A Review: The Development of Flapping Hydrodynamics of Body and Caudal Fin Movement Fishlike Structure

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Abstract – This paper presents the historical development of description on how the flapping hydrodynamics of fishlike structure grows from infancy to the current state, particularly for the structure in which propel using Body and Caudal Fin (BCF) locomotion. The paper divides the development into four phases; the Inception Phase, the Frozen Phase, the Revival Phase and the Discovery Phase. The key quintessential characteristics that mark the advancement of different phase would be highlighted and elucidated, with some consolidating comments on the future trends of research areas. Copyright © 2015 Penerbit Akademia Baru - All rights reserved.

Keywords: Flapping hydrodynamics, Fishlike structure, Aquatic animals, Underwater vehicle

1.0 INTRODUCTION AND INCEPTION TO FLAPPING HYDRODYNAMICS

The flapping hydrodynamics of BCF fishlike structure is one of the research spotlights internationally nowadays with a hefty amount of papers commenting and reporting the technical development of their own findings, especially since 1990. The research findings and developments are informative in both width and depth, and thus, it could be embattled for one to perceive the overall picture on how the research area develops. Therefore, this paper intends to provide a clear understanding on the historical development of the research domain from its inception in 1936 to the current situation. The history of flapping hydrodynamics of fishlike structure can be dated back to 1936 when Gray’s Paradox [1] was introduced. The theoretical framework of Gray’s Paradox conjectured that the drag coefficient of a swimming dolphin is sevenfold less than a towed rigid model, although the muscle of the dolphin is not able to produce the equivalent power. Gray explained this peculiarity with a notion that dolphin was granted with the ability to maintain a fully laminar boundary condition during swimming. The justified the reconciliation between these paradoxical phenomena of Gray’s Paradox cropped up as the focus of researches for the next 60 years [2-4], and Gray’s research was a contending issue discussed by much of the researchers [5, 6]. The research activities of flapping hydrodynamics of aquatic animals undergo three major phases of development: the Frozen Phase (1936 to 1970), in which the research activities were scarce and dormant; the Revival Phase (1970 to 1990), in which some hypothesis were generated based on mathematical justification; and the Discovery Phase (1990 to today), in which the research activities are blossoming in tandem with the rapid development in both experimental and computational approaches. These phases of development are shown in Figure 1.
2.0 THE FROZEN PHASE (1936 to 1970)

In the Frozen Phase, the trail of justification for Gray’s paradox floundered from 1936 to 1970 as very few papers could be found regarding this topic. Since most of the time the dolphins and other aquatic animals were swimming in the range of turbulence [4], the lack of understanding on turbulence flow and the dearth of efficacious experimental tools had hampered the development of the related researches for around three decades. The research activities flourished again after 1970 due to the entrenchment of Slender Body Theory [7, 8] and the rapid development of turbulence modelling [9, 10, 11], which provided a jump step to investigate the flapping hydrodynamics with turbulent basis.

The very few works existed during the Frozen Phase including further study on linearised body boundary conditions [12] and the development of dye visualization methods [13].

Nonetheless, in general, since the introduction of flapping hydrodynamics of aquatic animals become the interests for both scholars in marine biologist and engineering scientists on unravelling the myths of their swimming, and working on the feasibility to emulate those aquatic animals to design unmanned underwater vehicle (UUV). The research samples have been more inclusive and they comprise not only the locomotion of BCF swimmer especially dolphin, but also other aquatic animals with varied swimming styles such as Bird Wrasse [14], Atlantic Cownose Ray [15], eels and trouts [16].

3.0 THE REVIVAL PHASE (1970 to 1990)

In 1970, Lighthill proposed the Slender Body Theory [7], perceived as the emblematic work during the Revival Phase to examine and explain the performance of swimming fish. In his theory, he assumed the cross section of the fish body would give diminutive and ignorable effect on the fluid parameters and thus, the investigation followed would be fettered in two-dimensional notions. He explained that the body undulation with the fins would transfer the energy to the tail and the tail would further impart momentum to the trail of fish to generate the propulsion.

Under such purview, the concomitant topics of interest would develop into the self-displacement of a deformable body in a perfect fluid without shedding vorticity, the
development of associating drag, the possibility of the existence of optimal propulsion and large amplitude elongated body theory, as profoundly discussed by Sparenberg [17].

