The Deformation of Human Eyeball when undergoing Scleral Buckling

Zuhaila Ismail¹,a, Alistair Fitt²,b and Colin Please³,c

¹Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor, Malaysia.

²Faculty of Technology, Design and Environment, Oxford Brookes University, Headington Campus, Gipsy Lane, Oxford, OX3 OBP, United Kingdom.

³Mathematical Institute, University of Oxford, 24-29 St Giles’, Oxford, OX1 3LB, United Kingdom.

a zuhaila@utm.my, b afitt@brookes.ac.uk, c Colin.Please@maths.ox.ac.uk

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Abstract. Scleral buckling is a surgical technique to treat rhegmatogenous retinal detachment (RRD). Vision may be affected by the scleral buckle. Since the buckle is pushed into the sclera towards the detached retina, it may change the shape and the focal length of the eyeball. A paradigm mathematical model of human eyeball is set up to examine how the focal length of the eye is affected under the action of the external force. In particular, this model has been developed using the membrane equations of equilibrium for axisymmetric spherical shells. Using numerical analysis the resulting displacements of the eyeball will be examined. The results of the scleral buckle may prove useful to predict changes in focal length.

Introduction

The human eyeball is approximately a sphere 25mm in diameter with a volume of 6.5ml. In real life, the eyeball differs slightly from a sphere. It is composed of two spheres. The larger sphere is the sclera, which forms five-sixths of the circumference of the eyeball and has a radius of 11.5mm. The remaining one-sixth of the circumference of the eyeball is formed by the cornea. The cornea has a higher curvature than the sclera with a radius of curvature of 7.8mm [1]. Both the cornea and the sclera form the opaque strong fibrous outer layer of the eye, which acts as a shield to protect the ocular tissues from injury. From the outer to the innermost, there are four layers of the sclera; these are the episclera, the stroma, the lamina fusca and the endothelium. The stroma is a layer of collagen fibres that runs continuously through the sclera and the cornea, keeping the eyeball bound together [2]. The stroma of the sclera is continuous with the stroma of the cornea and plays an important role in the biomechanics of the eyeball. Due to the structure of the collagen fibrils that make up the stroma, we can model the human eyeball as a spherical shell with symmetry at the central axis and isotropic elasticity in the middle of the shell surface (i.e. the Young's modulus is the same in all directions tangent to the middle shell surface). Using the membrane theory of shells we attempt to develop a model of the eyeball with application to scleral buckling in order to examine the changes in shape of the eyeball.

A Mathematical Model of Human Eyeball

Membrane theory is a simplified version of shell theory and is based on neglecting bending and twisting moments in the stress analysis. Essentially, when applying the membrane theory of shells, only the normal and shearing forces, \( N_\theta, N_\phi, N_{\theta\phi} \) and \( N_{\phi\theta} \) and the loads \( P_\theta, P_\phi \) and \( P_\phi \), proportional to the area of the elements are considered. [3] stated that the main problem in this theory is therefore to determine the resultant forces for a given shape of shell, in terms of either the loads or the displacements applied to the edges and surface of the shell. A simplified human eyeball
model has been developed using the membrane theory of shells, by considering a sphere at the
centre, \( O \), which incorporates the cornea and the sclera, see Fig. 1. We assume the cornea is directly
attached to the sclera at the limbus and the sclera is considered to be continuous at the optic nerve.

Fig. 1 presents geometry of the cross section of the eyeball model in a spherical coordinate system \((\phi, \theta, R)\)
where \( \phi \) is the hoop angle, \( \theta \) denotes the meridian angle and \( R \) is the radius of the sphere. Here the Young's
modulus of the sclera and the cornea are denoted by \( E_s \) and \( E_c \), \( \delta \) denotes the angle of the cornea and \( \phi = 0 \)
is assumed to be the axis of symmetry.

