Numerical analysis on the effects of mixed convection of particles removal flow over heated cavity using Multi-Relaxation Time Thermal Lattice Boltzmann Method

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Keywords: Lattice Boltzmann Method, Multi-Relaxation-Time, Double-distribution function, Mixed convection, Grashof number, Removal efficiency.

Abstract

This paper provides numerical study of the effects of mixed convection on particles removal from a cavity using multi-relaxation time thermal lattice Boltzmann method (LBM) for compute the flow and isotherm characteristics in the bottom heated cavity located on a floor of horizontal channel. A point force scheme was applied for particles-fluid interaction and double-distribution function (DFF) was coupled with MRT thermal LBM to study the effects of various grashof number (Gr) and Aspect Ratio (AR) on the efficiency of particles removal. The results show that change in Grashof number and Aspect ratio causes a dramatic different in the flow pattern and particles removal efficiency.

Introduction

Lattice Boltzmann Method (LBM) is a powerful numerical technique for solving various fluid flow problems which was derived from lattice gas automata [1-2]. LBM is based on solving discrete-velocity Boltzmann equation in statistical physics. Due to its advantages of LBM; simple algorithm, it is easy to apply for complex geometry domain and efficient implementation for parallel computation [3]. LBM may lead to numerical instability when the relaxation time $\tau$ approaches 0.5 [4]. These deficiencies of the instability can be easily addressed by using Multi-Relaxation-time (MRT) model which is retains the simplicity and computational efficiency of the SRT Model [5].

Modeling of two-phase fluid flows-particles nowadays may have a concern in many applications in natural and industrial manufacturing process. During recent years, the process of cleaning pipelines and mechanical piping parts has become widely attention among researcher. On the other hand, the presence of recirculation vortices in cavity is one of the problem appear during the cleaning process due to difficulty to remove all the particles out from a cavity. There are numerous studies about fluid flow over a cavity where researchers were showed the behavior of fluid streamlines inside a cavity [6]. In addition, studies of mass transfer in a cavity have appeared in the pertinent literature [7-8]. As a result, most previous studies have assumed that the fluid flow in a cavity already achieved steady state condition.

Previously, azwadi et al. [9] have studied the behavior of fluid structure and isotherm structure of fluid flow over heated cavity with unsteady start up inlet velocity until steady state condition without particles removal inside a cavity. Therefore, the present work simulates a particles removal from heated rectangular cavity at the bottom by using double-distribution function (DDF) couple with MRT thermal lattice Boltzmann method. The main objective of the computation is to demonstrate the capability of the models for prediction of percentage of particles removal in a confined space various Grashof number (Gr), Reynolds number (Re) and Aspect Ratio (AR). For validation purpose, the obtained results will be compared with the previous experimental studies by Fang et al [10].
Thermal MRT-LBM Model

The thermal Lattice Boltzmann model utilizes two distribution functions, \( f \) and \( g \), for the fluid flow and temperature field respectively. Recently, Lallemand and Luo [11] suggested that the use of a MRT model could improve the numerical stability. The collision step in velocity space is difficult to perform; it is more convenient to perform the collision process in the momentum space [3]. The general form of multi-relaxation-time lattice Boltzmann equation with external forced can be written as, for the flow field and temperature field respectively

\[
\begin{align*}
  f_i(x + c_i \Delta x + \Delta t) - f(x,t) &= -M^{-1}S[m(x,t) - m^{eq}(x,t)] + F_k \\
  g_i(x + c_i \Delta x + \Delta t) - g(x,t) &= \frac{\Delta t}{\tau_T} [g_i^{eq}(x,t) - g_i(x,t)]
\end{align*}
\]

(1)

(2)

where \( f \) is the distribution function for particles with velocities \( c_i \) at time \( t \). Velocity \( c_i \) is equal to rate of change of position \( c_i \Delta t \) per unit time \( \Delta t \). \( m(x,t) \) and \( m^{eq}(x,t) \) are vectors of moments, \( m = (m_0, m_1, m_2, \ldots, m_n) \). The thermal diffusivity \( \alpha \) are defined in term of relaxation time, \( \alpha = c_s^2(\tau_T - 1/2) \) where \( \tau_T \) is relaxation time in temperature field, \( c_s \) is speed of sound. \( F_k \) is the external forced in direction of lattice velocity as follow:

\[
F_k = 3w_i g_x \beta \theta
\]

(3)

The relaxation matrix \( S \) is a diagonal matrix. In compact notation \( S \) can be written as;

\[
S = diag(1.0, 1.4, 1.4, 1.4, s_3, 1.2, s_5, 1.2, s_7, s_8)
\]

(4)

where \( s_7 = s_8 = 2/(l + 6\nu) \), \( s_3 \) and \( s_5 \) are arbitrary, can be set to 1.0.

