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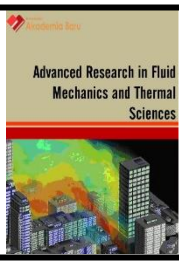

Thermal Effect of Welding Processes on A Steel Plate Subjected to Dynamic Load

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ABSTRACT

The article aimed to determine both numerical and theoretical solution regarding the deflection of a stainless-steel material type 316 in form of rectangular plate having a dimensions of 3 mm thickness, 250 length and 500 mm width of the plate for welding purpose. The theoretical approach which embraced in the current study is the simple plate theory (Kirchhoff's theory) and the derivation was started from this theory to predict the deflection that effect in the plate undergoing dynamic load which are represented by vibration frequency and welding process (thermal temperature) along the plate surface. A vibration action is always correlative to the welding process and causes a deflection in the plate. This deflection was calculated for four mode shape and under various thermal load (welding process). It was found that the maximum deflection during the welding process for the four mode shapes belong to the fourth one with a value of 0.0188 mm, while the lowest one is belonging to the first mode shape with a value of 0.00134 mm. The maximum deviation between the numerical and theoretical approach was found to be less than 5%.

1. Introduction

The welding is one of the important technologies process used in many branches of industry. It's simply defined as joining or cutting materials. More than 30 different types of welding exist, and they range from simple oxyfuel to high-tech processes such as laser beam welding. However, only four welding types are used commonly, and they are GMAW, TGAW, SMAW and FCAW. Each of them comes with its own advantages and disadvantages. In this article study the GMAW welding processes under influence of the dynamic load represented by forced vibration of the rectangular plate is an extremely important area owing to its wide variety of engineering applications such as in aeronautical, civil, and mechanical engineering. The physical and mechanical properties of the metal that affect the degree of distortion change with the application of heat so that different distribution

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residual stresses occur in a welded plate, these stresses are responsible for changing structure properties. Dynamic characteristics of structures are one of the most important properties; the presence of such residual stress in a weld plate affects its distortion behavior and ability to sustain applied loads while maintaining structural integrity [1]. In this paper force vibration analysis of friction stir spot welded plate for several support conditions under different welding parameters (pin shape and tool rotational speed) have been made. The welding processes of rigid plastic mechanism analysis and elastic large deflection analysis are combined in the derivation of the average stress average strain relationship of the rectangular plates [2]. Tensile residual stresses occur due to temperature gradient (welding and heat treatment) and mechanical working (milling, grinding and other operations) [3]. The tensile residual stresses are not preferred in any specimen because it's effective in fatigue life under dynamic loading, crack initiation and propagation and corrosion, which lead to failure [4]. The present paper described the modeling and simulation of the spot-welding process using the finite element modeling technique. The developing of this model could also help a welding engineering to produce welding schedules and to investigate the weld nugget formation for a variety of materials to be joined [5]. The derive exact solutions for the free vibration and buckling problems of the simply supported, clamped, simply supported and clamped plate. The case is solved by the classical power series method of Frobenius [6]. This paper presented the moment and shear force singularity in the vicinity also the free vibration of transversely isotropic sector plates by using the boundary layer function was investigated [7]. They developed the solution in terms of force vibration and free vibration of plates by boundary elements, where at this problem solved with the aid of the Laplace transform concerning time and the matrix coefficients are complex Bessel functions of the frequency parameters respectively [8].

Vibrating a metal during welding process represents a chronicle issue, where the sudden temperature rise causes a rapid expansion followed by rapid contraction. This phenomenon causes metal failure in most cases due to occurred deflections. Many efforts were paid in such studied, but the perfect solution did not attain yet. Hence, the current study embraces both theoretical and numerical study of the welding effect on metal plate to fill the gap in such important field.

2. Analytical models

2.1 Simple Plate Theory

The analytical solution that obtained in this paper on the simple plate theory is called Kirchhoff's theory this work deemed as an expansion of Euler Bernoulli beam theory (Figure 1) [9]

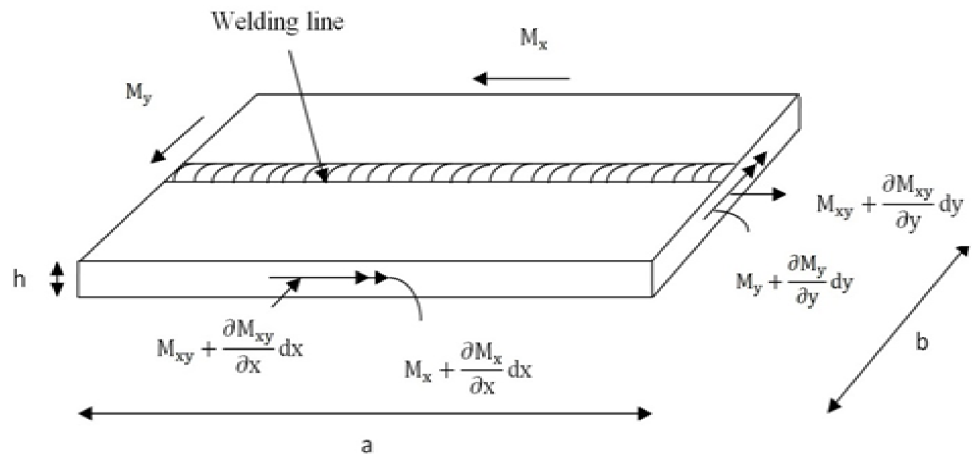

Fig. 1. The rectangular plate

2.2 The Stresses Equations

The stresses equation x and y directions can be derived. The strain in the plate x and y-direction can be expressed as