Further researches on the linearised boundary solution were continued by Blake [18], Ellington [19], Videler [20] and Ellington et al. [21]. Seminal works on its applications have been made, and the unsteady mechanism is identified as the imperative reason for achieving large loading on wings. However, the theory could be unsound when the extent of undulation spirals up to very large amplitude [22]. The over-prediction of the thrust production would be another problem using this linearised boundary condition. This theory is only suitable for a simple geometry slender body, and this was further justified by Hamid et al. [23] recently when they proved a good compromise between Slender Body Theory and CFD turbulence modelling for some simple slender bodies moving with constant speed.

Another illuminating research arena during this phase will be the foundation of introductory and fundamental inspection on fast start mechanism of fish. This mechanism plays its role when fish predates and escapes. Weihs [24] proposed that the hydrodynamic forces of fish could be harnessed from body segments undulation and the sum of the momentum circulatory forces occurring from each sharp-edged body segments. He had delineated the kinematics for the C-start of fish. Its performance is further improved by the lateral profile of its fins [25]. The related mechanisms will be further developed in the Discovery Phase.

In the point view of experimental works, the experimental techniques mainly applied would be visualization techniques, which encompassed dye visualization [26] and visualization using stratified layer [27]. These techniques are applied to observe the formation of vortices and acquire some macroscopic information, albeit with their two dimensional constraint.

At this stage, even though the discovery is confined to two dimensional dynamics, it engendered far-reaching effects to the future development of flapping hydrodynamics as it provides fundamental mathematical and conceptual understanding on how fish swims.

### 4.0 THE DISCOVERY PHASE (1990 to present)

Since the advent of turbulence modelling including Reynolds Averaged Navier Stoke (RANS) with its modelling, direct numerical simulation (DNS), large eddy simulation (LES) and many other computational techniques, the theoretical and mathematical development have brought the further research of perplexed flapping hydrodynamics into sphere to possibilities. In other words, the development of mathematical models has proceeded from linearised inviscid two dimensional flow assumption to increasingly more complex configurations and computer-intensive approaches, for instance by extending the linearised 2D methods to 3D wings and to nonlinear inviscid and viscous approaches [28].

The major characteristics of research trend in this phase are that, the likelihood to simulate and capture a three-dimensional view of study are conceived and developed. From 1990 to hitherto, much progress has been made, including myriad aspects of considerations such as optimization of swimming performance, development of vorticity control theory, and application of CFD and the contemporary experimental techniques and initial development of UUV.
4.1 Optimization of Swimming Performance

One of the key questions that the biologists and engineering scientists would like to access is the evaluation and optimization of swimming performance. The process requires the quantification of physical parameters and their correlations. The popular parameters involved may include Strouhal number ($St$), drag coefficient ($C_D$), Lighthill number ($Li$) and the propulsive efficiency ($\eta$), which can be defined as:

\[ St = \frac{fA}{U} \]  
\[ C_D = \frac{2F_D}{\rho u^2 A} \]  
\[ Li = \frac{SC_D}{h^2} \]  
\[ \eta = \frac{FU}{P} \]

In which $f$ is the tail beat frequency, $A$ is the mean lateral excursion of caudal fin at trailing, $U$ is the upstream velocity of the flow, $F_D$ is the drag force, $u$ is the instantaneous velocity of the flow, $S$ is the total surface area of the fish, $h$ is the vertical length of caudal fin, $C_D$ is the drag coefficient, $F$ is thrust force of the fish, and $P$ is the total power consumed.

In 1991, Tokamaru and Dimatakis [28] discovered that the rotary oscillation will give mass impact on the flow when the frequency is closed to Strouhal number. The separation points will move relatively much larger in a forced oscillation. Tyriantafyllou et al. [29,30] made their statement clearer by demonstrating that most of the cetaceans would swim within a narrow range of Strouhal number and a hypothesis had been deducted that the Strouhal number would correspond to the maximized stability of vortex wake thrust jet. In other words, the idea had emerged to claim that optimum efficiency can be obtained when the foil oscillation frequency coincides with the frequency of maximum amplification of disturbances, which can be determined from the detailed linear stability analysis of the wake.

