CFD Letters www.cfdl.issres.net Vol. 1 (2) – December 2009

Numerical Simulation of Confined Vortex Flow Using a Modified $k-\varepsilon$ Turbulence Model

Khalid M. Saqr[1C], Hossam S. Aly[2], Mazlan A. Wahid[1] and Mohsin M. Sies[1]
[1] *High-Speed Reacting Flow Laboratory, Faculty of Mechanical Engineering*
[2] *Department of Aeronautical Engineering, Faculty of Mechanical Engineering*
Universiti Teknologi Malaysia
81310 Skudai – Johor Bahru, MALAYSIA

Received: 7/8/2009 – Revised 22/9/2009 – Accepted 27/9/2009

Abstract

The turbulent flow in a tangential inlet / tangential outlet vortex tube is numerically simulated using a modified $k-\varepsilon$ turbulence model. The results are compared to experimental measurements from literature. The modified model shows better agreement with the local tangential velocity measurements compared to the standard and RNG $k-\varepsilon$ turbulence models. The flow structure is also demonstrated using the modified turbulence model.

Keywords: Turbulence modelling; dissipation rate equation; RNG k-epsilon; eddy viscosity models; Vortex flow

1. Introduction

The most well established eddy-viscosity turbulence model is the Launder and Spalding $k-\varepsilon$ model [1, 2]. The model has been subjected to rigorous experimental validations along four decades in a vast range of engineering and physical applications. The governing equations for the standard $k-\varepsilon$ turbulence model in tensor notations are:

Equation for the turbulence kinetic energy

$$\rho U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_i} - \rho\varepsilon + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_k}\right)\frac{\partial \varepsilon}{\partial x_j}\right] \qquad (1)$$

where U_i is the mean velocity, x_i is the position vector, τ_{ij} is the Reynolds stress, μ_T is the eddy viscosity, σ_k is a closure coefficient that has a unity value, and ε is the dissipation rate.

Equation for the dissipation rate of turbulence kinetic energy

$$\rho U_j \frac{\partial \varepsilon}{\partial x_j} = C_{\varepsilon 1}\frac{\varepsilon}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j} - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] \qquad (2)$$

The eddy viscosity is expressed as $\mu_T = \rho C_\mu \frac{k^2}{\varepsilon}$ (3)

[C] Corresponding Author: Khalid M. Saqr
Email: mmdmskhalid2@siswa.utm.my Telephone: +607 5534878

The model constants $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, C_{μ} and σ_{ε} values are 1.44, 1.92, 0.09 and 1.3, respectively, as reported in [1, 2].

Several modifications has been proposed to adapt the standard $k-\varepsilon$ for compressible and supersonic flows [3, 4], two phase flow [5], and jet flows [6]. However, for vortex/swirling flows, the performance of the standard $k-\varepsilon$ model was rather mystifying. In numerous cases the model showed excellent predictions for the flow structure and mean flow phenomena, such as recirculation zones and adverse pressure gradients [7-15]. On the other hand, several other researches reported poor performance of the standard $k-\varepsilon$ model in predicting swirling and vortex flows [16-20]. The authors believe that such discrepancy in the reported performance of the model comes from several combined reasons. The most important among these reasons are the validation parameters for each case and numerical methodology. This of course if the accuracy of the experimental data sets for the validation, in each case, is sufficient to consider the measurements for benchmarking. To overcome such inconsistency of the standard $k-\varepsilon$ model in predicting swirling flows, few attempts were proposed to add a swirl modification to the model. The first was proposed by Chenoweth et al [21] and it was based on a local value of the flux Richardson number which accounts for the azimuthal velocity and its variation. This modification, in fact, neglected the major drawback of the model, which lies in the ε equation. This equation in the standard model tends to predict higher dissipation rate than such physically developed. The second modification was presented by Wang and Liu [22] and it tended to modify the model based on the algebraic Reynolds stress model and Bradshaw's turbulent length scale. An earlier contribution in that field was proposed by Chang and Chen [23]. They have also established their modification on the direct expression of the Reynolds stress terms. However, the latter two modifications were designated for swirling flow which does not inherit recirculation phenomena, which are very common in most of the swirling and vortex flow applications.

