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Bionanofluid Flow Through a Moving Surface Adapting Convective Boundary Condition: Sensitivity Analysis

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ABSTRACT

The paper discusses a sensitivity analysis on a water-based bionanofluid having nanoparticles and living microorganisms flows over a moving surface. Mathematical model is constructed on the basis of Buongiorno model and a thermal convective boundary condition is taken over to replace the conventional fixed surface temperature. Numerical shooting technique is to follow closely behind the similarity transformation to solve the system of governing equations. The output parameters of interest are believed to be contingent on the governing input parameters. Obviously, the skin friction coefficient as well as the surface shear stress will only affected by the velocity ratio parameter; while the inclusion of thermal convective boundary conditions influenced the temperature profile, confined movement of the nanoparticles and living microorganisms. Focus on the response of local density of motile microorganisms to the variation of input parameters, an experimental scheme integrated with sensitivity analysis is designed. As highlighted, the local density of motile microorganisms is more sensitive to the bioconvection Péclet number rather than other governing parameters. The findings believed to offer preliminary guideline for further lab-based experiments.

Keywords:
Bionanofluid, Boundary layer, Convective boundary condition, Moving surface, Sensitivity analysis.

1. Introduction

Blasius flow refers to the boundary layer flow of a moving fluid against a stationary planar solid surface. Conversely, Sakiadis flow defines as the flow of a quiescent state fluid on a moving planar solid surface. The Sakiadis problem [1,2] was first advocated with theoretically idea and result. It was then extended by Tsou *et al.,* [3] with the aim of proving the corresponding analysis experimentally. The research of boundary layer flow on the basis of Sakiadis problem in a base fluid can be found in articles [4-7]. Those studies are valuable in providing primarily heat and mass transfer concept on wide-ranging field of applications. For instance, power generation system, transportation system, manufacturing system, etc.

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In those days, conventional heat transfer liquid can be water or oil exclusively. However, these fluids do not severe enough to have dramatic impact on heat and mass transfer intensification. Therefore, a base fluid equipped with nanophase particles has been proposed by Masuda *et al.,* [8] and then named as nanofluid by Choi and Eastman [9]. Obvious high thermal conductivity characteristics possessed by a nanofluid facilitated extensive applications such as microelectronic applications, microchannel applications, bio-microsystem applications, etc [10,11]. Exploration on boundary layer flow on the basis of Sakiadis problem immersed in a nanofluid has been done in different approaches but considering a fixed surface temperature, view [12-14]. Special note that, boundary layer flow of a nanofluid over a moving plate with convective boundary condition have been discussed in the literature of [15-17]. This viewpoint seem to be more practical and match to real life problems as heat transfer in a heat and cool exchanger system is mostly rely on the surface temperature which is vary from time to time. For this reason, customized fixed surface temperature condition in the heat transfer problem can be taken over by convective boundary condition.

Evolution over time, nanofluid appeared with hinder and difficulties. Instability features (aggregate and agglomerate) of nanoparticles in the base fluid is often an issue. Reduction of rate of heat and mass transfer, insufficient mixing, clogging and damaging samples in microchannel happened repeatedly [18,19]. In this respect, bionanofluid having both nanoparticles and living microorganisms has been introduced. Living microorganisms in bionanofluid performed bioconvection, a mechanism that believed to help in stabilizing nanoparticles behavior at the same time strengthening the thermal performance and mass transport susceptibility [20,21]. Bioconvection collaborate with the suspension of nanofluid is believed to attain perfection in biological technologies, biomedical engineering and environmental systems. But then, the study of bioconvection boundary layer flow in nanoparticle suspension is not fully explored.

The objective of this paper is to investigate the characteristics of bioconvection nanofluid flow over a moving planar surface with thermal convective boundary conditions arise. As usual, similarity transformation followed by shooting method programmed in Maple18 software is applied to solve the governing equations numerically. Meanwhile, an experimental scheme intimately linked to a sensitivity analysis is carried out to examine the dependency of output parameters of interest on the input governing parameters [22]. Noteworthy, sensitivity analysis performed by authors focused only for the local density of motile microorganisms. This investigation is associated with the possible guideline in future device fabrication. For the best comprehensive survey reveals to date, such study is new and unachieved.