An early experimental work by Fish [31] and Taylor et al. [32] reached a consensus that most of the fishes in the nature swim within the range of $0.2 < St < 0.4$. Experiment substantiation of the preferred Strouhal number of odontocete cetaceans was further carried out in 2004 for 248 species by Jim and Fish [33] as shown in Figure 2. The research found predominant constraints of locomotion which would only include one to two fluke-beat-swimming frequency normalized by the ratio of swimming speed to body length and the propulsive efficiency peaks at $St = 0.225$ to $0.275$. Further research by Triantafyllou et al. [29] also denoted that the vortices formed optimally when $St = 0.25$ to $0.35$. Godoy-Diana et al. [34] and Politis et al. [35] disclosed a similar conclusion that the optimum efficiency can be observed at $0.2 < St < 0.4$.

Eloy [36], nonetheless, questioned the findings that the optimum performance shall fall within the narrow range of Strouhal number. Using a new dimensionless parameter named as
Lighthill number ($Li$) which measures between the drag and typical thrust, he plotted a relationship between $Li$ and the Froude efficiency ($\eta_F$), and he discovered that the optimal swimming Strouhal number will be between 0.15 and 0.8 as shown in Figure 3. His research revealed that the propulsive performance of fish cannot simply be governed by mathematical equations as computed before this because there are still some other controllable parameters that a locomotors designer shall take into consideration.

Esfahani et al. [38] came out with a valuable finding that at $St < 0.2$ and $St > 0.6$, the hydrodynamic performance can reach its pinnacle when the length of caudal fin goes infinite, and the improvement of the hydrodynamics efficiency should be made within the range of $0.2 < St < 0.6$ by manipulating the length of the caudal fin.

![Figure 2: Propulsion efficiency of cetaceans as a function of Strouhal number ($St$). Colors indicate particular species: Tursiopstruncatus (red), Pseudorcacrassidens (green), Orcinus orca (blue) and Delphinapterusleucas (black) [33].](image)

Another alternative idea of determining the optimum swimming ability is through the biological measurement on the critical swimming speed. Plaut [39] executed an experiment to observe the fish swimming under controlled alteration of flow surrounding. The time required before fatigue sets in was recorded. Being distinct from the mathematical explanation, he concluded that the performance of swimming can also be measured based on its requirement to cater the needs of swimming, which comprise sustained swimming (> 200 min), prolonged (20s – 200 min) and burst (< 20s). Critical swimming speed is proportional to the routine activity levels. This can be viewed as a continuation of work from Brett [40] and Beamish [41] during the Revival Phase, and a consolidation of the advocates of Watkin [42] and Drucker [43] that swimming is the principal means for evading predators and any unfavourable condition.

Although varied point of views and research ideas coexisted and developed, an obvious conclusion can be clinched here. The prominent ability of fish to exploit the flow has led to its transcendence of human underwater locomotion in ways such that it generates large and short duration forces efficiently and coordinates the rhythmic unsteady body and tail motion to minimize energy consumed [44]. Some favourable advantages are recently identified that the flapping hydrodynamics mechanism possesses excellent locomotion performances to natural creatures including high efficiency, long endurance ability and low noise [45].
Figure 3: Strouhal number of 53 different species of aquatic animals as a function of the Lighthill number. These animals are divided in different categories corresponding to the different symbols displayed in the legend. The solid line is the predicted optimal Strouhal number and the dashed line corresponds to the interval for which efficiency is larger than $\eta_{\text{max}}-1$. The horizontal dotted lines correspond to the interval $0.25 < St < 0.35$ suggested by Triantafyllou et al. [29, 36].

4.2 Vorticity Control Theory

Scientific explanation of the consummated swimming performance of the fish due to its ecological evolution [46, 47] is one of the curiosities that human would like to bring to light. Derived from the quantification of fluid parameters as elucidated in previous section, engineering scientists started to summarize and figure out the physical mechanism that governs the flapping hydrodynamics of fish.

For the early development of the swimming mechanism theory, Ohmi et al. [48, 49] discovered that at large incidence, the patterns of vortex wakes hinge upon whether translational or rotational motion dominates the flow. When the flow is rotational, it is governed by a parameter proportional to Strouhal number.

With such backdrop and the conclusion reached by the Slender Body Theory, the mechanism of swimming fish is further clarified by other researchers such as Gopalkrishnan et al.[50], Streintlian et al. [51] and Anderson [52] that the active control of shear flow would induce unsteady flow using the oscillating foil.