Yakhot and Orszag presented a $k-\varepsilon$ model based on the application of the Renormalization group (RNG) theory [24, 25]. They have used a sophisticated scale elimination procedure to derive the two equations model, and its constants. These constants were the topic of some criticism such as that presented by Nagano and Itazu [26]. They have proved that all the constants in this model are invalid. However, the RNG/$k-\varepsilon$ model was reported to be successful in modelling different configurations of swirling flow [27-30]. Two reasons were given for such prevalence of the RNG/$k-\varepsilon$ model. The first is the swirl modification proposed by FLUENT® Inc. for their commercial solver, which is software advantageous modification of the turbulent viscosity equation. The essence of such modification is based on empirical correlations which are not revealed to users. The second reason is the additional production of dissipation term in the ε equation, which is the interest of the present paper.

The R_{ε} term in the dissipation rate equation produces higher dissipation rates that are able to take into account the effect of anisotropic turbulence in such cases where it is dominant. However, to examine the performance of such term, the strictly criticized aspects [31-35] of the RNG/$k-\varepsilon$ model has to be avoided. For this reason, a modified turbulence model, based on the standard $k-\varepsilon$ was built to include the R_{ε} term in its ε equation. The modified model has the model constants as reported by Launder and Spalding [1, 2] rather than the RNG model. The definition of the turbulent viscosity was also maintained as in equation (3), dropping the modifications for the turbulent Prandtl number proposed by the RNG model.

2. Developing the modified Model

By applying the RNG scale elimination procedure, Yakhot and Orszag [24, 25] derived a new term that accounts for the production of dissipation in the ε equation. The new term, R_{ε}, can be written in the following form:

$$R_\varepsilon = 2\nu_0 S_{ij}\overline{\frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_j}} \tag{4}$$

In order to solve the ε equation, the R_ε term has to be evaluated. It was shown that such closure can not be achieved using the methods based on ε expansion procedure, as described in [25]. Yakhot and Smith [36] postulated an expression for R_ε based on the additional expansion parameter η which is the ratio of the turbulent to mean strain time scale. The resulting expression is:

$$R_\varepsilon = \frac{\nu_T S^3 (1-\eta/\eta_0)}{1+\beta\eta^3} \tag{5}$$

Where $\eta = S\bar{k}/\bar{\varepsilon}$, $S = (2S_{ij}S_{ij})^{1/2}$, and $\eta_0 \approx 4.38$.

The constant β was chosen such as β=0.012 which results in a value for the Von Karman constant of 0.4; a recommended value for turbulent channel flows.

In the modified model, the R_ε term was integrated in the ε equation of the standard model, such that the modified ε equation can be expressed as:

$$\rho U_j \frac{\partial \varepsilon}{\partial x_j} = C_{\varepsilon 1}\frac{\varepsilon}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j} - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - R_\varepsilon \tag{6}$$

The addition of the R_ε term as a source term to the standard ε equation was based on the demand to damp the dissipation rate predictions by the standard model, in order to take into account the effect of elevated strain rates produced by anisotropic turbulence inherited by swirling flows. The modified model was used to predict the turbulent swirling flow in a horizontal tangential inlet/outlet cylinder, which was investigated numerically and experimentally by Gupta and Kumar [37]. Predictions by the modified model are compared to experimental measurements of tangential velocity at two radial stations in the cylinder, in addition to standard $k-\varepsilon$ predictions by the authors, and RNG predictions from [37]. The flow domain and dimensions of the vortex tube are illustrated in figure 1.

