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Local oscillator parameter estimation of collaborative beamforming nodes

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ABSTRACT

Parameter estimation of complex exponential signals corrupted by additive white Gaussian noise (AWGN) is crucial in the study of distributed beamforming in a practical scenario. Near zero (0) phase offset are expected at the receiver end which rely on the smoothing and correction of the frequency and phase estimates. Neither computational complexity nor the processing latency has an effect on the expected zero phase offset but the estimation accuracy does. Thus, the maximum likelihood estimator (MLE) using Fast Fourier Transform (FFT) approach is being considered for cases with none and post processing in locating of the maximum peaks. Details on how the phase estimates are arrived at is not always covered in literatures but explained in the article. Numerical results obtained showed that global maximum peaks are arrived at by employing a fine search with higher values of FFT.

1. Introduction

Longer directional transmission range, improved signal-to-noise ratio (SNR), shared energy among the collaborative nodes and nodes' redundancy are few advantages that are inherent in collaborative beamforming (CB). CB is achieved when two or more devices jointly send same signal message to a distant receiver for the purpose of the mentioned advantages [1,2]. Theoretical analysis of CB for the purpose of steering virtual antenna beams towards a receiver is considered in [3] for both linear and circular array configurations. Achieving CB practically comes with challenges in terms of transmission medium as well as the hardware devices themselves. While the medium affects the

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signal due to multipath/fading, the received signals are unsynchronized in terms of frequency and phase. Frequency drifts results from the frequency instability of transmitter and receiver local oscillators (LOs) and the Doppler effect of any or all the devices. The carrier phases of the received signal are unsynchronized due to LO phase instability, transmission delay and thermal noise (additive white Gaussian noise (AWGN)) within the devices.

The N210/NI USRP-2920 universal software radio peripheral (USRP) are mostly used as the hardware devices for the implementation of the CB. LO frequencies of CB nodes always drift off their carrier frequency periodically resulting in non-zero phase offset of the received signal, hence there is a need to eliminate this phase offset [4]. Combined LO (in the transmitters and receiver) parameter estimation algorithm will help to correct the offsets (frequency and phase) at the transmitting end through the help of feedback information from the receiver node. Depending on the extent of frequency drift on each node, this can be compensated for in software defined radio (SDR). The N210/NI USRP-2920 with no Discipline Oscillator has drifts of up to 2.5 ppm [5, 6]. These devices are often used for practical implementation of CB. Both devices use TCXO which accommodates frequency drifts of up to 2.5 ppm with an optional capability of using external 10 MHz reference and 1 pps input for lower drifts. The local oscillator (LO) in each device experiences frequency drift which affects the received signal strength (RSS) at the receiver. Hence, the need to estimate and correct the drifts becomes paramount. This means that for 900 MHz and 2.4 GHz, it is expected that drifts will seldom exceed 2.25 kHz and 6 kHz, respectively. The only setback will be if the drifts are more than 2.5 Hz for every 1 MHz. Phase only synchronization at the receiver end has been achieved by [7] in offline mode while assuming perfect frequency and timing synchronization.

The frequency and phase of complex received signals need to be estimated in order to eliminate or have a near zero phase offset of the received signal. Maximum likelihood estimation (MLE) of frequency in the presence of AWGN is treated in [8] with the mean square error (MSE) approaching Cramer Rao lower bound (CRLB). While the frequency estimation in [8] was based on DFT interpolation, though using phase and amplitude information of the signal, this article equally used three DFT points for global peak search for frequency estimation though not a sequel to the phase and amplitude information. This is because the algorithm in [8] will be highly impossible to be implemented practically due the high complexity of the estimator. A lower error was recorded in [9] with a fine search but affected the global maximum estimate due the error in the initial frequency coarse search. This can be attributed to the fact that lower computational complexity was given preference over estimation accuracy. Estimation using linear regression where the phase of the signal is used for calculating the frequency and phase estimate is considered in [10]. This is supposed to reduce the overall complexity of the algorithm in terms of latency, but rather increased it due to the complexity of the phase unwrapping algorithm. This was taken care of in [11] by eliminating the phase unwrapping algorithm, though the algorithm fails to attain the CRLB at low SNR.

