Numerical Simulation of Stress Shielding Induced by Crack Interaction in Human Phalanx Bone

S. A. Abdul Halim¹, a N. A. Md Zain², b, N. N. Mansor³, c, R. Daud⁴, d, Y. Bajuri⁵, e

¹-³Industrial Mathematics Research Group (IMRG), Institute of Engineering Mathematics, Universiti Malaysia Perlis (UniMAP), Arau, Perlis, Malaysia.
⁴-⁵Fracture & Damage Mechanical Research Group, School of Mechatronic Engineering, Universiti Malaysia Perlis (UniMAP), Arau, Perlis, Malaysia.

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Abstract. Bone fracture is an injury not uncommon to everyday life. Most of the time, it leaves permanent damage and a long period of recovery. This situation can be prevented if we understand the mechanics and the process of the bone fracture. This study aims to evaluate stress shielding induced by crack interaction using a simple model based on Linear Elastic Fracture Mechanics (LEFM). This simulation based on the determination of the Stress Intensity Factor (SIF) and the changes of stress shielding in different crack interval towards the human phalanx bone. Numerical simulation had been carried out in this project to understand the stress shielding induced by crack interaction. The results revealed that the interaction of two cracks is directly proportional to the SIF magnitude and interaction factor at the crack tips. The parallel cracks have experienced increasing shielding effect as the crack interval increase. The crack interaction limit (CIL) and crack unification limit (CUL) also had been accomplished for every range of crack interval in this project.

Introduction

Fracture bone can occur in all parts of human skeletal cortical bone including phalanx bones of finger bone. Fingers allow us to perform specialized function such as grasping a pen or manipulating small objects in our palm. This fracture bone seemingly insignificant among us, but it may lead to fine motor dysfunction and chronic problem [1]. Thus, the injury may not only limit the functionality of a finger’s usefulness in activities of daily living. Fracture of finger bone can be categorized into malunion, nonunion and posttraumatic osteoarthritis [2]. The fracture phenomena in cortical bone are accompanied by formation of microcrack or microcracks accumulation [3]. The objective of this study is to find the value of Stress Intensity Factor (SIF) on a human phalanx of finger bone within the variation of crack interval. This study required the use of Finite Element Analysis (FEA) for further analysis on the fracture behavior of double parallel edge cracks in phalanx bone. The two dimensional model will be constructed and the crack interval on a human phalanx of finger bone will be altered using ANSYS software.

Analytical model and procedure

The methodology of this study includes the model construction and development of the human phalanx bone finite analysis using the computational method. Two types of two-dimensional (2-D) human phalanx model were constructed in ANSYS APDL software. First type of model consists of two parallel straight edges. The crack length of Cr1 and Cr2 is denoted as a1 and a2 which occupies the segment of 0.05 ≤ a1/w ≤ 0.5 and 1 ≤ b ≤ 25. Second type of model consists of one straight edge
with segment of $0.05 \leq a / w \leq 0.5$. Both model had a constant thickness $t=1\,mm$, height $H=40\,cm$ and width $w=20\,cm$. Then, these models defined with Young’s Modulus and Poisson’s ratio of human phalanx cortical bone of the index finger; 0.35 and 2.0 GPa [4].

Next, the area plot was performed on 2-D model. The areas of 2-D model were meshed with a suitable size of meshing. After that, the whole plate of model was subjected to static uniaxial (Mode I) loading of magnitude 100 N. Analysis was conducted to find out the Stress Intensity Factor (SIF) at crack tips. Lastly, validation step was conducted by comparing results from previous results.

Results from this study include Stress Intensity Factor (SIF) and crack interaction factor. The analytical formulation can be expressed as

$$K_I = K_{a} F_n (a / w, b)$$

(1)

while analytical single edge crack SIF reference by Brown & Srawley [5],

$$K_{I,ref} = \sigma_0 \sqrt{\pi a} \left(1.12 - 0.23\left(\frac{a}{w}\right) + 10.6\left(\frac{a}{w}\right)^2 \right)$$

(2)

$$K_{I,ref} = \sigma_0 \sqrt{\pi a} \left(f_{I,ref, BS}\right)$$

(3)

Results and Discussion

There are three sections in this part. First section discussed on the numerical modeling validation for single edge crack. Mode I and Mode II fracture behavior were being discussed in the second section, whereas comparison between crack interaction factor and analytical data were discussed in the last section.