Subsequently, researchers including Koachesfahani and Dimantakis [53] and Cortilezzi [54] suggested that the body undulation mechanism could alter and reposition the oncoming vorticity, and Gopalkrishnan et al. [50] and Anderson [55] further developed the idea that the fish possesses the capabilities to recapture energy contained in the eddies of an incoming flow using vorticity control. Henceforth, the Vorticity Control Theory predominates the science of flapping hydrodynamics.

Wolfgang et al. [22] expounded an important finding that along the contraction region of the body, counterclockwise vorticity is fully developed, followed by the clockwise vorticity on the other side, and these vortices will be shed continuously to form a reverse von Karman street. Resulting in development of thrust jet, the process of formation of wake vortices
through the release of body generated vorticity and subsequent tail manipulation was revealed and justified.

Figure 4: The example of formation of vortices, or reverse Von Karman street, in the wake of undulating fish in a three-dimensional view [56].

Triantafyllou et al. [44] enhanced the vorticity control theory by giving a clear interpretation on how the oncoming vortices react with the foil-shed vorticity where the oncoming vortices are shed at the separation near the leading edge. Two years later, Zhu et al. [56] outlined the definition of Vorticity Control Theory as “the process of altering the position and strength of oncoming vortices and generation of additional vorticity, thereby affecting the load distribution on the body and the unsteady fluid dynamics”. Three modes of vorticity control are identified as illuminated in Figure 4:

1. Vorticity annihilation, where foil-generated vorticity interacts destructively with the oncoming vorticity, resulting in the generation of a weak vortex street downstream, and the propulsive efficiency is maximized.
2. Constructive interference, where foil-generated vorticity interacts constructively with the oncoming vorticity, resulting in the generation of a strong vortex street downstream and the drag, and thrust force production is maximized.
3. Vortex pairing, where the foil-generated vorticity interacts with the oncoming vorticity of opposite sign, resulting in pairs of vortices.

4.3 Application of CFD

One of the imminent aspects that differentiate the Revival Phase and Discovery Phase is the feasibility to apply computational fluid dynamics and the relatively more modern apparatus to study the flapping hydrodynamics of fishlike structure such as PIV and DPIV.

The sheer superiority that the researchers have during the Discovery Phase by using CFD approaches are the capabilities to study the fluid flow within a wide range of flow conditions, moving boundaries, unsteady flow mechanism and also to scrutiny even on the two-way fluid structure interactions [57]. The CFD provides a stronghold to develop the quantification of parameters and mathematical prediction of swimming capabilities. Application of CFD methods can be divided into a commercial software based simulation and self-developed coding based simulation.
4.3.1 Commercial software based simulation

For commercial software based simulation, the development of computer processors coupled with accessible turbulence models in commercial software namely COMSOL, OpenFOAM, ANSYS FLUENT, ANSYS CFX and STAR-CD has greatly brought advantages to delve into the research domain. Other than finite-volume-method-based ANSYS codes and other commercial software, there are quite a number of alternative techniques are being developed. Avalanche of researches based on CFD approaches are developed, and the advent of CFD techniques had made a leap for the deeper understanding of biohydrodynamics.

4.3.2 Self-developed coding based simulation

Since there are some computational hindrance existed in the commercial software based simulation such as the relatively high computational cost and encumbrance for computational modifications, researchers also developed some alternative numerical avenue to investigate the swimming hydrodynamics.

A review paper written by Deng et al. [45] have forged a framework about the available contemporary numerical methods, and the authors categorized the methods as the body conforming mesh methods, Cartesian mesh methods, overset grid methods and mesh free methods.

The body conforming mesh methods are meshing solutions generated to conform the instantaneous shape of moving structures. However, when the body conforming methods are not able to cater the mesh requirement, Cartesian mesh methods will be more compromising. Due to its effectiveness in incorporating multibodies and moving boundaries, overset grid method is a popular alternative. Meanwhile, the mesh free methods are developed currently as an accurate and stable numerical solution for integral equations and partial differential equations with a set of arbitrarily distributed particles without the application of meshing process. The examples of researchers that involved in the application of those methods will be underlined as follows.