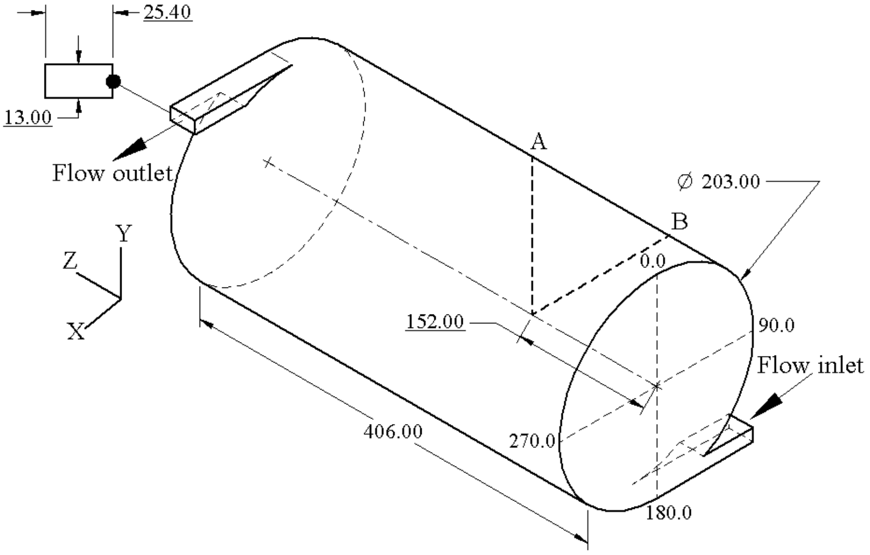

Figure 1. Tangential inlet / tangential outlet vortex tube after Gupta and Kumar [37]. Dimensions in mm

3. Results and Discussion

Particle tracking velocimetry (PTV) technique was used to measure the tangential component of velocity in the cylinder. Details on the experimental methodology and measurement locations can be acquired from [37]. Figures 2 and 3 illustrate the predicted and measured tangential velocity at two different radial locations on the cylinder. The measurements reported in [37] were normalized by the mean velocity and cylinder radius. In order to give more accurate comparison with our predictions, the measurements were denormalized to get the absolute values as in figure 2 and 3.

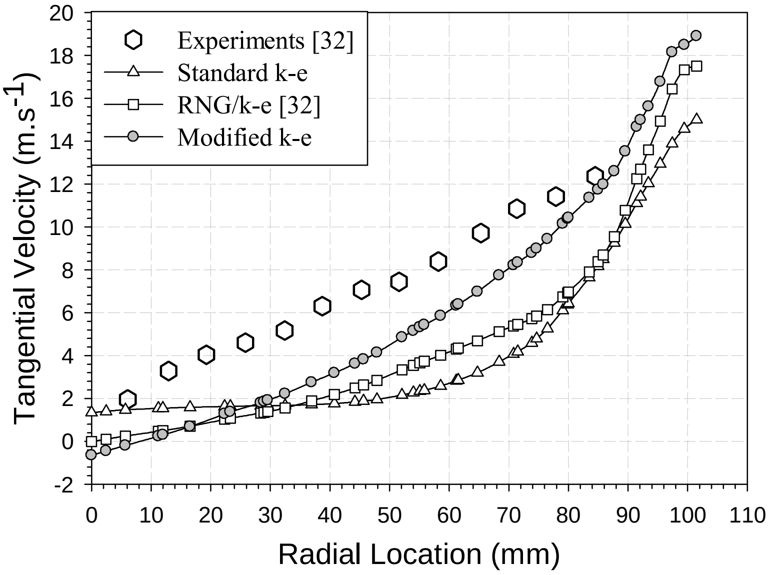

Figure 2. Comparison of the tangential velocity distribution on line A.