$$\varepsilon_x = \frac{z}{r_x} \tag{1}$$

where

$$\frac{1}{r_x} = \frac{\partial^2 w}{\partial x^2} \tag{2}$$

$$\varepsilon_y = \frac{z}{r_y} \tag{3}$$

where

$$\frac{1}{r_y} = \frac{\partial^2 w}{\partial y^2} \tag{4}$$

According to Werner and Frederiksen [10], Hooke's law supposable in the form of thermal load:

$$\varepsilon_x = \left(\frac{\sigma_x}{E} - v\ \frac{\sigma_y}{E}\right) + \frac{\alpha z T_{(x,y)}}{h} \tag{5}$$

By the same way

$$\varepsilon_y = \left(\frac{\sigma_y}{E} - v\ \frac{\sigma_x}{E}\right) + \frac{\alpha z T_{(x,y)}}{h} \tag{6}$$

By substitute Eq. (1) and (3), in Eq. (5) and (6) for x and y directions we get

$$\frac{z}{r_x} = \left(\frac{\sigma_x}{E} - v\ \frac{\sigma_y}{E}\right) + \frac{\alpha z T_{(x,y)}}{h} \tag{7}$$

By arranging Eq. (7) we may get

$$\frac{\sigma_x}{E} - v\ \frac{\sigma_y}{E} = \frac{z}{r_x} - \frac{\alpha z T_{(x,y)}}{h} \tag{8}$$

$$\sigma_x = ZE\left(\frac{1}{r_x} - \frac{\alpha T_{(x,y)}}{h}\right) + v\sigma_y \tag{9}$$

$$\sigma_y = ZE\left(\frac{1}{r_y} - \frac{\alpha T_{(x,y)}}{h}\right) + v\sigma_x \tag{10}$$

Final stresses equations σ_x with thermal effect on the rectangular plate, by substitute Eq. (10) in Eq. (9), we get:

$$\sigma_x = ZE\left(\frac{1}{r_x} - \frac{\alpha T_{(x,y)}}{h}\right) + v\left[ZE\left(\frac{1}{r_y} - \frac{\alpha T_{(x,y)}}{h}\right) + v\sigma_x\right] \tag{11}$$

$$\sigma_x = \frac{E\,Z}{1-v^2}\left(\frac{1}{r_x} + v\,\frac{1}{r_y}\right) - \frac{E\,Z\alpha\,T_{(x,y)}}{h\,(1-v^2)} \tag{12}$$

Also, final stresses equation σ_y with thermal effect on the rectangular plate by substitute Eq. (9) in Eq. (10) we get:

$$\sigma_y = \frac{E\,Z}{1-v^2}\left(\frac{1}{r_y} + v\,\frac{1}{r_x}\right) - \frac{E\,Z\alpha\,T_{(x,y)}}{h\,(1-v^2)} \tag{13}$$

2.3 The Bending Moment Equation

By using integration for stresses equations from Eqs. (12) and (13) (σ_x and σ_y) over the thickness z-direction gives the bending moment per unit length in the rectangular plate

$$\mathrm{M_x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x\,dz \tag{14}$$

$$\mathrm{M_x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E\,Z}{1-v^2}\left(\frac{1}{r_x} + v\,\frac{1}{r_y}\right) - \frac{E\,Z\alpha\,T_{(x,y)}}{h\,(1-v^2)}\,z\,dz \tag{15}$$

where

$$D = \frac{E\,h^3}{12\,(1-v^2)} \tag{16}$$

$$\mathrm{M_x} = -D\left(\frac{\partial^2 w}{\partial x^2} + v\,\frac{\partial^2 w}{\partial y^2}\right) - \frac{E\,h^2\,\alpha\,T_{(x,y)}}{12(1-v)} \tag{17}$$

$$\mathrm{M_y} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y\,dz \tag{18}$$

$$\mathrm{M_y} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E\,Z}{1-v^2}\left(\frac{1}{r_y} + v\,\frac{1}{r_x}\right) - \frac{E\,Z\alpha\,T_{(x,y)}}{h\,(1-v^2)}\,z\,dz \tag{19}$$

$$\mathrm{M_y} = -D\left(\frac{\partial^2 w}{\partial y^2} + v\,\frac{\partial^2 w}{\partial x^2}\right) - \frac{E\,h^2\,\alpha\,T_{(x,y)}}{12(1-v)} \tag{20}$$

The final equations for bending moment equations of the X direction and Y direction.

