DNS of Turbulent Boundary Layers: The Breakthrough That Opened a Can of Worms

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In the first issue of *CFD Letters*, the executive editor penned an editorial entitled "Towards a Better Understanding of Turbulence". He kindly asked me to author an editorial for the second issue of the journal. I wished to write about something else, but turbulence is quite an alluring subject, it warrants a second editorial albeit on a different streak: a comment on a recent article that describes the largest ever direct numerical simulations of an evolving boundary layer. But first a bird's-eye view of the seductive subject matter.

Romanticized since Leonardo da Vinci compared the motion of a water jet rapidly falling into a pool to the curls and waves of long, gorgeous hair, turbulence is a field of endeavor blessed with stunning images, elegant mathematics, intellectually fascinating physics, and vitally important applications. Its significance at the human, geologic and cosmologic scales can only be understated. Turbulent transport in plasma sustains the nuclear fusion process that in turn keeps the stars alive; the vigorous turbulent mixing in the atmosphere keeps megacities from suffocating under their own human-produced carbon dioxide; and a turbulent boundary layer allows an airfoil to generate more lift at larger angles of attack than a corresponding laminar flow. The darker facet of turbulence is its extreme complexity, preventing first-principles analytical solutions. Turbulence is also mostly responsible for the high fuel consumption of all air, land and sea transportation systems, a major concern nowadays for both the energy crisis and global warming.

The Navier–Stokes equations describe laminar and turbulent flows for most fluids under most circumstances. Exceptions exist for non-equilibrium flows such as those of polymers, for non-continuum flows such as those of highly rarefied gases, or, in the extreme, for fluid motions governed by quantum or relativistic effects. In any case, the instantaneous, three-dimensional, nonlinear Navier–Stokes equations have no known analytical solutions for the stochastic turbulent flows.

Those equations could be integrated numerically preserving all scales down to the Kolmogorov's scales, but such direct numerical simulations (DNS) is limited by today's computer memory and speed to modest Reynolds numbers (as an example, for a boundary layer $\mathrm{Re}_\theta = \mathcal{O}[1000]$, where Re_θ is based on the freestream velocity and momentum thickness).

Osborne Reynolds' turbulence decomposition results in a simpler set of equations for mean quantities, but again the nonlinearity of the root equations results in more unknowns than equations, and heuristic modeling is needed to close the equations. The closure models rely on one way or another on the Kolmogorov's universal equilibrium theory of turbulence spectrum, the universal logarithmic law of wall-bounded flows, or DNS validation. The foundering of any or all of the three cornerstones would be a blow to the modeling enterprise, desperately in need of accurately solving the numerous practical problems associated with high-Reynolds-number turbulent flows. Herein, we focus on DNS, leaving issues with the equilibrium theory and log law to another Hyde Park's soapbox.

The recent DNS calculations conducted by Xiaohua Wu and Parviz Moin (*J. Fluid Mech.*, vol. 630, pp. 5–41, 2009) do raise significant doubts about prior turbulent boundary layer simulations. Though Wu & Moin's study is sort of a breakthrough in our ability to compute turbulence, it opened a can of worms related to the validity of all prior boundary layer DNS results, upon which the modeling community depended for validation and refinement of competing closure models.

In the largest numerical simulations ever carried out of a spatially-evolving, nominally-zero-pressure-gradient boundary layer flow from laminar state, through bypass transition, and finally all the way to a significant region of fully-developed turbulence, Wu & Moin used a finite-difference scheme thus avoiding the unphysical periodic boundary conditions other researchers imposed in the past to be able to utilize the more efficient spectral methods. The new calculations used a whopping 210 million grid points, and were conducted on the IBM terascale parallel machines at the San Diego Supercomputing Center. A Blasius boundary layer of Re_θ=80 is prescribed at the inlet to the computational domain. Patches of isotropic, high-amplitude turbulence (intensity of the order of 6%) are introduced periodically from the freestream at this station in order to effect a bypass transition and reach a fully-developed state in a reasonably short distance from the virtual leading edge. Transition commenced at Re_θ=180, fully-developed turbulence was reached at Re_θ=750, and finally the computations ended at Re_θ=940. Congratulatory remarks for Wu & Moin's achievement were provided elsewhere (inaugural "Focus on Fluids" section of the *Journal of Fluid Mechanics*, penned by Ivan Marusic, vol. 630, pp. 1– 4, 2009).

Two issues stand out. The recent DNS exhibits striking preponderance of hairpin-shaped coherent structures in both the later parts of the bypass transition and in the fully-developed turbulent region of the boundary layer, consistent with at least indirect experimental observations. Prior DNS studies, limited by the more modest computational resources available at the time, were based, according to Wu & Moin, on a host of 'debatable' approximations. (I would have used 'questionable', 'contentious', 'disputable' or some other adjective, but Wu & Moin chose the quite mild adjective 'debatable.) The original computations exhibited hairpin vortices that were rarely in complete loops or quasi-symmetric, the notorious one-legged hairpins. Such vortices have never been observed experimentally. Secondly, there exist small but appreciable differences in total shear stress and other turbulence quantities between the 210-million-grid-point calculations and previous less-computer-intensive simulations. Since not everyone has access to, or can afford, computer resources capable of conducting DNS calculations with this enormity, the community is left with the question of what to do? A more ominous question is how to trust closure models that used the original DNS results for validation? A can of worms has been opened!