Numerical modeling had been used in validating the single edge crack by comparing them with analytical single edge crack SIF as proposed by Brown & Srawley [5]. Generally, both analytical and experimental values show increases of Mode I SIF as increasing of crack-to-width ratio, $a/w$. Fig. 1 illustrates the trend of Mode I SIF against crack-to-width ratio, $a/w$ for all crack interval, $b$. It can be observed that the trends are excellent agreed with the theoretical finite body trend as defined by Brown & Srawley.

![Graph Between Stress Intensity Factor versus a/w](image)

Figure 1: Variation of Mode I for $b = 1$ until $b = 25$

The two-dimensional (2-D) models in this study are subjected to static uniaxial (Mode I) loading of magnitude $\sigma_0$. The effect of Mode I loading had been studied by analyzing the Mode I and Mode II fracture behavior. Basically, the changes of Mode I and Mode II are largely due to the variation of crack interval, $b$. It can be seen that Mode I SIF directly proportional to crack interval, $b$; the
increase in $b$ results in an increase of Mode I SIF for $a/w = 0.05-0.50$. However, the value of Mode II SIF decrease as the value of $b$ increase for $a/w = 0.05$ until $a/w = 0.50$. Fig.2 and Fig.3 illustrates the trends of Mode I SIF and Mode II SIF against $b$ for $a/w=0.05$ and $a/w=0.50$, respectively.

In strong interaction $0 \leq b \leq 7$, the values of Mode I SIF increased rapidly for crack-to-width, $a/w=0.05-0.50$. However, the value of the Mode II SIF had been significantly declining at $a/w = 0.05-0.50$. For weak interaction area $b \geq 8$ of $a/w = 0.05-0.50$, it can be seen that the value of Mode I increase slightly before maintaining at the same level. Meanwhile, the value of the Mode II SIF slightly decreases at $b \geq 8$ for $a/w=0.05-0.50$. Mode I SIF is more influenced by damaging shielding effect rather than Mode II SIF [6]. In other words, the crack opening is highly affected by the damage shielding effect.

**Figure 2: Variation of Mode I SIF and Mode II SIF for $a/w = 0.05$**

**Figure 3: Variation of Mode I SIF and Mode II SIF for $a/w = 0.50$**

In this section, the Mode I fracture of elastic interaction $\gamma_{1,D}(\gamma_{1,D,Cr1}, \gamma_{1,D,Cr2})$ for crack interval are evaluated from $b = 1$ until $b = 25$ and $a/w = 0.05 – 0.5$. Fig.4 shown the variation of $\gamma_{1,D}$ against $a/w$ for range $b = 14 – 16$ with $\gamma_{1,BS}$. Overall, it can be seen that interaction factor directly proportional with $a/w$; the $a/w$ increases, the crack interaction factor increases and vice versa. The $\gamma_{1,D}$ prediction line is compared with the predicted result of single edge crack $\gamma_{1,BS}$. The point of intersection can be observed in this study for all the $b$ values. For ranges $b = 14 – 16$, the minimum or error differences is about 0.304% to 0.504%. While, the maximum of error differences within these ranges is about 2.896% to 0.531%.

Based on error differences, the trends are closer converging towards the predicted result of single edge crack $\gamma_{1,BS}$. There is a crack interaction limit (CIL) and crack unification limit (CUL) at these ranges of $b$ values as there is an intersection point at the lowest value of $a/w$ and the highest value of $a/w$. The double edge crack is about to equivalent to single edge crack at crack interval, $b = 16$. The trends agree with theoretical finite body trends as presented by Brown & Srawley.
Conclusion

In present work, the objective is to gain the value of Stress Intensity Factor (SIF) in numerical simulation based on the shielding stress caused by parallel edge crack interaction in human phalanx bone. The results of this study provide much significant information for understanding the shielding effect which will mainly occur in parallel edge cracks of finite body. The finite element results revealed the interaction of two cracks is directly proportional to the SIF magnitude and interaction factor $\gamma_{I}$ at the crack tips. The parallel cracks have experienced increasing shielding effect as the crack interval, $b$ increase. The crack interaction limit (CIL) and crack unification limit (CUL) had been accomplished for every range of $b$ in this study. The Mode I of SIF is more significant than Mode II of SIF in higher or lower crack interval, $b$ and crack-to-width ratio, $a/w$. Therefore, the Mode II of SIF can be neglected due to its small magnitude of stress shielding effect. Lastly, the corresponding numerical computational results of crack propagation have been attained and verified using theoretical results proposed by Brown & Srawley’s theory [5].

References