For the governing equation of the plate the Eq. (17) and (20) are not enough to generate it, because its need the torquing moment of the bending in X and Y direction, the following procedure was used to derive the torquing moment:

$$\gamma_{xy} = -2\,Z\,K_{xy} \tag{21}$$

where

$$K_{xy} = \frac{1}{r_{xy}} = \frac{\partial^2 w}{\partial x \partial y} \tag{22}$$

By using the thermal load to the Eq. (20) obtained

$$\gamma_{xy} = -2\, Z \left(\frac{1}{r_{xy}}\right) + \frac{\alpha\, Z\, T_{xy}}{h} \tag{23}$$

The torqueing moment

$$\mathrm{M_{xy}} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy}\ z\, dz \tag{24}$$

$$\tau_{xy} = \mathrm{G}\, \gamma_{\,xy} \tag{25}$$

Substitute Eq. (23) in Eq. (24), we get:

$$\tau_{xy} = -2\, G\, Z \left(\frac{\partial^2 w}{\partial x \partial y}\right) + \frac{\alpha\, Z\, T_{xy}}{h} \tag{26}$$

Substitute Eq. (25) in Eq. (23) and used integration in the torquing moment we get:

$$\mathrm{M_{xy}} = \int_{-\frac{h}{2}}^{\frac{h}{2}} -2\, G\, Z \left(\frac{\partial^2 w}{\partial x \partial y}\right) + \frac{\alpha\, Z\, T_{xy}}{h}\ z\, dz \tag{27}$$

The integration of Eq. (26) gives

$$\mathrm{M_{xy}} = \left[-2\, G\, \frac{Z^3}{3} \left(\frac{\partial^2 w}{\partial x \partial y}\right) + \frac{Z^3}{3}\, \frac{\alpha\, Z\, T_{xy}}{h}\right]_{-\frac{h}{2}}^{\frac{h}{2}} \tag{28}$$

where

$$G = \frac{E}{2(1+v)} \tag{29}$$

Substitute Eq. (28) in Eq. (27), the final torqueing moment is obtained:

$$\mathrm{M_{xy}} = -D\,(1-v)\, \frac{\partial^2 w}{\partial x \partial y} + \frac{h^2 \alpha\, T_{xy}}{12} \tag{30}$$

The governing equations are generated from bending moment they consist the Laplace power four ∇^4 of the x and y direction also the torquing moment equation the starting drives equations $\mathrm{M_x}$, $\mathrm{M_y}$, $\mathrm{M_{xy}}$

$$\frac{\partial^2 \mathrm{M}}{\partial x^2} = D \left(\frac{\partial^4 w}{\partial x^4}\right) - \frac{E\, h^2\, \alpha}{12\,(1-v)}\, \frac{\partial^2 T}{\partial x^2} \tag{31}$$

$$\frac{\partial^2 \mathrm{M}}{\partial y^2} = D \left(\frac{\partial^4 w}{\partial y^4}\right) - \frac{E\, h^2\, \alpha}{12\,(1-v)}\, \frac{\partial^2 T}{\partial y^2} \tag{32}$$

For the twisting moment, we may get

$$\frac{\partial^2 \mathrm{M}}{\partial x\, \partial y} = D\,(1-v)\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{h^2\,\alpha}{12}\,\frac{\partial^2 T}{\partial x \partial y} \tag{33}$$

According to Chakraverty [11], by using the thermal load and dynamic load for three Eqs. (29-31) we obtained.

$$\left[D\,\left(\frac{\partial^4 w}{\partial x^4}\right) - \frac{E\,h^2\,\alpha}{12\,(1-v)}\,\frac{\partial^2 T}{\partial x^2}\right] W_{(x,y,t)} + \left[D\,(1-v)\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{h^2\,\alpha}{12}\,\frac{\partial^2 T}{\partial x \partial y}\right] W_{(x,y,t)} + \left[D\,\left(\frac{\partial^4 w}{\partial y^4}\right) - \frac{E\,h^2\,\alpha}{12\,(1-v)}\,\frac{\partial^2 T}{\partial y^2}\right] W_{(x,y,t)} + \rho h\,\frac{\partial^2 w_{(x,y,t)}}{\partial t^2} = q_{(x,y)}\;e^{i\omega t} \tag{34}$$

By finding the biharmonic operator which is Laplace operator ∇^4 that equal to:

$$\nabla^4 = \frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \tag{35}$$

The final equation expressed the thermal and load effect on the rectangular plate

$$DW_{(x,y,t)}\nabla^4 - \frac{E\,h^2\,\alpha}{12\,(1-v)}\,\frac{\partial^2 T}{\partial x^2} + \frac{h^2\,\alpha}{12}\,\frac{\partial^2 T}{\partial x \partial y} - \frac{E\,h^2\,\alpha}{12\,(1-v)}\,\frac{\partial^2 T}{\partial y^2} + \rho h\,\frac{\partial^2 w_{(x,y,t)}}{\partial t^2} = q_{(x,y)}\;e^{i\omega t} \tag{36}$$

To solution Eq. (34) that assume according to Chakraverty [11] for vibration characteristics with the natural frequency

$$w = W\,e^{i\omega t} \tag{37}$$

Substitute Eq. (35) in Eq. (34) we get:

$$D\nabla^4\,W\,e^{i\omega t} + \rho h\,(-\omega^2) W\,e^{iwt} = q_{(x,y)}\,e^{i\omega t} + T_{(x,y)} e^{i\omega t} \tag{38}$$

