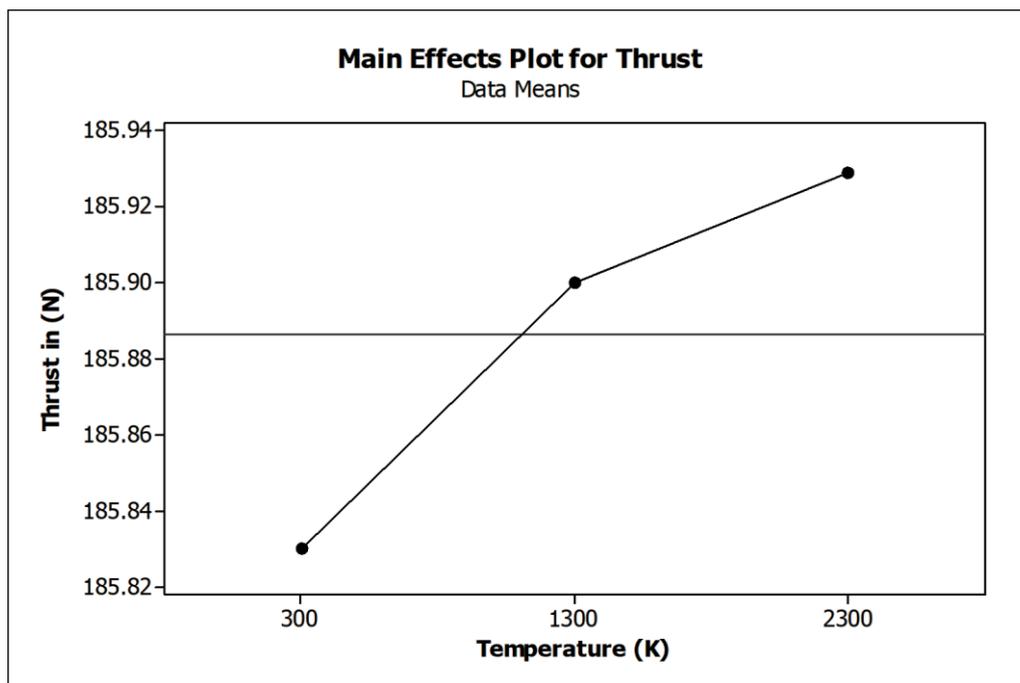
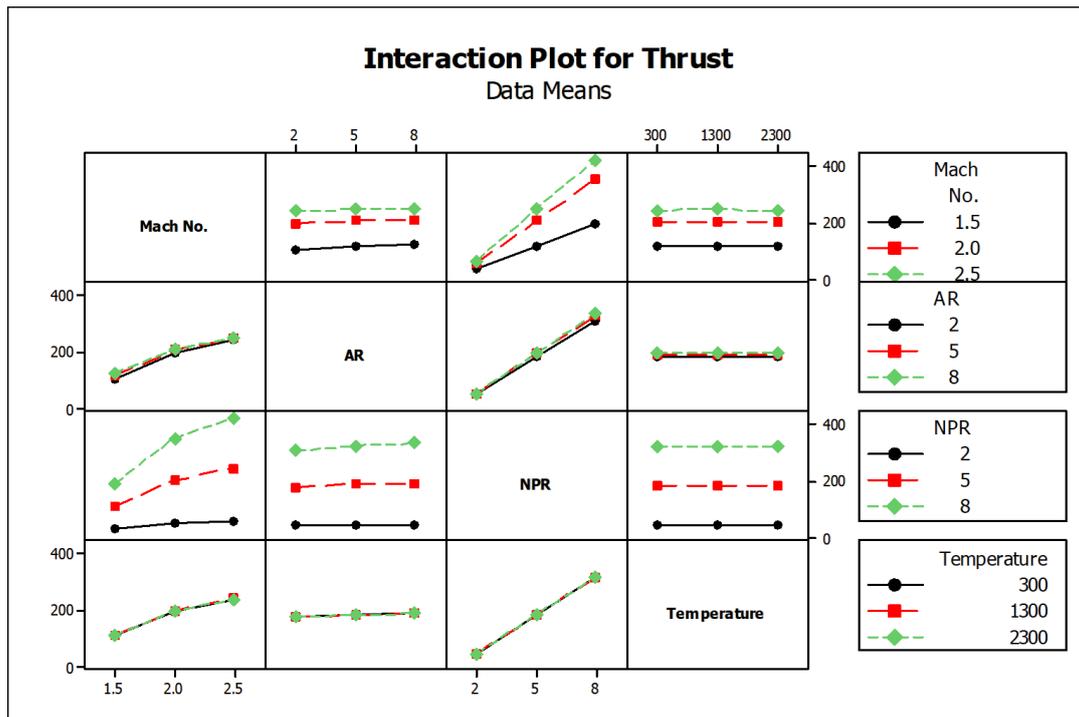