In this eyeball model, we consider the eyeball to be an elastic membrane shell, isotropic in the
shell surface and a hollow sphere in which the thickness and Young's modulus various. The corneal thickness, \( \alpha_c \) is assumed to be thin compared to the thickness of the
sclera, \( \alpha_s \). However, both the corneal and scleral thickness are considered to be very small. The
vitreous humour is assumed to be an incompressible Newtonian fluid with constant density and
viscosity. It is assumed that a pressure \( P_s \) is applied to the shell in the direction normal to the
surface. This pressure is equal to the pressure difference between the pressure inside the eyeball and
the pressure outside the eyeball such that, \( P_s = P_{in} - P_{out} \). The load component tangent to the
shell, \( P_{\theta} \) is assumed to be zero. The pressure inside the eyeball, \( P_{in} \), consists of the atmosphere
pressure, \( P_{atm} \) and the intraocular pressure, \( P_{IOP} \). The pressure outside the eyeball, \( P_{out} \) is equal to the
atmosphere pressure, \( P_{atm} \). Thus in the normal human eyeball, under no external outside pressure, it
is assumed that the pressure on the eyeball equals the intraocular pressure such that, \( \Delta P = P_{IOP} \). In
this study we examine the shape of the eyeball under a variation of loading conditions depending on
the outside pressure that is exerted from the scleral buckle.

The Governing Equations. Under the condition of static equilibrium and assuming the sphere is
axisymmetric, the general equations of spherical membrane shells have been simplified in order to
analyse the elastic membrane of the human eyeball under the effects of external outside pressure
which are from the scleral buckling. These are:

\[
\frac{d}{d\theta} \left( R \sin \theta N_\phi \right) - RN_\phi \cos \theta = 0,
\]

\[
\frac{N_\theta + N_\phi}{R} = P_\theta,
\]

\[
\frac{1}{E_c \alpha_c} \left( N_\theta - \nu N_\phi \right) = \frac{1}{R} \left( w + \frac{dv}{d\theta} \right),
\]

\[
\frac{1}{E_s \alpha_s} \left( N_\theta - \nu N_\phi \right) = \frac{1}{R \sin \theta} \left( v \cos \theta + w \sin \theta \right).
\]

\( N_\theta \) and \( N_\phi \), are the stress resultants and the displacements, \( v \) (\( \theta \)-direction) and \( w \) (\( R \)-direction).
Here \( \nu \) denotes the Poisson's ratio. The eyeball has radius \( R = a \), Young's modulus \( E = E(\theta) \),
thickness \( \alpha = \alpha(\theta) \) where \( E \) and \( \alpha \) are defined by

\[
E(\theta) = \frac{E_c + E_s}{2} + \frac{E_c - E_s}{2} \tanh \left(K(\theta - \delta)\right) \quad \text{and} \quad \alpha(\theta) = \alpha_c + \left( \alpha_s - \alpha_c \right) \frac{\theta}{\pi}
\]

respectively. \( E = E(\theta) \) is defined to be the hyperbolic equation because of the different values
between the Young's modulus of the sclera and the cornea. \( K \) denotes a constant and is assumed to
be large. The thickness distribution of the eyeball, $\alpha(\theta)$ is considered to be a straight line in this problem. Notice that the derivations of the governing equations, Eq. 1 to Eq. 4 are based on the studies of direct stresses in shells of revolution by [4] and [5]. Eq. 1 and Eq. 3 are first order differential equations and both equations require two boundary conditions. The boundary conditions are

$$N_\alpha(\theta = 0) = \frac{aP_s(\theta = 0)}{2} \quad \text{and} \quad u(\theta = 0) = q.$$  \hspace{1cm} (6)

where $q$ could be any number of constants providing the meridian displacement, $v$ is zero at $\theta$ equals to zero and $\pi$. Notice here different constants of $q$ will lead to different solid body translations along the $z$-axis. In physical circumstances the eyeball remains fixed as it is supported by the eye socket at the back of the eye. Therefore in the modelling assumption, it would be necessary to fix the eyeball from translations by assuming the displacements at the back of the eye to be zero. We note that [6] also developed these two boundary conditions given in Eq. 6.