This article considers an FFT based MLE algorithm for the estimation of the combined LO's frequency drift and phase offsets of the receiver signal relative to individual CB nodes in real-time. The order (*O*) of operation in this algorithm is $O\left(N\log_2 N\right)$ [12, 13] when compared to a more complex order $O\left(N^2\right)$ used in Discrete Fourier transform (DFT) for frequency estimation. The FFT algorithms are faster ways of implementing the DFT thus making the order of computational complexity less when compared to the DFT. To be able to practically implement the algorithm in the near future, the Cramer-Rao lower bound (CRLB) is first examined for the parameters in questions and then develop/analyze the estimation algorithm.

2. Cramer-Rao lower bound (CRLB)

Let $C_{TR}[n]$ be the discrete frequency domain samples of the combined received signal

$$C_{TR}[n]=s[n;a_0,f_0,\phi_0]+w[n] \tag{1}$$

$w[n]$ is the additive white Gaussian noise (AWGN) which is random and leading to the different realization of $C_{TR}[n]$ any time $s[n;a_0,f_0,\phi_0]$ is being observed. Frequency, f_0, and phase, ϕ_0, are the only parameters of interest as the amplitude, a_0, does not have a negative effect on the received signal. The deviation of the PDF should minimize the mean square errors of the estimates of the frequency, $\sigma^2_{\hat{f}_0}=E\left\{\left(\hat{f}_0-E\left[\hat{f}_0\right]\right)^2\right\}$, and phase, $\sigma^2_{\hat{\phi}_0}=E\left\{\left(\hat{\phi}_0-E\left[\hat{\phi}_0\right]\right)^2\right\}$, respectively. The unbiased estimates of the frequency and phase are thus $\hat{f}_0$ and $\hat{\phi}_0$. The minimum variance unbiased estimator (MVUE) of the received signal as described is shown in Fig. 1.

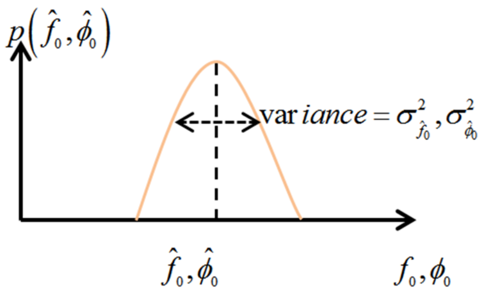

Fig. 1. PDF for frequency and phase estimate [14,15]

The variance of the parameters of interest, $\sigma^2_{\hat{f},\hat{\phi}}$, is related to the Cramer-Rao lower bound (CRLB) by the relationship in (2), that is, the CRLB is the lower bound on the variance of any unbiased estimator [16].

$$\sigma_{\hat{f},\hat{\phi}}\geq\sqrt{CRLB_{\hat{f},\hat{\phi}}} \tag{2}$$

The sample of the complex signal in (1) is

$$x[n]=a_0\exp\left(j\left(2\pi f_0\left(t_0+nT\right)+\phi_0\right)\right)+w[n] \tag{3}$$

where t_0 is the time of arrival of the first signal for samples $n=0,1,2,.....,\mathrm{N}-1$. The CRLB of the signal which is the inverse of the Fisher information matrix (FIM) is [17]

$$\operatorname{var}\left\{\hat{a}_0\,\hat{f}_0\,\hat{\phi}_0\right\} \geq \frac{1}{\left[\frac{1}{\sigma^2}\left(\frac{\partial s_{a_0,f_0,\phi_0}}{\partial(a_0,f_0,\phi_0)_i}\right)\left(\frac{\partial s_{a_0,f_0,\phi_0}}{\partial(a_0,f_0,\phi_0)_j}\right)^T\right]_{ij}} \geq \frac{1}{\sigma^2}\sum_{n=0}^{N-1}\frac{\partial s\left[n;(a_0,f_0,\phi_0)\right]}{\partial(a_0,f_0,\phi_0)_i}\frac{\partial s\left[n;(a_0,f_0,\phi_0)\right]}{\partial(a_0,f_0,\phi_0)_j} \tag{4}$$