2. Methodology

2.1 Governing Equations

A steady two-dimensional boundary layer flow of a water-based bionanofluid flowing through a moving planar surface is physically modelled in Figure 1. $x-$ and $y-$axis in the Cartesian coordinate system customized with velocity components u and v are elongated simultaneous and vertical to the moving planar surface, respectively. u_w and u_∞ are constants that referring to the moving surface velocity and the free stream velocity. Here, the variables of the uniform temperature T, nanofluid volume fraction C and concentration of microorganisms N, attached with subscript w and ∞ represents corresponding values near and far from the surface, respectively. While, T_f signifies the convective hot fluid temperature below the moving surface.

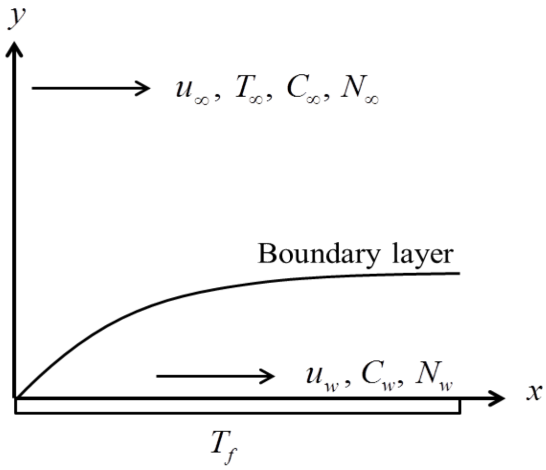


Fig. 1. Physical model and coordination system

The system of governing partial differential equations accounting on the Brownian motion and thermophoresis effects are constructed as follows

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

Equation of momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}\,, \tag{2}$$

Equation of thermal energy

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

Equation of nanoparticle volume fraction

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}\,, \tag{4}$$

Equation of microorganisms density

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{\partial}{\partial y}\left(N\tilde{v}\right)=D_n\frac{\partial^2 N}{\partial y^2}\,. \tag{5}$$

The boundary conditions are annotated as

$$v=0,\ u=u_w,\ -k\frac{\partial T}{\partial y}=h_f\left(T_f-T\right),\ C=C_w,\ N=N_w \text{ at } y=0,$$

$$u=u_\infty,\ T=T_\infty,\ C=C_\infty,\ N=N_\infty \text{ as } y\to\infty. \tag{6}$$

From the above expressions, υ denotes as the kinematic viscosity, α stands for the thermal diffusivity of nanofluid, τ expresses the ratio of effective heat capacity of the nanoparticle to the base fluid, D_B recognizes as the Brownian diffusion coefficient, D_T conveys the thermophoretic diffusion coefficient while D_n represents the diffusivity of microorganism. Then, $\tilde{v}=\left(bW_c/\left(C_w-C_\infty\right)\right)\left(\partial C/\partial y\right)$ where b and W_c are exemplify as the chemotaxis constant and maximum swimming speed of microorganisms. Whereas, the symbol of thermal conductivity k, and the heat transfer coefficient h_f arises with thermal convective boundary conditions.

Hereafter, similarity transformation comes with the similarity variables and a stream function ψ is implicated to solve Eq. (1) - (5). Notable, $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ is proved to be satisfied Eq. (1) while U is the sum of velocity u_w and u_∞.

$$\eta=\left(\frac{U}{\upsilon x}\right)^{1/2}y,\ \psi=\left(U\upsilon x\right)^{1/2}f(\eta),\ \theta(\eta)=\left(\frac{T-T_\infty}{T_f-T_\infty}\right),\phi(\eta)=\left(\frac{C-C_\infty}{C_w-C_\infty}\right),\chi(\eta)=\left(\frac{N-N_\infty}{N_w-N_\infty}\right). \tag{7}$$