This equation has still had the partial differentiation of the rectangular plate, so that gives another condition to solve this partial which is simulated by the Eigenfunction for the plate :

$$W_{(x,y)} = W_0 \sin\frac{\mathrm{m\pi x}}{a} \sin\frac{n\pi y}{b} \tag{39}$$

Where $\mathrm{W_0}$: maximum deflection
Substitute Eq. (37) in Eq. (36) we obtained:

$$D\left[W_0\left(\frac{\mathrm{m\pi}}{a}\right)^4 \sin\frac{\mathrm{m\pi x}}{a}\sin\frac{n\pi y}{b} + 2\,W_0\left(\frac{\mathrm{m\pi}}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 \sin\frac{\mathrm{m\pi x}}{a}\sin\frac{n\pi y}{b} + W_0\left(\frac{n\pi}{b}\right)^4 \sin\frac{\mathrm{m\pi x}}{a}\sin\frac{n\pi y}{b}\right] - \rho h\omega^2\,W_0\,\sin\frac{\mathrm{m\pi x}}{a}\sin\frac{n\pi y}{b} = q_{(x,y)} + T_{(x,y)} \tag{40}$$

After the simplification solve to Eq. (38) we get

$$W_0 \sin\frac{\mathrm{m\pi x}}{a}\sin\frac{n\pi y}{b}\left[D\left[\left(\frac{\mathrm{m\pi}}{a}\right)^4 + 2\left(\frac{\mathrm{m\pi}}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right] - \rho h\omega^2\right] = q_{(x,y)} + T_{(x,y)} \tag{41}$$

2.4 Dynamic Load Analysis

In this work the dynamic load is divided to two branches, firstly vibration characteristics which as represented of frequency and the effect of the rectangular plate, the secondly the thermal load by using welding processes on the rectangular plate. According to the reference [9] the harmonic motion is defined by a frequency which effects on the surface of the rectangular plate

$$q_{(x,y)} = F_0\,\delta\left(x - \frac{a}{2}\right)\delta\,\left(y - \frac{b}{2}\right)\sin\frac{\mathrm{m}\pi\mathrm{x}}{a}\sin\frac{n\pi y}{b} \tag{42}$$

By setting the point load $\mathrm{x} = \mathrm{x}_0$ and $\mathrm{y} = \mathrm{y}_0$ and at the same time $\mathrm{x}_0 = \frac{\mathrm{a}}{2}$ and $\mathrm{y}_0 = \frac{\mathrm{b}}{2}$, the equation become

$$q_{(x,y)} = \int_0^a \int_0^b F_0\,\delta\left(x - \frac{a}{2}\right)\delta\,\left(y - \frac{b}{2}\right)\sin\frac{\mathrm{m}\pi\mathrm{x}}{a}\sin\frac{n\pi y}{b} \tag{43}$$

When $\mathrm{x} = \frac{\mathrm{a}}{2}$ and $\mathrm{y} = \frac{\mathrm{b}}{2}$ we fined the final load equation

$$q_{(x,y)} = F_0 \sin\frac{\mathrm{m}\pi}{2}\sin\frac{\mathrm{n}\pi}{2} \tag{44}$$

The welding process is applied to a rectangular plate represented by high temperature and normal distribution along with the surface plate thickness. In this work assumption for the thermal distribution of x and y directions as elliptic shape

$$T_{(x,y)} = T_0\,(\frac{x^2}{a} + \frac{y^2}{b} - 1) \tag{45}$$

According to references [12-13] that used assumption distribution temperature ellipsoid model for welding process to one dimensional.

Substitute Eq. (43) into Eq. (39) and after substitute all parameters (external force and thermal load) into Eq. (39) we obtained as

$$W_0 \sin\frac{\mathrm{m}\pi\mathrm{x}}{a}\sin\frac{n\pi y}{b}\left[D\left[\left(\frac{\mathrm{m}\pi}{a}\right)^4 + 2\left(\frac{\mathrm{m}\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right] - \rho h\omega^2\right] = F_0 \sin\frac{\mathrm{m}\pi}{2}\sin\frac{\mathrm{n}\pi}{2} + \left[\frac{Eh^3\alpha}{12(1-v)}\,T_0\left(\frac{a}{2}\right) + \frac{Eh^3\alpha}{12(1-v)}\,T_0\left(\frac{b}{2}\right)\right] \tag{46}$$

After simplification of the equation we can be written as final equation :

$$W_{(x,y)}\ \frac{F_0 \sin\frac{\mathrm{m}\pi}{2}\sin\frac{\mathrm{n}\pi}{2} + \frac{Eh^3\alpha}{12(1-v)}T_0\left(\frac{a}{2}+\frac{b}{2}\right)}{D\left[\left(\frac{\mathrm{m}\pi}{a}\right)^4 + 2\left(\frac{\mathrm{m}\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right] - \rho h\omega^2} \tag{47}$$

3. Results and Discussion

The specimen utilized was made of austenitic stainless-steel type 316 ASTM, plates with a nominal thickness of (3 mm) the mechanical properties as shown in Table 1 The nonlinear transient heat transfer FE analysis requires multiple sub-steps within each load step to cover the welding process time and the cooling downtime until the plate reached the ambient temperature. The elapsed analysis time was 600 sec. seven different times of welding were chosen from (0.01 to 600 sec). The heat source to assess the effect on the results of natural frequency and deflection 30.625 KJ.