Fig. 11. The main effect plot of temperature for thrust

Figure 12 shows the interaction plots between all the factors, the M, AR, NPR, and the inlet temperature of the air. Based on these figures, it can be concluded that the factors like; M, AR, and NPR have significant interaction with each other. The interaction of the inlet temperature of air with any other factor is relatively small, as shown in Figure 12.



**Fig. 12.** The interaction effect plots for thrust at  $L/D = 5$

## 6. Analysis of Variance (ANOVA)

Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures used to analyze the differences among group means in a sample. The ANOVA is used to analyze the effectiveness of parameters based on two different outputs namely base pressure and the thrust.

### 6.1 ANOVA Based on Base Pressure

The calculations are done for analysis of variance (ANOVA), and the ANOVA is summarized in Table 3. From ANOVA Table 3, it is seen that the factors which significantly affect the base pressure are the NPR, the AR, and the inertia level (i.e., Mach number M). F-value for the interactions between Mach number & area ratio, Mach number & NPR, area ratio & NPR and Mach number, area ratio, & NPR have P-values 0.000, indicating significant interactions between these factors. The other parameters and their interactions with having P-value of more than 0.05 may be considered as insignificant parameters [18].

### 6.2 ANOVA Based on Thrust

The calculations are done for analysis of variance (ANOVA), and the ANOVA is summarized in Table 4. From ANOVA Table 4, it is seen that the factors which significantly affects the thrust are the NPR, M, and AR. F-value for the interactions between Mach number & area ratio, M & NPR, AR & NPR, and M, AR & NPR have P-values 0.000, indicating significant interactions between these factors. The other parameters and their interactions with having P-value of more than 0.05 may be considered as insignificant parameters [18].

**Table 2**  
 Analysis of Variance for Base Pressure, using Adjusted SS for Tests at L/D = 5

Source	DF	Seq SS	Adj SS	Adj MS	F	P
M	2	0.051	0.051	0.026	70.04	0.000
AR	2	0.554	0.554	0.277	758.74	0.000
NPR	2	2.584	2.584	1.292	3537.9	0.000
Temp.	2	0.000	0.000	0.000	0.11	0.896
M × AR	4	0.624	0.624	0.156	427.41	0.000
M×NPR	4	0.346	0.346	0.086	236.84	0.000
M×T	4	0.000	0.000	0.000	0.15	0.959
AR×N	4	0.438	0.438	0.109	299.67	0.000
AR×T	4	0.000	0.000	0.000	0.12	0.975
N×T	4	0.001	0.001	0.000	0.96	0.458
M×AR×N	8	0.931	0.931	0.116	318.77	0.000
M×AR×T	8	0.001	0.001	0.000	0.2	0.986
M×N×T	8	0.002	0.002	0.000	0.72	0.673
AR×N×T	8	0.001	0.001	0.000	0.45	0.872
Error	16	0.006	0.006	0.000		
Total	80	5.540				

**Table 3**  
 Analysis of Variance for Thrust, using Adjusted SS for Tests at L/D = 5

Source	DF	Seq SS	Adj SS	Adj MS	F	P
M	2	253803	253803	126901	107830.3	0.000
AR	2	2741	2741	1371	1164.56	0.000
N	2	1026494	1026494	513247	436114.8	0.000
T	2	0	0	0	0.06	0.942
M×AR	4	804	804	201	170.86	0.000
M×N	4	97458	97458	24365	20703.01	0.000
M×T	4	2	2	0	0.33	0.854
AR×N	4	1709	1709	427	362.99	0.000
AR×T	4	1	1	0	0.3	0.871
N×T	4	4	4	1	0.86	0.508
M×AR×N	8	610	610	76	64.76	0.000
M×AR× T	8	3	3	0	0.33	0.942
M×N× T	8	7	7	1	0.7	0.687
AR×N×T	8	4	4	0	0.42	0.893
Error	16	19	19	1		
Total	80	1383658				