The body conforming methods are quite commonly applied due to its wide coverage including interpolation method, the arbitrary Lagrangian-Eulerian method (ALE), the deforming spatial domain or stabilized space-time finite element method and finite volume method (DSD/SST). The researchers who applied the interpolation method include Tang [58], Di et al. [59] and Zhang [60]. ALE is also described and applied by Ramamurti et al. [61], Ramamurti and Sandberg [62], Ramamurti et al. [63], Heil et al. [64] and Sahin and Mohseni [65]. The DSD/SST finite element method is popularly used by quite a number of researchers including Tezduyar et al. [66], Tezduyar [67, 68], Tezduyar et al. [69, 70], Wang et al. [71], Takizawa and Tezduyar [72, 73], Tian et al. [74] and Bazilevs et al. [75].

The examples of CFD-based researches under the scope of applying Cartesian mesh methods include the application of immersed boundary methods [76, 77, 78, 79] and Lagrangian approaches using regularized fluid or vortex particles [80, 81]. Cartesian mesh methods also embrace the Lattice Boltzmann method (LBM) which considered the kinematics of particles [82, 83, 84]. Deng et al. [44] recommended the combination of LBM and immersed boundary methods as an alternative for CFD simulation due to its aptness to inhibit the fluctuation of force acting on the deforming structure. Several papers using this method are listed including the work done by Tian et al. [84], Sui et al. [85], Hao et al. [86], Zhu et al. [87], Zhang et al. [88], Xu et al. [89], Xu et al. [90].
For overset grid methods, Sun et al. [91], Liu et al. [92] and Liu [93] have adopted this method in their simulation, while the authors who applied mesh free methods are also listed here for some references such as the work by Eldredge et al. [94], Hieber and Koumoutsakos [79], Cohen et al. [95] and Kajtar et al. [96, 97].

Apart from body conforming mesh methods, Cartesian mesh methods, overset grid methods and mesh free methods, there are also papers that report their application on direct numerical simulation (DNS) to enhance the understanding on biological fluid dynamics are Borazjani and Sotiropoulos [78], Dauptain et al. [98], Mittal et al. [99], Herschlag and Miller [100] and Nakata and Liu [101].

Rather than the finite domain methods as pontificated above, Boundary Element Method (BEM) cropped up to be a robust way of simulation due to its simplicity in discretisation effort, and thus negated the necessity to undertake the complex remeshing processes. The BEM-based works reported include Bose [102], Liu et al. [103], Politis et al. [104], Mantia and Dabnichki [105] and Floc'h et al. [106].

### 4.4 Application of the contemporary experimental techniques

The advent of varied experimental methods dotted the Discovery Phase of biohydrodynamics as well. More specifically the advancing experimental apparatus contributes to the research of two principal arena: observation of the wake pattern of flapping structure during propulsion, maneuvering, hovering or fast-start, and recording for the interaction of foils with the oncoming eddies [107].

The researchers who deployed experimental PIV in locomotion capturing include Stamhuis and Videler [108], Anderson [55], Muller et al. [109], Epps and Techt [110] and Wolfgang et al. [111]. Besides that, the visualization techniques were further improved by employing digital particle image velocimetry (DPIV), and some researchers that involved in this undertaking include Anderson [52], Muller et al. [109], Peng et al. [112] and Drucker and Lauder [113]. The sheer advantages of applying DPIV are its capabilities to capture the instantaneous movement in a smaller time step and the corresponding wake structure in three-dimensional views.

Camcorder was also used by Jim and Fish [33] to capture swimming sequences of cetaceans at a rate of 30 frames per second. Moreover, a new design of apparatus configuration was proposed by James et al. [114] to capture the wake patterns of the wake of flapping foils. High-speed digital vidicon is made up of high-speed mega-pixel CMOS camera (Mikrotron MC1310), digital video capture card (IO industries CLFC) and high-speed hard-disk were deployed by Qin et al. [115] to study the Carangiform robotic fish. Chadwell et al. [116] also deployed the capturing camera to expand the analysis of median fin function during the escape response of bluegill sunfish (Lepomismacrochirus), particularly the soft dorsal and anal fin surfaces.