In Figure 2 and Figure 3, showing the temperature distribution at a path along with the rectangular plate through the spot with different elapsed times, during and after welding. The thermal distributions with a different time step whereat seen that the maximum temperature reached (1514 ºC) in the center.

Table 1
Mechanical properties for stainless steel type 316

Materials properties	Symbols	Stainless steel 316	Units
Modules of elasticity	E	205	GPa
Poisson's Ratio	υ	0.275	
Density	ρ	8.07	mg/m^3
Thickness	t	3	mm
Width	a	250	mm
Length	b	500	mm

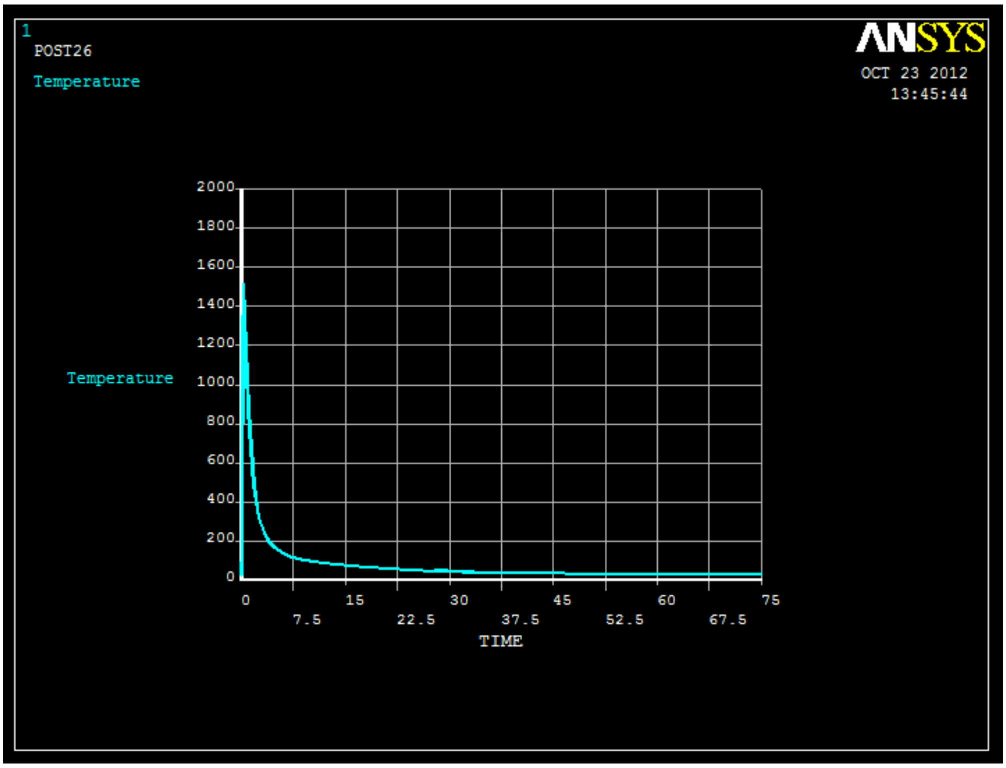

Fig. 2. Temperature time history for the welded plate

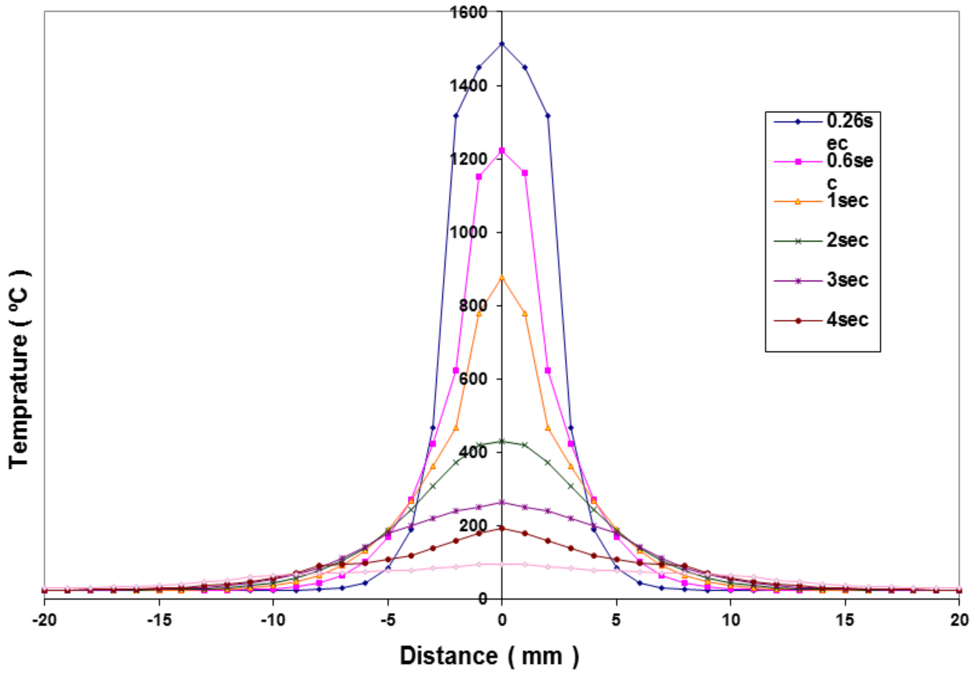

Fig. 3. Temperature Distribution in x-direction for rectangular plate

The dynamic load is an important topic of structural investigation and design. Determining the force vibration characteristics of the structural system. In this paper the natural frequency and mode shapes of the welded processes rectangular plate was the boundary condition Clamed-Free-Clamed-Free. The comparison between the theoretical and numerical results showed in Table 2, the different natural frequency between predicted theoretical and numerical (FEM) data are found to be less than 6%, so the natural frequencies depend on Young's modulus, Poisson's ratio and density of the material, for different mode shapes to same material as shown in Figure 4 to 7, where at the theoretical results compared with numerical solution and found to be satisfactory. The Figure 8 the relation between nature frequency with respect mode shapes so show the increase in natural frequency with an increase in the mode shapes. The deflection from final equation 45 is in direct proportion with the welding temperature. Where, a perfect match was detected between the numerical and theoretical results as they compared, see Figure 9 to 12.

Table 2
Mode shapes and natural frequency of the welding rectangular plate

Mode shapes	Numerical	Theoretical	Error (%)
1st	113.45	109.6	3.39
2nd	140.005	131.7	5.93
3rd	295.16	280.56	4.95
4th	311.78	296.67	4.85