Afterward, a set of transformed ordinary differential equations are obtained.

$$f'''+\frac{1}{2}ff''=0, \tag{8}$$

$$\frac{1}{\Pr}\theta''+\frac{1}{2}f\theta'+Nb\theta'\phi'+Nt\theta'^2=0, \tag{9}$$

$$\phi''+\frac{1}{2}Lef\phi'+\frac{Nt}{Nb}\theta''=0, \tag{10}$$

$$\chi''+\frac{1}{2}Scf\chi'-Pe\left[\phi'\chi'+\left(\sigma+\chi\right)\phi''\right]=0. \tag{11}$$

The boundary conditions (6) change into

$$f(0)=0,\ f'(0)=\lambda,\ \theta'(0)=Bi\left[\theta(0)-1\right],\ \phi(0)=1,\ \chi(0)=1,$$

$$f'(\infty)=1-\lambda,\ \theta(\infty)=0,\ \phi(\infty)=0,\ \chi(\infty)=0. \tag{12}$$

Here, λ indicates as the velocity ratio parameter and decodes into three different cases. $0<\lambda<1$ designates the case when the surface and the fluid act in the same direction. Later, $\lambda<0$ specifies the case of the free stream moves along the positive $x-$direction, while the surface shifted towards the negative $x-$direction and vice versa for the case of $\lambda>1$. Other than that, Bi, $\Pr$, Le, Pe, Sc, Nb, Nt and σ are the governing parameters named Biot number is the convective parameter, Prandtl number, Lewis number, bioconvection Péclet number, Schmidt number, Brownian motion parameter, thermophoresis parameter and a dimensionless constant respectively conveyed as follow, respectively.

$$\lambda=\frac{u_w}{U},\ Bi=\frac{h_f}{k}\left(\frac{\upsilon x}{U}\right)^{1/2},\ \Pr=\frac{\upsilon}{\alpha},\ Le=\frac{\upsilon}{D_B},\ Pe=\frac{bW_c}{D_n},\ Sc=\frac{\upsilon}{D_n},\ Nb=\frac{\tau D_B\left(C_w-C_\infty\right)}{\upsilon},$$

$$Nt=\frac{\tau D_T\left(T_f-T_\infty\right)}{\upsilon T_\infty},\ \sigma=\frac{N_\infty}{\left(N_w-N_\infty\right)}. \tag{13}$$

Then, skin friction coefficient C_f, local Nusselt number Nu_x, local Sherwood number Sh_x and local density of the motile microorganisms Nn_x which also symbolizes as $f''(0)$, $-\theta'(0)$, $-\phi'(0)$ and $-\chi'(0)$ are the parameters of interest with the following definition:

$$C_f=\frac{\tau_w}{\rho U^2},\ Nu_x=\frac{xq_w}{k\left(T_f-T_\infty\right)},\ Sh_x=\frac{xq_m}{D_B\left(C_w-C_\infty\right)},\ Nn_x=\frac{xq_n}{D_n\left(N_w-N_\infty\right)}, \tag{14}$$

where surface shear stress as $\tau_w=\mu\left(\partial u/\partial y\right)_{y=0}$, wall heat flux as $q_w=-k\left(\partial T/\partial y\right)_{y=0}$, the wall mass flux and the wall motile microorganisms flux as $q_m=-D_B\left(\partial C/\partial y\right)_{y=0}$ and $q_n=-D_n\left(\partial N/\partial y\right)_{y=0}$ respectively.