where the denominator is the FIM, $\boldsymbol{I}\left(a_0\, f_0\, \phi_0\right)$, and thus becomes [17]

$$\boldsymbol{I}\left(a_0\, f_0\, \phi_0\right) = \frac{1}{\sigma^2}\begin{bmatrix} a_0^2T^2\left(n_0^2N+2n_0P+Q\right) & 0 & a_0^2T\left(n_0N+P\right) \\ 0 & N & 0 \\ a_0^2T\left(n_0N+P\right) & 0 & a_0^2N \end{bmatrix} \tag{5}$$

where $P=\sum_{n=0}^{N-1} n = \frac{N\left(N-1\right)}{2}$ and $Q=\sum_{n=0}^{N-1} n^2 = \frac{N\left(N-1\right)\left(2N-1\right)}{6}$

From equation (5), the variances of (4) which are the diagonals of the inverse of the FIM and known to be the unbiased CRLB for frequency and phase are therefore given as

$$\operatorname{var}\left\{\hat{f}_0\right\} \geq \frac{12\sigma^2}{a_0^2T^2N\left(N^2-1\right)} \tag{6}$$

$$\operatorname{var}\left\{\hat{\phi}_0\right\} \geq \frac{12\sigma^2\left(n_0^2N+2n_0P+Q\right)}{a_0^2N^2\left(N^2-1\right)} \tag{7}$$

If the frequency and phase to be estimated are unknown, the frequency is estimated first and later the phase from the received samples [17]. The maximum likelihood frequency and phase are given as

$$\hat{\omega} = \max_{\omega}\left|A\left(\omega\right)\right| \tag{8}$$

$$\hat{\phi} = \angle\left\{\exp\left(-j\hat{\omega}t_0\right)A\left(\hat{\omega}\right)\right\} \tag{9}$$

where

$$A\left(\omega\right) = \frac{1}{N}\sum_{n=0}^{N-1} x\left[t\right]\exp\left(-jn\omega T\right) \tag{10}$$

It can be seen from [18, 19] that the right hand side of equation (10), except for the $1/N$ is related to the discrete Fourier transform (DFT). Hence, FFT can be developed for solving the frequency estimation problem.

3. Parameter estimation

3.1. Maximum peak search methods

Periodic packets of complex signals are supposed to be received by each CB node as feedbacks from the receiver node. The packets contain information about the LO frequency and phase offsets of the CB nodes relative to the receiver. As mentioned earlier, frequency is estimated first and later the phase of the combined received signal using equations (8) and (9). In estimating the frequency of the LO, the algorithm computes the complex forward FFT in equation (10) for every M entry and at each time selects that with a maximum value. This is referred to as the *coarse search* of $\max|A(\omega)|$ which can be solved by coarse methods described in [20] at frequencies $\omega = 2\pi k/MT$ for $k = 0,1,.....M-1$. The index at the maximum value is selected for calculation of the frequency offset estimate of equation (8). The *fine search* (using three DFT points) method further locates the global maximum closest to the ω value of the depicted *coarse search*. The three DFT points with the coarse search maximum (k) is depicted in Fig. 2. This has further shown the need for a fine search in order to locate the global maximum of the signal.