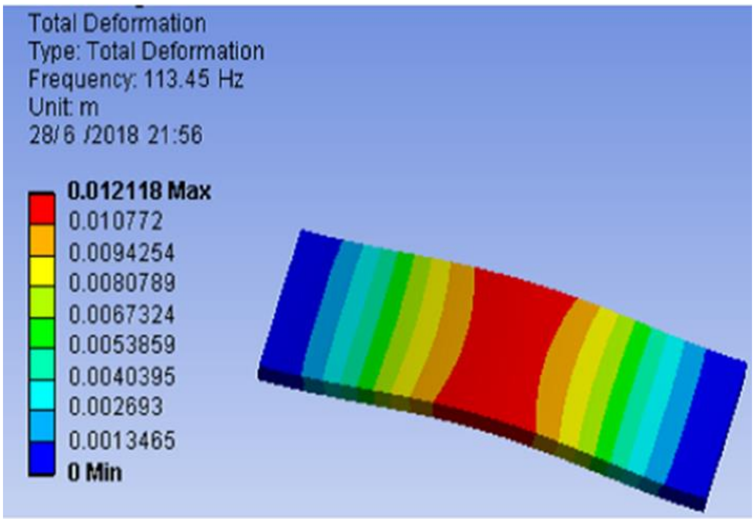

Fig. 4. First mode shape of the welding process for rectangular plate

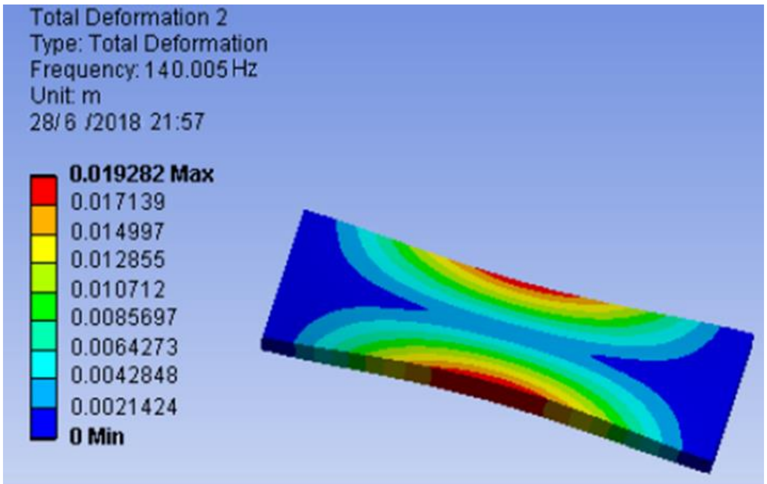

Fig. 5. Second mode shape of the welding process for a rectangular plate

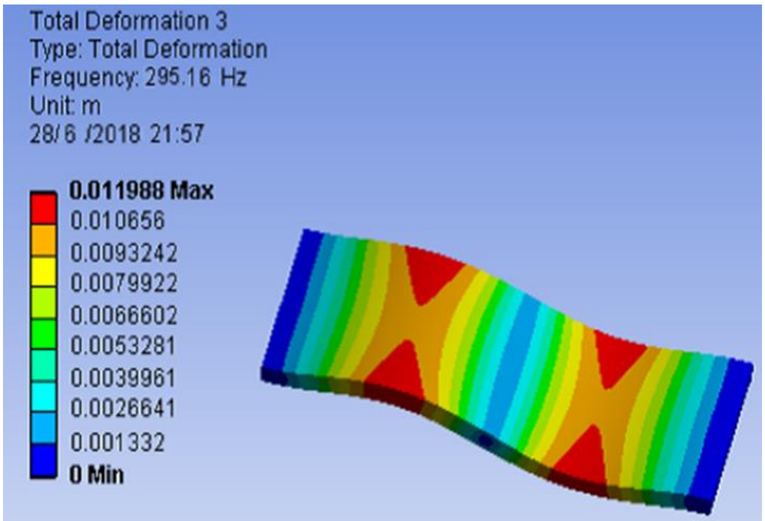

Fig. 6. third mode shape of the welding process for a rectangular plate

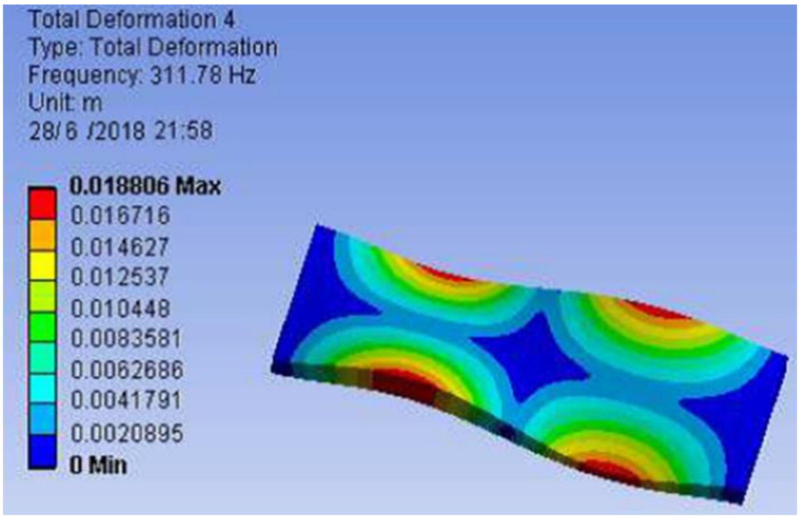

Fig. 7. fourth mode shape of the welding process for a rectangular plate

The mentioned figures surely indicated that the temperature is the master key that controlling the deflection, where the last one increase in rapid rate as the temperature increase. This behavior was noted for all mood shape without any odd behavior.