The macroscopic density, velocities and temperature can be computed simply by moment integration as

\[
\begin{align*}
  \rho &= \sum_i f_i \\
  u &= \frac{1}{\rho} \sum_i c_i f_i \\
  T &= \sum_i g_i
\end{align*}
\]

(5)

(6)

(7)

In the present investigation, we only consider point force acting on the particle in a cavity and assume the presence of solid particle gives no effect to the fluid flow. The equation of motion for solid particle is written as

\[
m_p \frac{dv_p}{dt} = F_p
\]

(8)

where \( m_p \), \( v_p \) and \( F_p \) are the mass of particle, its velocity and drag force acting on particle due to surrounding fluid. Here, the drag force can be written as follow

\[
F_p = C_p A_p \rho \left| \frac{v_p - u}{2} \right|
\]

(9)

where \( A \) is the projected area of solid particle and \( C_p \) is the drag coefficient which is defined as

\[
C_p = \frac{24}{\text{Re}_p} \left( 1 + \frac{1}{5} \text{Re}_p^{0.687} \right)
\]

(10)

where, the \( \text{Re}_p \) is the Reynolds number of solid particle and the value of drag coefficient in equation (10) for Reynolds number, \( 1 < \text{Re}_p < 1000 \).
Numerical Modeling

In this section, the physical domain of the problem is show in Figure 1 with length $L$, velocity inlet $U_{in}$, inlet height $H$, cavity height $D$ and cavity width $W$. The inlet flow applied with different Reynolds number within 50 to 400, different Grashof Number (Gr) between 1000 to 3000 and different Aspect ratio (AR) between 1 to 4 with $N_p$ number of particles where AR and Gr are defined as:

$$AR = \frac{W}{D} \quad Gr = \frac{Ra}{Pr}$$

where, $Ra$ and $Pr$ is the Rayleigh number and Prandtl number (7 for water) respectively.

Results and Discussion

Fluid flow over a rectangular cavity and constant heat flux from a bottom heated cavity were investigated. The effect mixed convection of the removal efficiency from a cavity by using thermal MRT-LBM was studied and validated with the Fang et al. [10] at different Grashof number (Gr = 1000 and Gr = 3000) and aspect ratio (AR = 3 and AR = 4). Figure 2 and 3 shows the flow structure and isotherm flow of the bench mark results and current results under the effect of heat convection at Reynolds number is 50, AR 1 and 4, and Gr 1000 and 3000. Figure 4 show the isotherm flow pattern under constant Gr is 1000, different Re 50 and 100 with aspect ratio (AR) AR 1, 2, 3 and 4.

Figure 2, Flow and isotherm structure Illustration under the effect of heat convection at Re = 50

a) the results of [10] b) the results of current study

Figure 3, Particles removal from a cavity illustration with AR = 4, Re = 50, Gr = 3000
Figure 4, Removal efficiency of particles from a cavity at constant Re = 50 with different Aspect ratio; a) AR = 4 b) AR = 3

According to the figures above, by increasing Grashof number and Aspect ratio will affect the patterns of flow, isotherm structure and percentage of particles removal. It can be seen that the percentage of particles removal from a cavity increases with increase of Gr and AR due to a stronger buoyancy effect. According to figure 4(a) the percentage of particle removal for Gr=1000 and Gr=3000 are 61.77% and 62.97% respectively and due to figure 4(b) at Gr = 1000 percentage of particles removal was 61.77% and 44.68% for AR = 4 and AR = 3 respectively.

Conclusion

The particles removal efficiency from a cavity and the structure of fluid flow and isotherm were obtained for numerical simulation of particles removal from a cavity by using MRT thermal LBM with applied effects of mixed convections at the end of this study. Result from the work presented in this study shows that by increasing Grashof number influence higher efficiency of particle removal from a cavity. For future work, various type solid fluid flow couple with heat convection needs to be extended and cover present formulation.

References