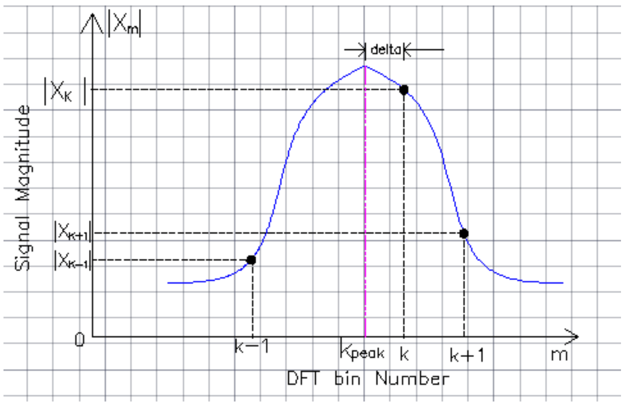

Fig. 2. Coarse Search output of frequency estimation

3.2. Numerical results and discussion

Some numerical examples of the CRLB in equations (6) and (7) and those of the unknown parameters in equations (8) and (9) for coarse and fine search are examined. The unknown parameters were carried out for $M = 2^{10}, 2^{12}, 2^{14}$ and $M = 2^{16}$ values of FFT. A complex observation $x[n]$ with AWGN and SNR of $10\log_{10}\left(b_0^2/2\sigma^2\right)$ for 257 N samples is considered. The sampling time, T , was 1 second and the first sample index, n_0, is -128. Random independent uniform distribution $\omega_0 \sim U(0.07*2*\pi, 0.12*2*\pi)$ and $\phi_0 \sim U(-\pi,\pi)$ was used for the frequency and phase respectively.

3.2.1. Case 1

Firstly, consider an FFT method without post processing (coarse search only). The MLE FFT with no post processing for the frequency estimation in equation (8) is shown in Fig. 3a. It could not track completely the CRLB for all values of FFT. However, it tracked the CRLB at -5dB for M=2^{10} and M=2^{12}. The estimator with M=2^{14} tracked the CRLB from -5dB up until 10dB before finally deflecting. Though with an unstable damping, the M=2^{16} estimator had a close range and longer CRLB tracking from the

-5dB threshold[2] until the 34dB SNR mark. The slight instability in the decay that is experienced by the M=2^{16} estimator is attributed to the number of realizations used. The higher the number of realizations used, the better the stability of the decay. The phase estimations were realized from the frequency estimate values using equation (9) with t_0 being $-\left(\frac{N-1}{2}\right)T_s$. It can be observed that they have the same threshold with that of the frequency MSE but were able to track the CRLB from the threshold to the last SNR.

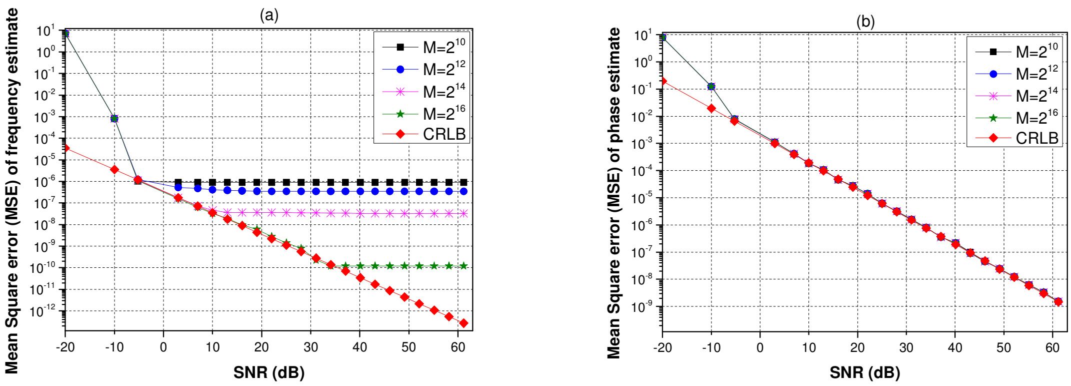

Fig. 3. Mean squared error of an observed complex signal using FFT only processing (a) frequency estimate (b) phase estimate

3.2.2. Case 2

The ‘fine search’ employed by the quadratic interpolation is considered next where equations (12) and (13) from a quadratic fit of equation (11) is used to estimate the frequency and that of the phase of the signal from the neighbourhood of the maximum peak.