## 7. Effect of Length to Diameter Ratio

In this section, the three levels of length to diameter (L/D) ratio, along with the NPR and M, are considered for the analysis. The area ratio of the duct was kept constant at 5. Table 5 shows the parameters and their levels considered for the CFD analysis. As the inlet temperature of the air does not have a significant effect on base pressure and thrust, the inlet temperature of the air is kept constant at 300 K.

**Table 4**  
 Parameters and Levels for Area Ratio = 5

Parameter	Levels	Values
Mach number (M)	3	1.5, 2.0 and 2.5
Nozzle pressure ratio (N)	3	2, 5 and 8
Length to diameter ratio of the enlarged duct (L/D)	3	2, 5 and 8

### 7.1 Factorial Design for Three Factors with Three Levels

A full factorial experimental design is used to make all the possible combinations of the factors/parameters and levels. In each trial of the analysis, all the possible combinations of the levels of the factors are investigated. The main effect and interaction effects of each factor are studied. The CFD analysis was carried out for a total of 27 different combinations of factors. The responses in the form of thrust and the total base pressure are recorded for all the cases.

### 7.2 The Main Effect and the Interaction Effects Based on Base Pressure

The main effect of factor; the length to diameter (L/D) ratio on the base pressure is plotted using Minitab software to assist in the practical interpretation of the results obtained from CFD analysis. Figure 13 presents the main effect plot of length to diameter (L/D) ratio on the dimensionless base pressure. The main effect plots are the marginal response averages at each level of the L/D ratio.