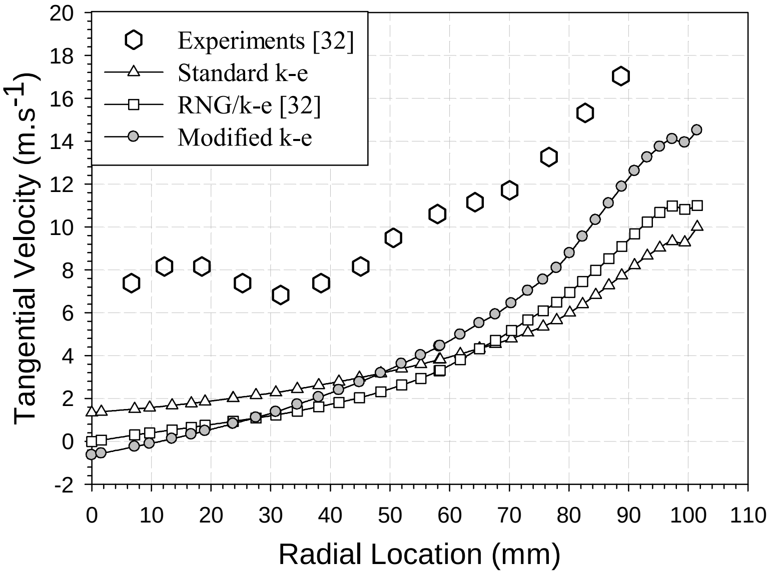

Figure 3. Comparison of the tangential velocity distribution on line B.

In figure 2, it is shown that the R_ε term, with the modified standard model, results in better quantitative agreement with the measurements, compared to the standard and RNG $k-\varepsilon$ models. However, the latter and the modified model predict negative tangential velocity near to the axis, while the standard model maintains its predictions to positive values which agree better with measurements. Similar behaviour is also depicted in figure 3, where the modified model yields better agreement as well. From both figures, the standard and RNG models produce predictions that are similar in qualitative and quantitative aspects. These results suggest that the R_ε term elevates the accuracy of predicting vortex/swirling flows, since it represents, physically, the effect of the elevated strain rate in such flows.

Another important measure of the performance of the modified model is the distribution of turbulent viscosity. Figure 3 shows the gradient of turbulent viscosity predicted by the standard, RNG and modified $k-\varepsilon$ models. The R_ε term enhances the predictions of turbulent viscosity of the standard model (i.e. the modified $k-\varepsilon$ model). While such predictions in the R_ε parent model (i.e. RNG/$k-\varepsilon$) remains at minimum. These results might lead to the formulation of a new two equation turbulence model, specifically designed for vortex/swirling flows.

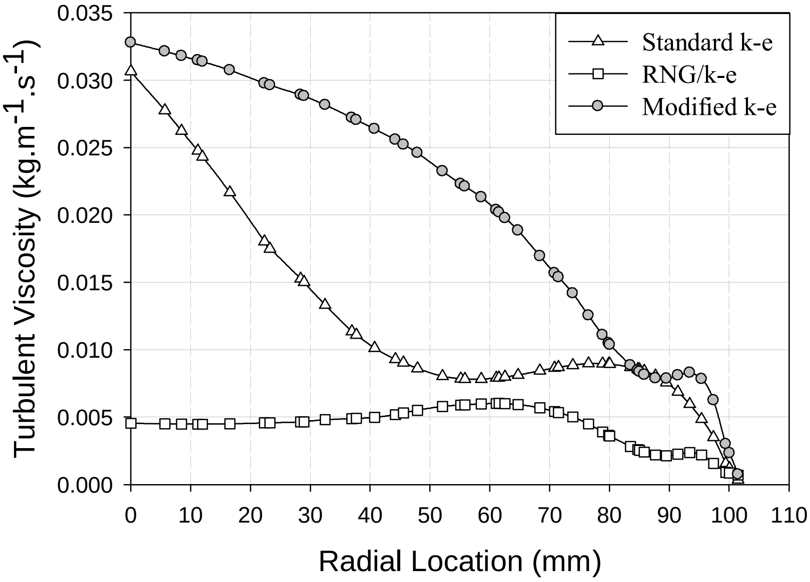

Figure 4. . Predictions of the turbulent viscosity using three turbulence models

The velocity magnitude is plotted on XY planes over different axial distances from the inlet location in figure 5. It is obvious that the vortex core movement around the geometric axis of the tube is captured using the modified turbulence model. This agrees with the results obtained by Gupta and Kumar [37]. They reported that in such flow, the vortex core revolves around the geometrical axis of the tube in a helical locus. The vortex core region is denoted by zero velocity magnitude near the centre of the tube.