By referring Eq. (7), (13) and (14), we possessed expressions embedded the local Reynolds number $\mathrm{Re}_x=Ux/\upsilon$.

$$\mathrm{Re}_x^{1/2}C_f=f''(0),\ \mathrm{Re}_x^{-1/2}Nu_x=-\theta'(0),\ \mathrm{Re}_x^{-1/2}Sh_x=-\phi'(0),\ \mathrm{Re}_x^{-1/2}Nn_x=-\chi'(0). \tag{15}$$

2.2 Experimental Elaborations

In mathematics, an experiment is mainly a numerical computational simulation deals with the logic of quantity. It is a technique consists of series of data tests or runs act of imitating the behaviour of real world situation whereby a computer program is being used. A numerical experiment is carrying out for the purpose of determine the output response of a computer code change due to the several input factors. Also, conclusion about the factors prominence and eminence can be deduced in the final analysis. Response Surface Methodology (RSM) is a mathematical and statistical technique useful in designing, formulating, improving and optimizing processes in an experiment. The origin of RSM is by Box and Wilson [23], a seminal paper discussed the model's uncertainty and sensitivity for chemical processes.

In this paper, authors absorb RSM to identify the model dependency in term of the relationship between the parameters of interest (output response) and the governing parameters (input factors). There are four parameters of interest and a total of eight independent input governing parameters throughout the research. Yet, authors performed sensitivity analysis focus only for the parameter of interest named the local density of motile microorganisms. Also, only selective input governing parameters which are expected to have major variability on the local density of motile microorganisms will be considered. The model can be simplified with the aid of data interpretation. The fundamental of RSM are well set in the books of [24, 25].

A mathematical experiment on identifying the correlations between the output response parameter to the independent input parameters could be carried out based on a standard non-linear polynomial model.

$$\mathrm{Re} = \gamma_0 + \gamma_1 A + \gamma_2 B + \gamma_3 C + \gamma_{11} A^2 + \gamma_{22} B^2 + \gamma_{33} C^2 + \gamma_{12} AB + \gamma_{13} AC + \gamma_{23} BC. \qquad (16)$$

Note that, one intercept term, three linear terms, three two-factor bilinear terms and three quadratic terms are embedded in the mathematical equation. Whereas, Re denotes the response local density of motile microorganisms rely on the three independent input governing parameters of the Schmidt number, bioconvection Péclet number and dimensionless constant coded with symbol A, B and C respectively.

For a lab-based experiment, each unit cost of runs or tests is relatively high and takes times. Hence, an appropriate design of runs and levels is necessary. At the same time, to fit a second-order model in an experiment design, three and above levels of each input factors is a must. Thus, authors assign three levels of design parameters namely low (-1), medium (0) and high (1) is optimum. Furthermore, for a numerical or computational experiment to be carried out, a Central Composite Design (CCD) employed in MINITAB-18 is widely-utilized. The series of experiment with 20 runs is planned by referring the expression of $2^F + 2F + P$, whereby $F = 3$ is the number of factors and $P = 6$ is the number of center points.

ANOVA analysis also named as the analysis of variance is a statistical technique significance for the variability performance of particular variables' dependency on the RSM model. ANOVA analysis synthesis criterion on the RSM model in terms of degree of model accuracy whereby Degree of Freedom (DOF), sum of squares, adjusted mean square, F-value and P-value are the computing estimators. An attempt to explain the impact of parameters as well as model fittest would probably deduced.

Sensitivity is generally defined as the quality of sense. Literally, sensitivity is a proportion on the derivation of the response function with respect to the model parameters. Thus, sensitivity analysis is the measure of a model output performance yield due to the different distribution of input factors. Such analysis synthesis information about the key drive parameters on the model's result which to access the model robustness. This concept promotes beneficial manner for engineers in future designing processes.

3. Results and Discussion

The set of transformed ordinary differential Eq. (8)-(11) assigned with the boundary conditions (12) are solved numerically where shooting method programmed in computer software (Maple18) is taken into practice. In this study, $0 < \lambda < 1$ showing both the surface and the fluid move in same direction; is the only case discussed. Then, thickness of the boundary layer takes a constant value of

$\eta_\infty = 8$. Also, to provide a desirable water-based conductivity medium for the living microorganisms, Prandtl number $\Pr$ is customized at 6.2 signified water at a temperature of $25°C$. Table 1 recorded the shooting values of the skin friction coefficient $f''(0)$, local Nusselt number $-\theta'(0)$, local Sherwood number $-\phi'(0)$, and local density of the motile microorganisms $-\chi'(0)$ for different values of Bi, λ, Le, Nt, Nb, Sc, Pe and σ.