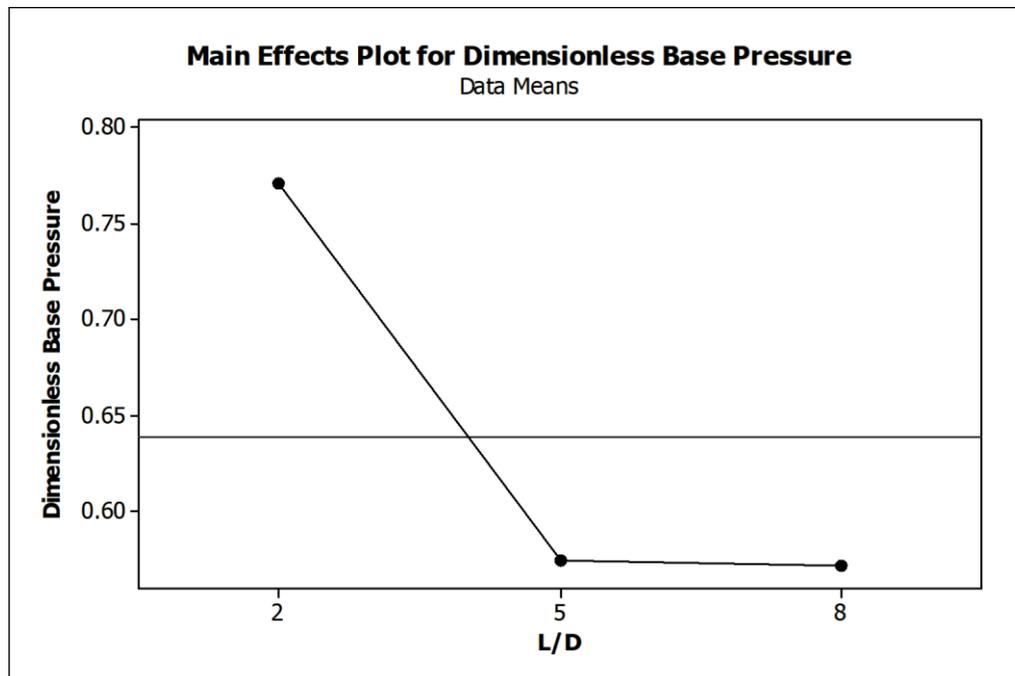
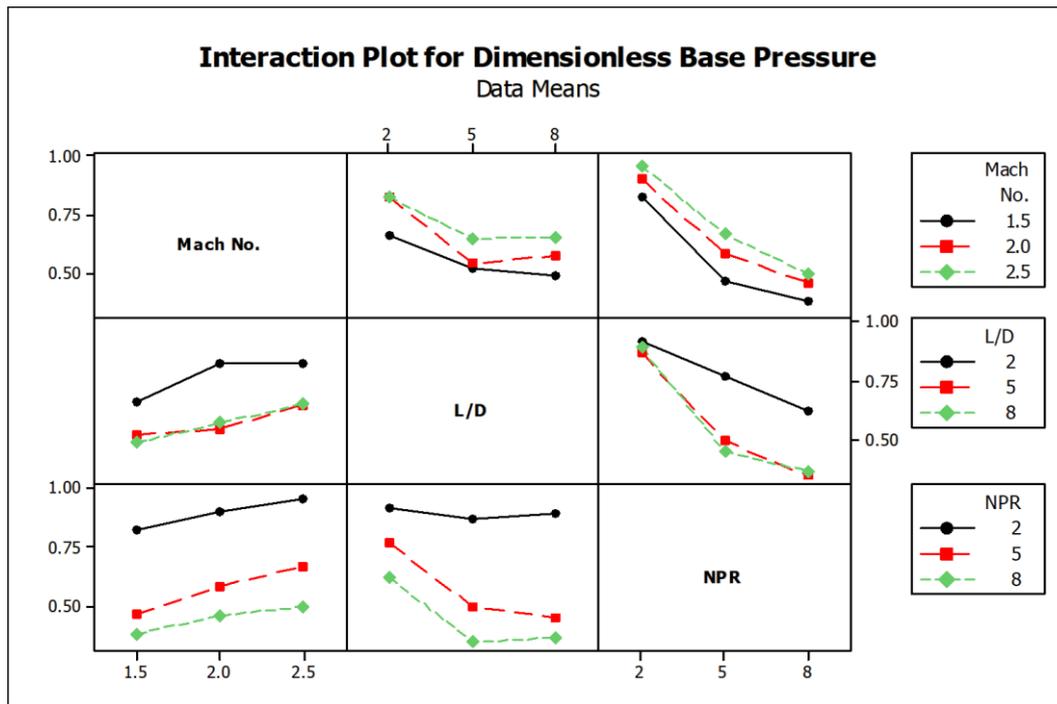


Fig. 13. The main effect plot of L/D ratio for dimensionless base pressure

Figure 13 is plotted by considering mean values of the base pressure for all the cases at the values of L/D ratios as 2, 5, and 8. From the figure, it is observed that the base pressure decreases with the extension in length of the duct (L/D ratio). With the increment in the duct length, the flow reattachment possibility increases and hence close to L/D = 5, in most of the cases the flow is reattached to the duct. As the flow reattached to the duct, the base pressure reduces below the atmospheric pressure. When the length of an enlarged duct again increases from L/D = 5 to 8, the decrement in base pressure is marginal. Figure 13 is plotted by considering mean values of the base pressure for all the cases at the values of L/D ratios as 2, 5, and 8.