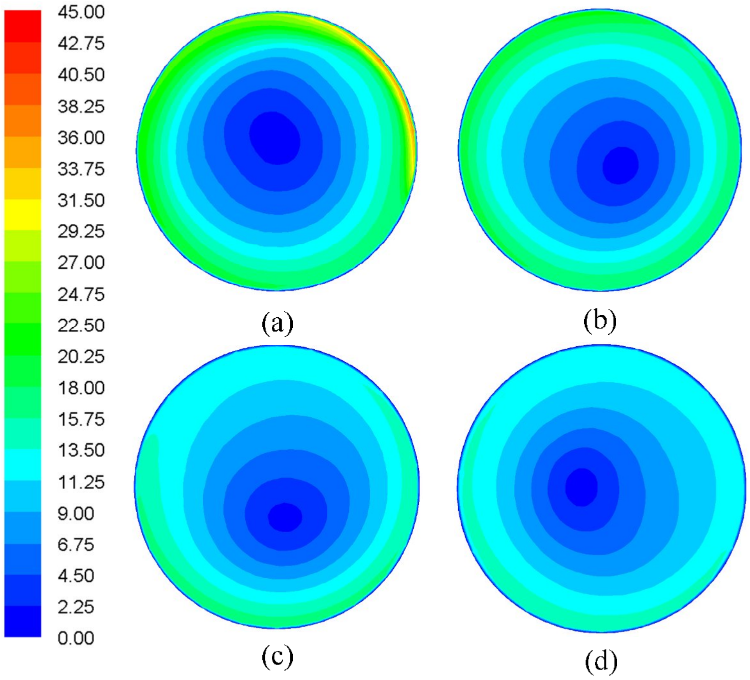

Figure 5. XY Contours of velocity magnitude at axial distance of (a) 80 mm (b) 160 mm) (c) 240 mm and (6) 320 mm.

Conclusion

The vortex flow in a tangential inlet/outlet cylinder was numerically simulated using a modified turbulence model. The proposed turbulence model is actually a variant of the standard $k-\varepsilon$ model with an additional source term in the dissipation rate equation. This term is adopted from the RNG/$k-\varepsilon$ turbulence model in order to take into account the augmented dissipation rate in highly strained flows. The predicted tangential velocity showed good agreement with measurements obtained from literature. This agreement is noticeably better than such provided by the predictions of standard and RNG $k-\varepsilon$ models. The velocity distribution along the cylinder was illustrated and the predicted vortex core movement was found to conceptually agrees with the literature. Future research shold investigate the capabilities of the proposed turbulence model in simulating vortex/swirling flows with different strengths and configurations.