Table 1
The values of skin friction coefficient $f''(0)$, local Nusselt number $-\theta'(0)$, local Sherwood number $-\phi'(0)$, and local density of motile microorganisms $-\chi'(0)$ for different values of Bi, λ, Le, Nt, Nb, Sc, Pe and σ when $\Pr = 6.2$ (water)

Bi	λ	Le	Nt	Nb	Sc	Pe	σ	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
0.1 1 10	0.2	2	0.5	0.5	1	1	1	0.2173	0.0556 0.0703 0.0704	0.5391 0.6249 0.6364	1.1197 1.2970 1.3203
0.1	0.4 0.6	2	0.5	0.5	1	1	1	0.0775 -0.0819	0.0637 0.0693	0.5680 0.6000	1.2093 1.3043
0.1	0.2	3 4	0.5	0.5	1	1	1	0.2173	0.0472 0.0414	0.6590 0.7528	1.3353 1.5078
0.1	0.2	2	0.6 0.7	0.5	1	1	1	0.2173	0.0522 0.0482	0.5610 0.5877	1.1657 1.2211
0.1	0.2	2	0.5	0.6 0.7	1	1	1	0.2173	0.0438 0.0324	0.5562 0.5658	1.1508 1.1671
0.1	0.2	2	0.5	0.5	3 5	1	1	0.2173	0.0556	0.5391	1.2419 1.3409
0.1	0.2	2	0.5	0.5	1	3 5	1	0.2173	0.0556	0.5391	2.8947 4.8509
0.1	0.2	2	0.5	0.5	1	1	2 3	0.2173	0.0556	0.5391	1.4615 1.8033

Obviously, the velocity ratio parameter λ influences only for the skin friction coefficient with Eq. (8) - (12) as the factual evidence. As both the surface and the fluid move in similar way, the surface shear stress reduces. Next, based on the transformed thermal energy Eq. (9) and nanoparticle volume fraction Eq. (10), we can deduce that the local Nusselt and Sherwood number change with Le, Nt and Nb. Experimentally proof, $-\theta'(0)$ decreases with increasing Le, Nt and Nb; opposite trend for $-\phi'(0)$. Furthermore, the multivariate local density of motile microorganisms happens with all the governing input parameters. Such situation caused troublesome for the lab-based experimenters to control the independent input parameters in order to obtain a favourable yield. So, numerical experiment is the best interest in such a case.

To conduct a numerical experiment, authors used to identify the parameters first. As mention earlier, local density of motile microorganisms varies with all the independent input parameters but only selective input governing parameters which are expected to have major variability will be considered. The elimination of the input governing parameters done by the fact that λ disturbs only for the skin friction coefficient; Le, Nt and Nb brings major effect on the local Nusselt and Sherwood number; while large range of Bi (1 to 10) seem to have a minor impact on the local density of motile microorganism (increase of 1.8%). Finally, the local density of motile microorganisms owned a significant change with only three input governing parameters (Schmidt number, bioconvection

Péclet number and dimensionless constant). The reasonable interpretation is absorbed and a fitted into Eq. (16).

Second, the three input governing parameters with their corresponding symbols and levels are shown in Table 2. The experiment values are determined based on process knowledge. Thereafter, Table 3 organized with the experiment set of 20 runs. Real values are integrated with coded values for the reason of calculations simplification.