$$y = cx^2 + bx + a \tag{11}$$

$$\hat{\omega}_q = -b/2c \tag{12}$$

$$\hat{\phi}_q = \angle\left\{\exp\left(-j\omega_q t_0\right)A\left(\omega_q\right)\right\} \tag{13}$$

where $x \in \left\{2\pi\left(\hat{k}_{before}\right)/MT,\ 2\pi\left(\hat{k}\right)/MT,\ 2\pi\left(\hat{k}_{after}\right)/MT\right\}$ consists of the maximum and its nearest two neighbouring magnitudes and $y = \left|A(x)\right|$ is the magnitude of the post processing of the FFTin equation (8), instead of the coarse search only [21]. Thus equation (11) can be rewritten as

[2] The region at low SNR where the MSE rises very rapidly as SNR decreases

$$\begin{bmatrix} \left|A\left(2\pi\left(\hat{k}_{before}\right)/MT\right)\right| \\ \left|A\left(2\pi\left(\hat{k}\right)/MT\right)\right| \\ \left|A\left(2\pi\left(\hat{k}_{after}\right)/MT\right)\right| \end{bmatrix} = \begin{bmatrix} \left(2\pi\left(\hat{k}_{before}\right)/MT\right)^2 & \left(2\pi\left(\hat{k}_{before}\right)/MT\right) & 1 \\ \left(2\pi\left(\hat{k}\right)/MT\right)^2 & \left(2\pi\left(\hat{k}\right)/MT\right) & 1 \\ \left(2\pi\left(\hat{k}_{after}\right)/MT\right)^2 & \left(2\pi\left(\hat{k}_{after}\right)/MT\right) & 1 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} \tag{14}$$

The quadratic interpolation post processing is expected to further lower the variances (MSE) of the estimates, moving it closer to the CRLB.

Fig. 4a depicts the frequency estimation results to decay in the MSE estimate, as expected, due to the additional fine search. The quadratic interpolation shows that FFT value for $M=2^{10}$ is able to successfully track the CRLB for SNR between the threshold of -5dB and 16dB and deflected afterwards. That of $M=2^{12}$ tracked for a wider range from -5dB until approximately 55dB while CRLB for $M=2^{14}$ and $M=2^{16}$ was able to successfully track the CRLB for the entire positive SNR from the same threshold level. This shows that increasing M allows the algorithm to successfully attain CRLB though at the expense of the execution time.

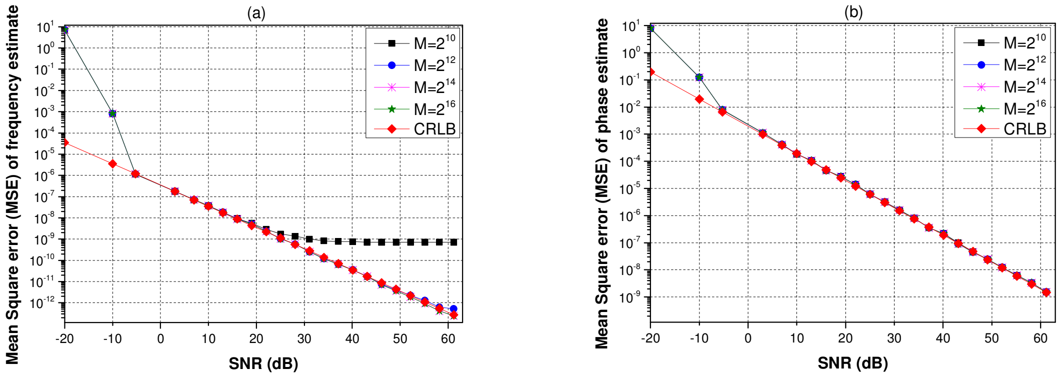

Fig. 4. Mean squared error of an observed complex signal using FFT with quadratic interpolation (a) frequency estimate (b) phase estimate

4. Conclusion

The article presents the mean square of frequency and phase estimate errors of a complex exponential signal in AWGN. The threshold is seen to be the same for the cases considered for the frequency estimates with the MSE for the quadratic case for the FFT values being closer to the CRLB. In addition, attainment of the CRLB has been enhanced by the fine search routine as well as increasing the size of the FFT.

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