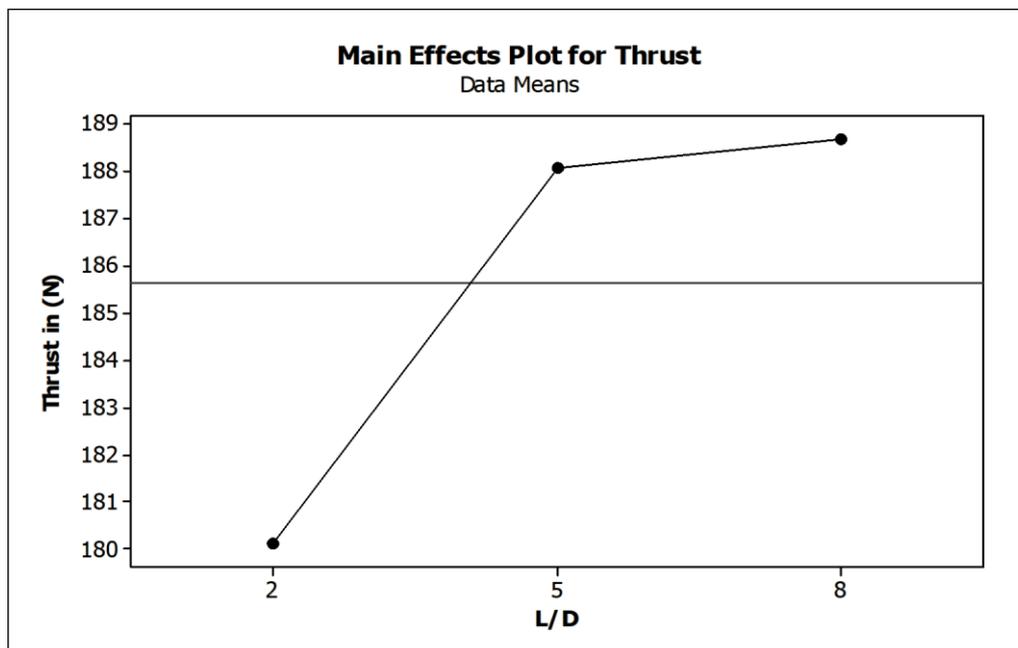
Figure 14 shows the interaction plots between all the factors, the Mach number (M), nozzle pressure ratio (NPR), and L/D ratio. Based on the plots, it is observed that the factors Mach number, nozzle pressure ratio, and L/D ratio have significant interaction between each other.



**Fig. 14.** The interaction effect plots for dimensionless base pressure at AR = 5

### 7.3 The Main Effect and Interaction Effects on Thrust

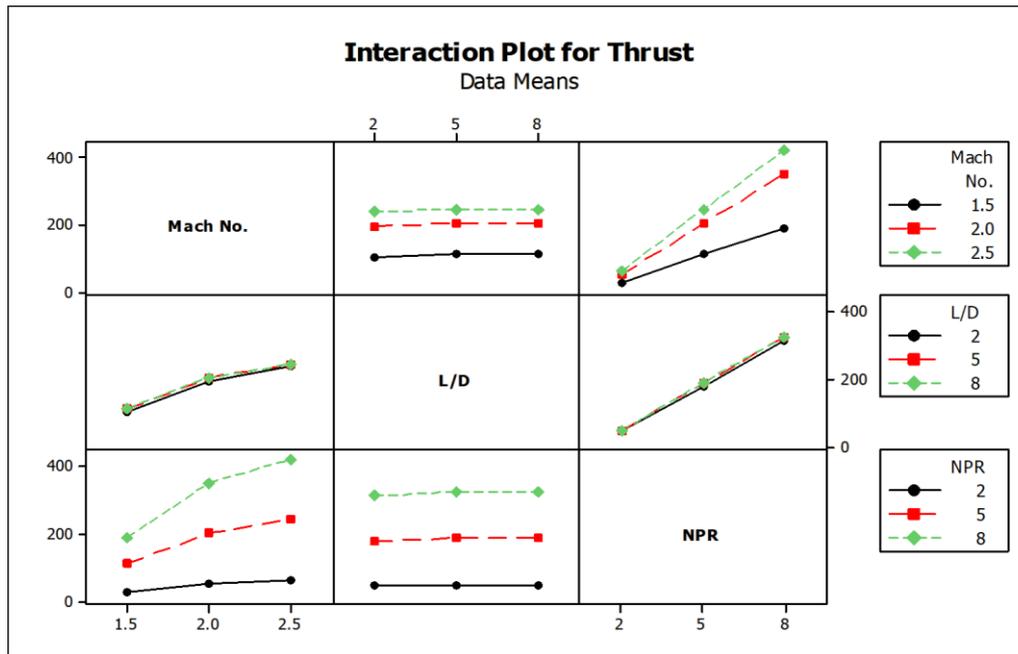
The main effect of factor length to diameter (L/D) ratio on the thrust is plotted using Minitab software to assist in the practical interpretation of the results obtained from CFD analysis. Figure 15 presents the main effect plot of length to diameter (L/D) ratio on the dimensionless base pressure. The main effect plots are the marginal response averages at each level of the L/D ratio.



