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Unsteady Flow of Bingham Model: Investigating the Impact of Arterial Inclination on Solute Transport in Stenosed Artery

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ABSTRACT

The present research investigates the impact of arterial inclination and body acceleration force on the blood flow and solute dispersion through a stenosed artery using the Bingham model as a representation of the blood rheology. The problem is formulated using the momentum equation with the presence of inclination and body force acceleration parameter; and solved numerically for the blood velocity using the perturbation method with the stenosis size as the boundary condition. The solute dispersion aspect is formulated using the unsteady convective-diffusion equation and solved using the Generalized Dispersion Model (GDM) for the steady dispersion function. The solutions are graphically plotted and analysis of the blood flow and solute dispersion under the influence of arterial inclination, yield stress, time and stenosis size are conducted. Results show that the increase in arterial inclination increases the blood velocity and steady dispersion function. However, the increment in yield stress and stenosis size shows a contradictory effect by decreasing the blood velocity and steady dispersion function.

1. Introduction

The study of hemodynamics regarding a diseased artery; specifically, an artery experiencing atherosclerosis have a positive contribution to the biomedical field in terms of improving treatment, refining medical devices and providing a good quality of life for patients. Atherosclerosis patients have a narrowed artery at any point within the cardiovascular system due to the deposits of lipids and cells on the arterial wall known as stenosis. Without further preventative care or treatment, the stenosis size can potentially grow and lead to a total blockage within the artery. This will cause further health complications such as heart attack, stroke, embolism and many more. Hence, the study of blood flow behavior and solute dispersion within a stenosed artery is important to reduce the risk of further medical problems. Administering drugs intravenously is one of the treatments for a stenosed artery. It is necessary to understand how drug solutes disperse within the bloodstream, as many medications are therapeutic at low concentrations but can be harmful at higher levels.

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Consequently, gaining further insight into solute dispersion within narrow arteries can assist healthcare professionals in determining the appropriate dosage and infusion rate for patients while minimizing the risk of toxicity. This study primarily focuses on the dispersion of solutes in blood flow, particularly within an inclined artery due to many physiological systems involve blood vessels with some degree of inclination rather than being entirely horizontal. Consequently, introducing an inclination factor to the examined artery provides valuable insights into blood behavior when gravity is considered.

Many researchers have conducted studies to understand the drug delivery and blood dynamic behavior through an artery using various types of fluid model. By mathematically analyzing the solutions of a mass transport and momentum within an artery, the behavior of solute dispersion and blood flow can be observed under influence of internal or external factors. Yadeta and Shaw [1] studied the blood flow of Jeffrey model transporting magnetic nanoparticles through an inclined stenosed artery and utilized the Caputo–Fabrizio time fractional derivative. They highlighted that increase in stenosis height decreases the volumetric flow rate while increasing the flow resistance. Jamil *et al.,* [2] also utilized the Caputo–Fabrizio time fractional derivative to solve the magnetic blood flow through an inclined stenosed artery; but using the Casson model instead. However, they also discovered that increase in stenosis height decreases the blood flow velocity while increasing the flow resistance. Sharma *et al.,* [3] investigated the effect of stenosis height on the hemodynamics of unsteady blood flow through an inclined stenosis with the presence of overlapping stenosis modelled by the Casson fluid and solved using the Crank-Nicolson scheme. They discovered that the increase in Reynolds number drops the blood flow velocity. Bunonyo *et al.,* [4] discovered that both velocity and concentration decrease with increasing height of stenosis in their research of blood flow through a microchannel with magnetic field presenting. Awasthi and Siddiqui [5] studied the mechanics of unsteady blood flow through an inclined artery with the presence of growth at the arterial wall using the Casson model. They highlighted that the plug core velocity exhibits a decrease when both the yield stress and stenosis height are heightened. Conversely, the plug core velocity experiences an increase with elevated Reynolds numbers, inclined angles, slip velocity, and body acceleration. Das *et al.,* [6] studied the nanoparticles hemodynamics within an inclined stenosed artery with the presence of magnetic field. They adopted the homotopy perturbation method (HPM) to solve the momentum equation. Umadevi *et al.,* [7] investigated blood flow behavior flowing through an inclined artery with the presence of overlapping stenosis and magnetic field.

In observing the solute dispersion behavior, many methods have been adopted by researchers to obtain the solution that explains the behavior of dispersion. Ratchagar and VijayaKumar [8] conducted a study on the solute dispersion in a steady Newtonian model flow through a mild stenosed inclined artery under the influence of externally applied magnetic field and chemical reaction by adopting the Taylor's dispersion model to solve for the solute dispersion behavior. Bég and Roy [9] conducted a study regarding a dual species drug delivery by solving the bi-component species transport using the perturbation method for obtaining axial velocity and Aris-Barton approach for solving solute dispersion. They find that increase in yield stress increases the peak mean concentration for both two types of solute species.