Table 2
Experimental parameters and their level

Parameter	Symbol	Level		
		Low (-1)	Medium (0)	High (1)
Sc	A	1	3	5
Pe	B	1	3	5
σ	C	1	2	3

Table 3
Design of experiments and response result

Runs	Coded values			Real values			Response
	A	B	C	Sc	Pe	σ	Nn_x
1	-1	-1	-1	1	1	1	1.1197
2	1	-1	-1	5	1	1	1.3409
3	-1	1	-1	1	5	1	4.8509
4	1	1	-1	5	5	1	4.4939
5	-1	-1	1	1	1	3	1.8033
6	1	-1	1	5	1	3	1.8342
7	-1	1	1	1	5	3	9.0399
8	1	1	1	5	5	3	7.6236
9	-1	0	0	1	3	2	4.0604
10	1	0	0	5	3	2	3.6685
11	0	-1	0	3	1	2	1.5185
12	0	1	0	3	5	2	6.3394
13	0	0	-1	3	3	1	2.8173
14	0	0	1	3	3	3	4.7250
15	0	0	0	3	3	2	3.7711
16	0	0	0	3	3	2	3.7711
17	0	0	0	3	3	2	3.7711
18	0	0	0	3	3	2	3.7711
19	0	0	0	3	3	2	3.7711
20	0	0	0	3	3	2	3.7711

Table 4 demonstrated the ANOVA analysis for the local density of motile microorganisms. From the statistical point of view, large F-value and small P-value (≤ 0.05) designates a significant results and model adequacy. Hence, linear, two-factor bilinear and squared terms are statistically significant for the response parameter Nn_x. Generally, residual error and lack-of-fit exhibits when there are unknown data points and irrelevant input parameters to the output response. Thereupon, a normal probability plot of the residual for the local density of motile microorganisms is demonstrated in Figure 2. A straight line is clearly presented in the plot, showing the errors are normally distributed and the regression model is well fitted.

Besides, the estimated regression coefficients for the local density of motile microorganisms Nn_x that fitted the polynomial Eq. (16) is jotted in Table 5.

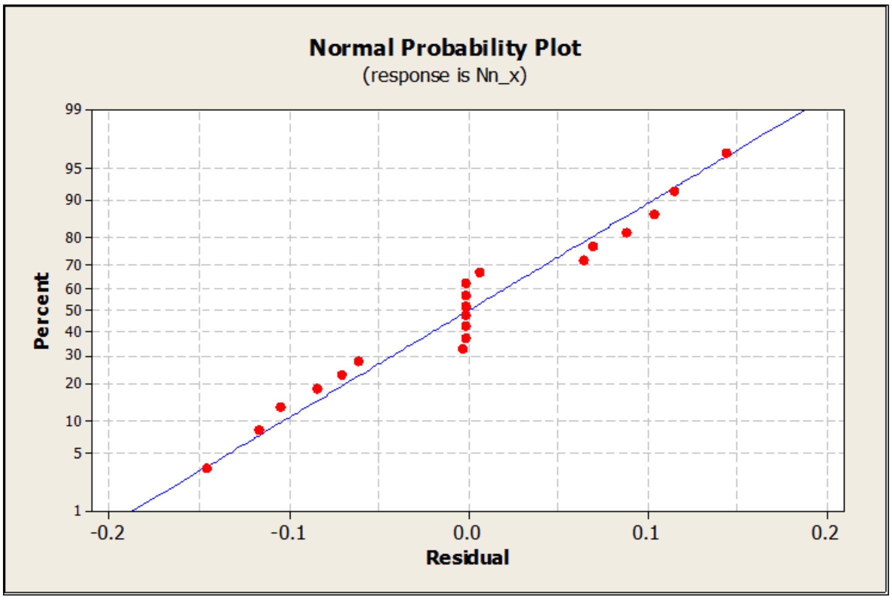


Fig. 2. Normal probability plot of the residual for the local density of motile microorganisms, Nn_x

Table 4
ANOVA analysis for the local density of motile microorganisms, Nn_x

Source	DOF	Sum of squares	Adj. mean square	F-value	P-value
Nn_x					
Model	9	78.0136	8.6682	704.74	0
Linear	3	72.3516	24.1172	1960.79	0
Square	3	0.2388	0.0796	6.47	0.01
Interaction	3	5.4232	1.8077	146.97	0
Residual error	10	0.123	0.0123	-	-
Lack-of-Fit	5	0.123	0.0246	-	-
Pure error	5	0	0	-	-
Total	19	-	-	-	-