References

1. Launder, B.E. and D.B. Spalding, *Lectures in Mathematical Models of Turbulence.* . 1972: Academic Press, London, England.
2. Launder, B.E. and D.B. Spalding., *The Numerical Computation of Turbulent Flows.* Computer Methods in Applied Mechanics and Engineering, 1974. **3**: p. 269-289.
3. Xingsi, H., Y. Taohong, Z. Minming, and C. Yiliang. *A modified k-ε turbulence model for compressible flow application*. in *Collection of Technical Papers - AIAA Applied Aerodynamics Conference*. 2008. Honolulu, HI.
4. Han, X.S., T.H. Ye, M.M. Zhu, Y.L. Chen, and W.Q. Shao, *Numerical simulation of supersonic H2/air combustion appling modified k-ε turbulence model.* Tuijin Jishu/Journal of Propulsion Technology, 2008. **29**(2): p. 158-162.
5. Hu, H.G. and C. Zhang, *A modified k-خμ turbulence model for the simulation of two-phase flow and heat transfer in condensers.* International Journal of Heat and Mass Transfer, 2007. **50**(9-10): p. 1641-1648.
6. Georgiadis, N.J., D.A. Yoder, and W.A. Engblom. *Evaluation of modified two-equation turbulence models for jet flow predictions*. in *Collection of Technical Papers - 44th AIAA Aerospace Sciences Meeting*. 2006. Reno, NV.
7. Menzies, K.R., *An Evaluation of Turbulence Models for The Isothermal Flow in A Gas Turbine Combustion System*, in *6th International Symposium on Engineering Turbulence Modeling and Experiments*, W. Rodi, Editor. 2005: Sardinia, Italy
8. Hsiao, G. and H. Mongia, *Swirl Cup Modeling Part III: Grid Independent Solution with Different Turbulence Models*, in *41th Aerospace Sciences Meeting & Exhibit*. 2003, AIAA: Reno, Nevada
9. Smirnov, A., A. Lipatnikov, and J. Chomiak. *Simulations of swirl-stabilized premixed combustion*. in *American Society of Mechanical Engineers, Pressure Vessels and Piping Division (Publication) PVP*. 1998. San Diego, CA, USA: ASME.
10. Smirnov, A.V. and J. Chomiak, *Computations of swirling flow using different turbulence models*, in *Doktorsavhandlingar vid Chalmers Tekniska Hogskola*. 1998, Chalmers Tekniska Hogskola.http://www.scopus.com/inward/record.url?eid=2-s2.0-0031676239&partnerID=40
11. Tarr, S.J., G. Allen, A. Aroussi, and S.J. Pickering. *Interaction of swirling flows from two adjacent coal burners*. in *American Society of Mechanical Engineers, Fluids Engineering Division (Publication) FED*. 1997. Vancouver, Can: ASME.
12. Erdal, F.M. and S.A. Shirazi. *Local velocity measurements and computational fluid dynamics (CFD) simulations of swirling flow in a cylindrical cyclone separator*. in *Proceedings of the Engineering Technology Conference on Energy*. 2001. Houston, TX.