**Numerical Analysis of Scleral Buckling**

We now examine and discuss in detail the elastic membrane of the human eyeball under the effect of the scleral buckle. The scleral buckling is one of the procedures used in treating treat rhegmatogeneous retinal detachment. Usually the scleral buckle stays attached permanently after the treatment and in the long term may cause a risk to changes of the focal length of the eyeball. Thus, in this study we are interested in examining the deformation of the eyeball and the changes in the focal length due to pressure provoked by the scleral buckle. Here we examine the case where the scleral buckle is placed in general around the eyeball depending on the location of the retinal detachment. In this case we introduce the load component normal to the shell for the general position of the scleral buckle is given as follows:

$$P_s(\theta) = \begin{cases} P_{IOP} & \text{if } 0 \leq \theta \leq \delta, \ \delta < \theta < \phi - \beta, \ \phi + \beta < \theta \leq \pi - \kappa, \\ P_{IOP} + P \frac{\cos \beta - \cos(\theta - \phi)}{1 - \cos \beta} & \text{if } \phi - \beta \leq \theta \leq \phi + \beta, \\ P_{IOP} + Q \left(\cos (\pi - \kappa) - \cos \theta\right) & \text{if } \pi - \kappa < \theta \leq \pi, \end{cases}$$  \hspace{1cm} (7)

where $\phi$ is the general angle that the scleral buckle is going to be placed around the eyeball and $Q$ is the equilibrium pressure which is needed, in this case, in order to maintain the equilibrium condition. We now examine the case of the scleral buckling by substituting Eq. 5 and Eq. 7 into Eq. 1-4, and solve the equations numerically. For the purpose of numerical calculations, we are assuming $\phi = \pi/3$, $\beta = \pi/10$ and the remaining parameter values are based on [8]. We then carried out an analysis of the shape of the eyeball by reducing the value of the Young's modulus for the sclera and the cornea, and the results of this have been shown in Fig. 2. Fig. 2 shows that if the rigidity of the elastic membrane of the eyeball is reduced, then the deformation under the region of contact between the surface of the eyeball and the scleral buckle is increased. In the study of the scleral buckle near the equator, we also examine the changes in the focal length under the action of the scleral buckle, by predicting the values of the displacements.
at $\theta = 0$. We then plotted and fitted the changes in the focal length, $FL$ against the indentation, $I$ exerted by the scleral buckle using the linear least squares method, see Fig. 3. The intraocular pressure, $P_{IOP}$ is taken to be 15mmHg and the pressure provoked the indentation at $\phi = \pi/3$ are taken to be varied between 10mmHg to 60mmHg. Fig. 3 shows the focal length againts the indentation for each different material value of the scleral and the corneal Young's modulus is in linear relation. Fig. 3 is given the similar linear relations which mean the changes in the focal length are quite small, however the modified focal length seems to be more affected by the indentation suffered by the surface of the eyeball when the scleral buckle is placed near the equator.

Summary

We developed a mathematical model of a scleral buckle and examined the outer surface of the membrane eyeball when the scleral buckle has been placed anywhere around the eyeball due to the location of the retinal detachment. We then examined the changes in focal length against the indentation provoked by the scleral buckle. The results presented in this study, show that the modified focal length against the indentation in the scleral buckle case is in linear relation. These results have shown that the indentation exerted by the scleral buckle may be affected the changes in focal length. The numerical results also have shown that the indentation affected by the scleral buckle depends on the modulus of rigidity of the membrane eyeball itself. This means that a smaller indentation corresponds to a stiffer eyeball. Notice that when we older the modulus of rigidity of the eyeball increases. This may affect the changes in shape of the eyeball and also reduce vision. However, the bigger indentation might influence in changes in the focal length of the eyeball. Due to this risk, complications such as astigmatism and direct injury to a rectus muscle may occur.

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