Table 5
Estimated regression coefficients for the local density of motile microorganisms, Nn_x

Term	Coefficient	P-value
Nn_x		
Constant	3.7724	0
A	-0.1913	0
B	2.4731	0
C	1.0403	0
A^2	0.0901	0.208
B^2	0.1546	0.043
C^2	-0.0032	0.962
AB	-0.2532	0
AC	-0.1562	0.003
BC	0.7677	0
	$R^2 = 99.84\%$	$R^2 - \text{adj} = 99.70\%$

Attention that, all the terms are important and have associate change to the response function; exclude the term of C^2. The reason lies in that, the term linked to the P-value of (≤ 0.05) is statistically significant and vice versa. Therefore, Eq. (16) turns into

$$Nn_x = 3.7724 - 0.1913A + 2.4731B + 1.0403C + 0.0901A^2 + 0.1546B^2 - 0.2532AB - 0.1562AC + 0.7677BC. \tag{17}$$

It is worth to mentioning that, the coefficient of determination R^2 and $R^2 - adj$ described proportion of variability in the data points and revealed RSM model adequacy. Nearly 100% of the R^2 and $R^2 - adj$ values depicted good prediction of correlation between parameters and response in the RSM model.

Lastly, in regards to Eq. (17), sensitivity functions indicated the partial derivatives of the local density of motile microorganisms to the three independent input parameters; Schmidt number (A), bioconvection Péclet number (B) and dimensionless constant (C) are deduced.

$$\frac{\delta Nn_x}{\delta A} = -0.1913 + 0.1802A - 0.2532B - 0.1562C. \tag{18}$$

$$\frac{\delta Nn_x}{\delta B} = 2.4731 + 0.3092B - 0.2532A + 0.7677C. \tag{19}$$

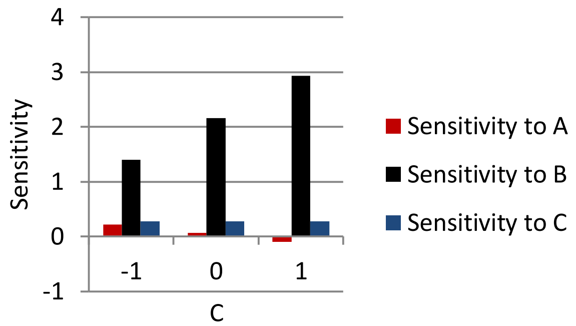
$$\frac{\delta Nn_x}{\delta C} = 1.0403 - 0.1562A + 0.7677B. \tag{20}$$

In accordance to the sensitivity functions (18)-(20), sensitivity of the response Nn_x relative to the selected governing parameters $(A$, B and $C)$ are measured in Table 6. Appreciable, positive sensitivity value characterized response function growth with the input parameter. Conversely, negative sensitivity value conveyed response function decay with the input parameter.

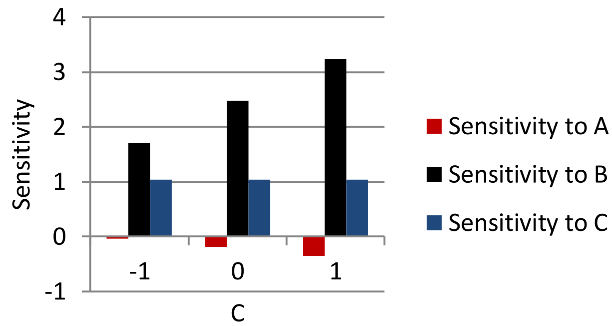
Table 6
Sensitivity analysis for the local density of motile microorganisms, Nn_x when $A = 0$