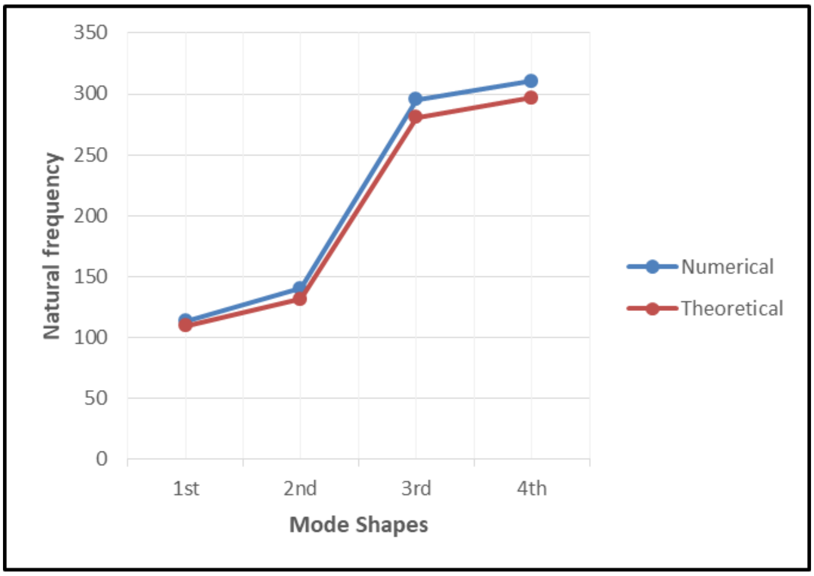

Fig. 8. Natural frequencies for mode shapes of the welding process

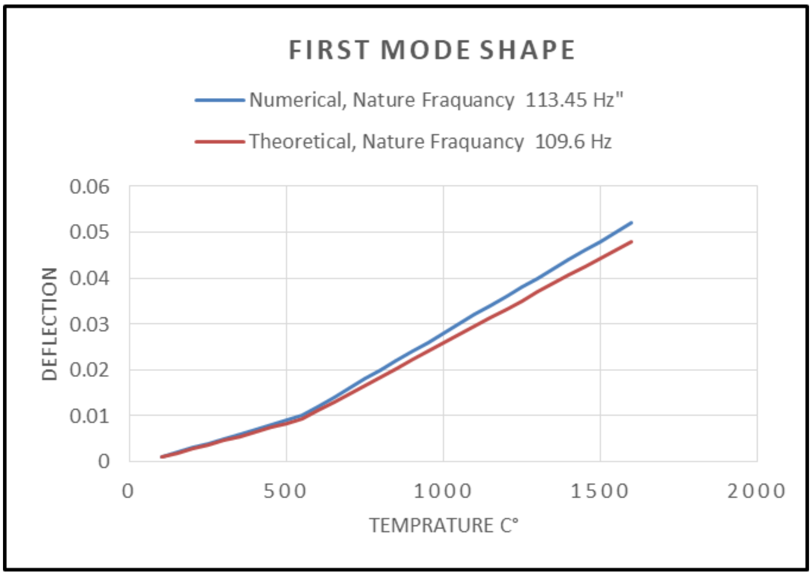

Fig. 9. Deflection and temperature of the rectangular plate for first mode shape

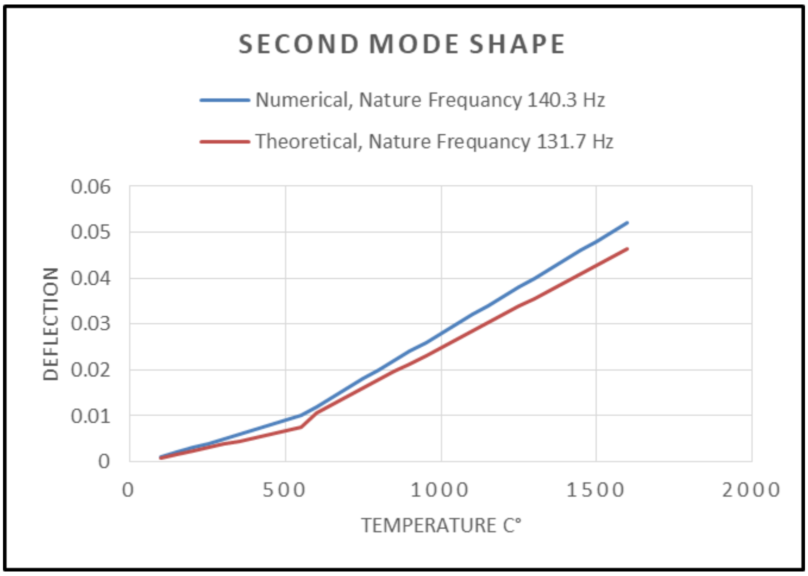

Fig. 10. Deflection and temperature of the rectangular plate second mode shape

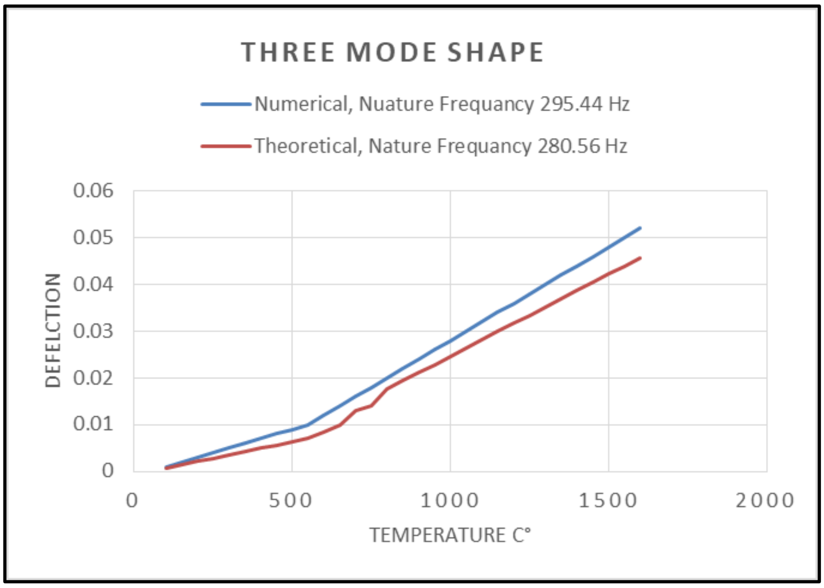

Fig. 11. Deflection and temperature of the rectangular plate for three-mode shape

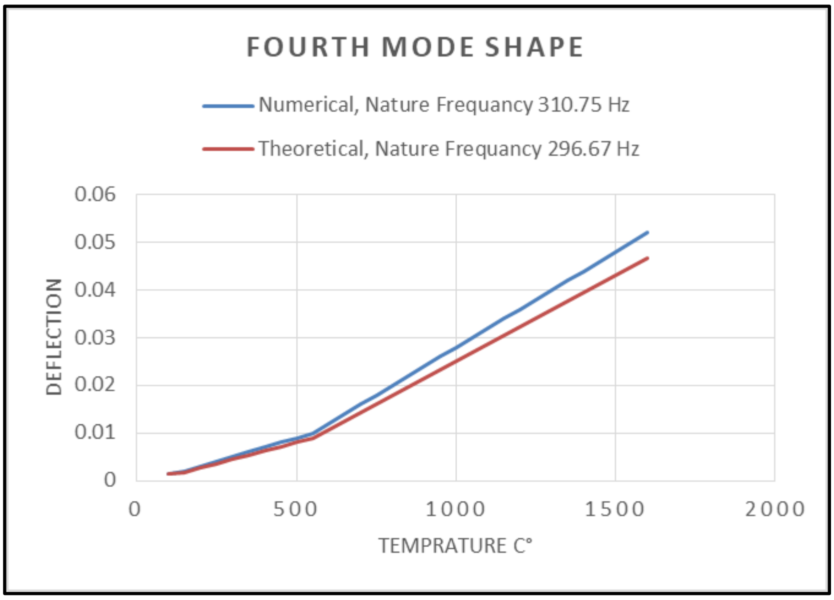

Fig. 12. Deflection and temperature of the rectangular plate for fourth mode shape

4. Conclusion

An analytical model for the deflection due to from high temperature (welding processes) and dynamic load to applied of a stainless steel material type 316 in form rectangular plate, the analytical that used classical plate theory after that obtained equation of rectangular plate which represents all the parameters that act on the plate through of time welding process are external force and high temperature and nature frequency. The analytical results compared with the finite element method, so results showed a good agreement

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