13. Erdal, F.M. and S.A. Shirazi, *Local velocity measurements and computational fluid dynamics (CFD) simulations of swirling flow in a cylindrical cyclone separator.* Journal of Energy Resources Technology, Transactions of the ASME, 2004. **126**(4): p. 326-333.
14. Engdar, U. and J. Klingmann. *Investigation of two-equation turbulence models applied to a confined axis-symmetric swirling flow*. in *American Society of Mechanical Engineers, Pressure Vessels and Piping Division (Publication) PVP*. 2002. Vancouver, BC.
15. Halder, M.R. and S.K. Som, *Numerical and experimental study on cylindrical swirl atomizers.* Atomization and Sprays, 2006. **16**(2): p. 223-236.
16. Kaya, F. and I. Karagoz, *Performance analysis of numerical schemes in highly swirling turbulent flows in cyclones.* Current Science, 2008. **94**(10): p. 1273-1278.
17. Armfield, S.W. and C.A.J. Fletcher, *Comparison of k-خμ and algebraic Reynolds stress models for swirling diffuser flow.* International Journal for Numerical Methods in Fluids, 1989. **9**(8): p. 987-1009.
18. Xia, J.L., G. Yadigaroglu, Y.S. Liu, J. Schmidli, and B.L. Smith, *Numerical and experimental study of swirling flow in a model combustor.* International Journal of Heat and Mass Transfer, 1998. **41**(11): p. 1485-1497.
19. Ridluan, A., S. Eiamsa-ard, and P. Promvonge, *Numerical simulation of 3D turbulent isothermal flow in a vortex combustor.* International Communications in Heat and Mass Transfer, 2007. **34**(7): p. 860-869.
20. Chenoweth, J.D., B. York, and A. Hosangadi. *Turbulence model upgrades for swirling flows*. in *2007 Proceedings of the 5th Joint ASME/JSME Fluids Engineering Summer Conference, FEDSM 2007*. 2007. San Diego, CA.
21. Chenoweth, J.D., B. York, and A. Hosangadi. *Modification of the standard K-ε model for swirling flows*. in *Collection of Technical Papers - 43rd AIAA/ASME/SAE/ASEE Joint Propulsion Conference*. 2007. Cincinnati, OH.
22. Wang, Z. and W.M. Liu, *Two modificatory K-ε turbulence models for turbulent swirling flows.* Journal of Hydrodynamics, 2003. **15**(2): p. 51-57.
23. Chang, K.-C. and C.-S. Chen, *Development of a hybrid k-ε turbulence model for swirling recirculating flows under moderate to strong swirl intensities.* International Journal for Numerical Methods in Fluids, 1993. **16**(5): p. 421-443.
24. Yakhot, V. and S.A. Orszag, *Renormalization-group analysis of turbulence.* Physical Review Letters, 1986. **57**(14): p. 1722-1724.
25. Yakhot, V. and S.A. Orszag, *Renormalization group analysis of turbulence. I. Basic theory.* Journal of Scientific Computing, 1986. **1**(1): p. 3-51.
26. Nagano, Y. and Y. Itazu, *Renormalization group theory for turbulence: Assessment of the Yakhot-Orszag-Smith theory.* Fluid Dynamics Research, 1997. **20**(1-6): p. 157-172.
27. Secchiaroli, A., R. Ricci, S. Montelpare, and V. D'Alessandro, *Numerical simulation of turbulent flow in a Ranque-Hilsch vortex tube.* International Journal of Heat and Mass Transfer, 2009. **52**(23-24): p. 5496-5511.
28. Jawarneh, A.M., H. Tlilan, A. Al-Shyyab, and A. Ababneh, *Strongly swirling flows in a cylindrical separator.* Minerals Engineering, 2008. **21**(5): p. 366-372.
29. Karagoz, I. and F. Kaya, *CFD investigation of the flow and heat transfer characteristics in a tangential inlet cyclone.* International Communications in Heat and Mass Transfer, 2007. **34**(9-10): p. 1119-1126.
30. Escue, A. and J. Cui. *Numerical investigation of swirling pipe flows*. in *American Society of Mechanical Engineers, Fluids Engineering Division (Publication) FED*. 2006. Chicago, IL.
31. Eyink, G.L., *The renormalization group method in statistical hydrodynamics.* Physics of Fluids 1994. **6**: p. 3063-3078.
32. McComb, W.D., *The Physics of Fluid Turbulence* 1990, U.K: Oxford University Press.
33. Teodorovich, E.V., *On the Yakhot-Orszag theory of turbulence.* Fluid Dynamics, 1995. **29**(6): p. 770-779.

34. Sukoriansky, S., B. Galperin, and I. Staroselsky, *Cross-term and ε-expansion in RNG theory of turbulence.* Fluid Dynamics Research, 2003. **33**(4): p. 319-331.
35. Wang, X.H. and F. Wu, *One modification to the Yakhot-Orszag calculation in the renormalization-group theory of turbulence.* Physical Review E, 1993. **48**(1).
36. Yakhot, V. and L.M. Smith, *The renormalization group, the e{open}-expansion and derivation of turbulence models.* Journal of Scientific Computing, 1992. **7**(1): p. 35-61.
37. Gupta, A. and R. Kumar, *Three-dimensional turbulent swirling flow in a cylinder: Experiments and computations.* International Journal of Heat and Fluid Flow, 2007. **28**(2): p. 249-261.