B	C	Sensitivity $\frac{\partial Nn_x}{\partial A}$	$\frac{\partial Nn_x}{\partial B}$	$\frac{\partial Nn_x}{\partial C}$
-1	-1	0.2181	1.3962	0.2726
	0	0.0619	2.1639	0.2726
	1	-0.0943	2.9316	0.2726
0	-1	-0.0351	1.7054	1.0403
	0	-0.1913	2.4731	1.0403
	1	-0.3475	3.2408	1.0403
1	-1	-0.2883	2.0146	1.8080
	0	-0.4445	2.7823	1.8080
	1	-0.6007	3.5500	1.8080

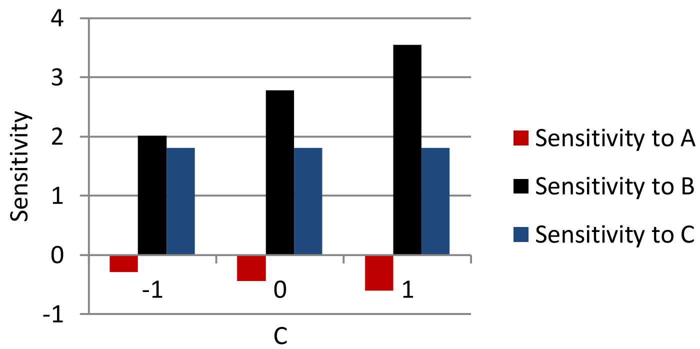
For better understanding, results of the sensitivity analysis are then illustrated with vertical bar charts; Figure 3(a)-(c).



(a) Sensitivity of Nn_x, for A=0 and B=-1



(b) Sensitivity of Nn_x, for A=0 and B=0



(c) Sensitivity of Nn_x, for A=0 and B=1

Fig. 3. Results of sensitivity analysis for the local density of motile microorganisms, Nn_x

From this perspective, an upright bar showing a positive sensitivity value while inverted bar presenting a negative sensitivity value. As highlighted, Figures 3(a)-(c) showed a general trend where positive sensitivity of the local density of motile microorganisms Nn_x to B (bioconvection Péclet number) increases with increasing C (dimensionless constant) from -1 to 1. At the same instant, positive sensitivity of the local density of motile microorganisms to C increases with increasing B but the sensitivity value fixed for all values of C. On the other hand, local density of motile microorganisms computed major negative sensitivity values to A (Schmidt number). Figure 3(a) for the case of low bioconvection Péclet number displayed sensitivity of Nn_x to A dropped from $C=-1$ to $C=1$. Moreover, Figures 3(b) and 3(c) plotted inverted bars for all value of C signified negative sensitivity values of Nn_x to A. Yet, the sensitivity of Nn_x to A raises when dimensionless constant increases from 1 to 3 $\left(C=-1 \text{ to } 1\right)$ and bioconvection Péclet number increases from 3 to 5 $\left(B=0 \text{ to } 1\right)$. Prominently, the local density of motile microorganisms is most sensitive to governing parameter B rather than A and C.

4. Conclusions

The problem of bionanofluid flow over a moving surface associated with thermal convective boundary conditions has been studied numerically. The mathematical models constructed with governing partial differential equations are changed into ordinary differential equations using similarity transformation. Numerical results obtained by shooting technique are then being performed with sensitivity analysis to infer the dependency of output parameters of interest on the input governing parameters. The findings offered preliminary guideline for lab-based experimenters in future device fabrication.

- Skin friction coefficient disturbs only by the velocity ratio parameter where surface shear stress reduces for the case of the surface and fluid move in the similar way.
- Adoption of thermal convective boundary condition alters the surface temperature makes the problem match to real life situation. Hence intensifies of Biot number mainly affects the temperature profiles.
- Local Nusselt number stands for the rate of heat transfer while local Sherwood number holds for the nanoparticle concentration; gains major impact from the Lewis number, thermophoresis and Brownian motion parameter.
- Local density of motile microorganisms signifies motility of living microorganisms in the bionanofluid increases with bioconvection Péclet number and dimensionless constant. But it decreases with Schmidt number.
- The local density of motile microorganisms is more sensitive to the bioconvection Péclet number instead of the Schmidt number and dimensionless constant.

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