Besides the observation on the living and swimming fishes using various kind of video recorder, models emulating the fish are also devised to simplify and quantify the fluid fields. For instance, the tests of single degree of freedom (DOF) flapping foil using MIT Ship Model Testing Tank were reported by Licht et al. [117]. Moreover, aquasi-steady aerodynamic model was developed by Menozzi [118] based on unsteady aerodynamic mechanisms. Ding et al. [119] established a 3 degree of freedom model of turtle with a heaving motor, and his work was followed by the design of circulating water tunnel for future experimental research.
in flapping hydrodynamics. The similar research methodology was applied to Zhou et al. [120] who studied the backward swimming for unsymmetrical structure bio-inspired robotic fish using the gaits planning method and a prototype of robotic fish. The related investigation based on self-designed flapping structures resembling fish locomotion were also reported by Muijres and Lentink [121], Beal and Bandyopadhyay [122],

In short, the development of digital and electrical technology has improved the experimental feasibility to provide a real-world observation of swimming fish in a detailed manner.

4.5 Initial development of UUV

Notwithstanding the flaw in theoretical development and detailed study on the correlation between various fluid parameters, some researchers have started to engage in the design and development of UUV. Even though there are some biological limitation on the fishes, those fishes are widely justified as competent and high-efficiency swimmers due to ecological evolution [123]. The activities of incorporation of novel structures and mechanisms from nature into the design and function of engineered system, or known as biomimetics, are carried out during the Discovery Phase as well.

The initial research and development of UUV could be an auspicious way to examine the effect of flapping UUV under the controlled and simplified locomotor. It is possible to take a range of studies from simple physical models to more highly biomimetic robotic devices that allow one to investigate fundamental questions about propulsion with much greater control that can be achieved by the living animals [124].

Barrett et al. [125] constructed a robotic underwater hull vehicle and his works showed a high propulsive efficiency through boundary relaminarization vorticity control by the tail notwithstanding the restricted range of variables.

Nakashima et al. [126] are inspired by dolphin to develop a UUV that is able to move in three dimensions, while Kemp et al. [127] also proposed a prototype propelled by flapping fins. Xu et al. [128] developed Flapping-Fins Propelled UUV prototype “Robo-Manta I”, inspired from the swimming pattern of batoid species. Besides that, James et al. [129] also developed a biorobotic flapping fin for propulsion and maneuvering.

John et al. [130] and Patar et al. [131] designed a robotic fish in which the movement is controlled by pectoral fins. Liang et al. [132] also devised a two-joint robotic fish for real world exploration.

Hultmark et al. [133] engineered robotic lamprey for scientific investigation, and it was improvised by Root et al. [134] for further experimental and mathematical studies.

Hu et al. [135] constructed water-walking inspired by insects and spiders, meanwhile Samuel [136] envisaged an idea of designing a UUV whereby its functions are a hybrid of walking and swimming.

Despite a broad range of UUV have been designed, the development of UUV is also subjected to the examination of propulsion optimization and the energy required for turning and fast-start, as extensively reported by Triantafyllou et al. [137].
5.0 FUTURE DEVELOPMENT

Although the kinematics and mechanical systems developed from the previous researches are referred to engineer the UUV, but it is still unable to lead to propulsive advantages all the time [138, 139]. More improvements are direly required to clarify the underlying mechanisms and mathematical description of biohydrodynamics.

Lauder [118] listed some questions that the future researchers shall be heeding, which encompass the plotting of 3D biological geometry, the equations for varied type of locomotion, the 3D wake structures of vortex wake, the corresponding flow effect due to different types of locomotion and the response to the natural surroundings [140]. The development trends proposed here are legitimately perfect as a reference for future research. Laura et al. [141] also figured out some attributes that future researchers can work on which may encompass the accurate simulation of deforming structure associated with the fluid structure interaction, the necessity to discover tailored mathematical equations and numerical approaches pertinent to different cases and the development of vortex sheet method to examine the particular difference existed in flapping hydrodynamics.

To recategorise the ideas outlined by previous reviews, it can be summarized in a simpler way that the future development of flapping hydrodynamics of fishlike structure can be divided into three domains of research: the effect of morphological difference, locomotion diversities and the boundaries conditions to the swimming performance of fishlike structure. These three facets of biohydrodynamics research interests are expected to be the spotlight of researchers in the near future, disclosed by the development in numerical, computational and experimental facets.

REFERENCES