**Fig. 15.** The main effect plot of L/D ratio for thrust

From Figure 15, it is observed that the thrust increases rapidly with the increase in the L/D ratio of the duct from 2 to 5. When the length of the duct again increases from L/D = 5 to 8, the increase in the thrust is marginal.

Figure 16 shows the interaction plots between all the factors, the Mach number, nozzle pressure ratio, and L/D ratio. Based on the plots, it is found that the factors like Mach number, nozzle pressure ratio, and L/D ratio have significant interaction between each other from L/D = 2 to 5. The interaction of L/D ratio from 5 to 8 with any other factor is marginal, as shown by the similar shape of the lines in Figure 16.



**Fig. 16.** The interaction effect plots for thrust at AR = 5

#### 7.4 Analysis of Variance (ANOVA) Based on Base Pressure

The calculations are completed for analysis of variance (ANOVA), and the ANOVA table is summarized in Table 6 for area ratio = 5.

**Table 5**  
 Analysis of Variance for Base Pressure, using Adjusted SS for Tests at AR = 5

Source	DF	Seq SS	Adj SS	Adj MS	F	P
M	2	0.10442	0.10442	0.05221	3.29	0.091
L/D	2	0.23624	0.23624	0.11812	7.43	0.015
NPR	2	0.9495	0.9495	0.47475	29.88	0.000
M×L/D	4	0.01857	0.01857	0.00464	0.29	0.875
M×N	4	0.00707	0.00707	0.00177	0.11	0.975
L/D×N	4	0.08721	0.08721	0.0218	1.37	0.325
Error	8	0.12711	0.12711	0.01589		
Total	26	1.53011				

From ANOVA Table 6, it is seen that the factors which significantly affect the base pressure are the nozzle pressure ratio (NPR), and L/D ratio. F ratio for the Main effect of Mach number (M) and the interactions between all the factors have P-values more than 0.05, indicating there are no significant interactions between these factors [18].

### 7.5 Analysis of Variance (ANOVA) Based on Thrust

The calculations are done for analysis of variance (ANOVA), and the ANOVA table is summarized in Table 7 for AR = 5.

From ANOVA Table 7, it is seen that the factors which significantly affect the thrust are the nozzle pressure ratio (NPR), Mach number (M), and the L/D ratio. F-ratio for the interactions between Mach number & NPR has P-values 0.000, indicating significant interactions between these factors. The other parameters and their interactions having a P-value of more than 0.05 may be counted as insignificant parameters.

**Table 6**  
Analysis of Variance for Thrust, using Adjusted SS for Tests at AR = 5

Source	DF	Seq SS	Adj SS	Adj MS	F	P
M	2	83954	83954	41977	1914.19	0.000
L/D	2	411	411	205	9.36	0.008
NPR	2	340884	340884	170442	7772.3	0.000
M×L/D	4	39	39	10	0.45	0.772
M×NPR	4	31676	31676	7919	361.12	0.000
L/D×NPR	4	169	169	42	1.93	0.199
Error	8	175	175	22		
Total	26	457308				

## 8. Conclusions

From the above discussions and the results obtained, it is found that the base pressure and thrust created by the flow are significantly influenced by the parameters NPR, M, and AR. The L/D ratio has a significant effect on base pressure and thrust from L/D = 2 to 5. The parameter inlet temperature of the air has no significant effect on the base pressure and the thrust.

The base pressure increases with an increase in Mach number and area ratio and reduces with an increase in nozzle pressure ratio. The thrust increases with an increase in Mach number, area ratio and nozzle pressure ratio. The base pressure reduces with an increase in L/D ratio from 2 to 5. The thrust increases with an increase in L/D ratio from 2 to 5.

Based on the acquired results, the parameters NPR, M, AR, and L/D ratio can be considered for the study of suddenly expanded flows. However, the parameters like area ratio and L/D ratio should be optimized.

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