Abidin *et al.,* [10] adopted the Generalized Dispersion Model (GDM) to solve for the diffusion coefficient in a Herschel-Bulkley flow through a catheterized stenosed artery and found that the diffusion coefficient increases to a constant value as the time increases. Elias *et al.,* [11] also utilized the GDM method to obtain the dispersion function and mean concentration of solutes in a Casson model flow through a stenosed artery with the presence of body acceleration and slip velocity. Similarly, Tiwari *et al.,* [12] implemented the GDM method in their research of solute dispersion within a two-fluid model flowing through tubes with absorptive wall to solve for the convection, dispersion and mean concentration. Thus, while all methods are useful to solve for the solute dispersion behavior, the GDM method is chosen for this present research as it yields the desired dispersion function solutions.

In this present analysis, a mathematical model designed to mathematically examine the unsteady dispersion of solutes within an unsteady flow of blood has been developed. This flow is characterized by the Bingham model flowing through a stenosed artery with the artery inclined to a certain angle. The perturbation method is adopted in solving for the blood velocity and the GDM method is utilized in solving the steady dispersion function. This study holds relevance for understanding numerous physiological processes such as the dispersion behavior when solutes are first introduced into the bloodstream and the subsequent dispersion of drugs or nutrients in the circulatory system. The primary objective is to investigate how various physical parameters, such as stenosis height and arterial inclination, exert an effect on the blood velocity and steady dispersion function within the artery. This present research offers theoretical insights into the dynamics of solute dispersion within the circulatory system and can potentially benefit the medical practices and treatments.

2. Methodology

2.1 Mathematical Formulation

The problem considered the blood flow modelled by the Bingham model through an inclined stenosed artery experiencing a body force acceleration to be an unsteady, axisymmetric, laminar and fully-developed unidirectional flow. The graphical depiction of the problem geometry is shown in Figure 1 where $\bar{r}$ is radial coordinate, $\bar{w}(\bar{z})$ is the axial velocity, g is the gravitational acceleration force and θ is the angle of arterial inclination. Hence, the gravitational force in the axial direction is expressed as $\bar{g}\sin\theta$.

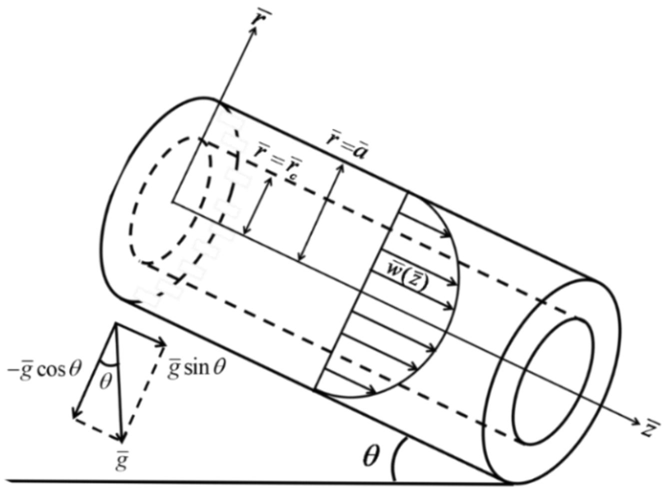


Fig. 1. Geometry of hemodynamics within an inclined stenosed artery

The problem is formulated and solved in the cylindrical coordinates of $(\bar{r},\bar{z},\bar{\varphi})$ where the simplified continuity is given as,

$$\frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{1}$$

where $\bar{w}$ is the axial velocity. The momentum equation considered in the axial $\bar{z}$ direction, with the artery inclined to an angle of θ degree is simplified to,

$$\bar{\rho}\frac{\partial \bar{w}}{\partial \bar{t}} = -\frac{d\bar{p}}{d\bar{z}} - \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\bar{\tau}\right) + \bar{\rho}\bar{g}\sin\bar{\theta}, \tag{2}$$

where $\bar{p}$ is the pressure, $\bar{\tau}$ is the shear stress, $\bar{g}\sin\theta$ is the gravitational force parameter axially and $\bar{\rho}$ is the density of the fluid. The pressure gradient $d\bar{p}/d\bar{z}$ is considered as a constant. The boundary condition for Eq. (2) is given as,

$$\bar{\tau} \text{ is finite at } \bar{r} = 0. \tag{3}$$

According to Abidin *et al.,* [13], the constitutive equation of the Bingham model is described by,

$$\frac{d\bar{w}}{d\bar{r}} = \begin{cases} -\dfrac{\left(\bar{\tau} - \bar{\tau}_y\right)}{\bar{\mu}_B}, & \text{if } \bar{\tau} \ge \bar{\tau}_y, \\ 0 \quad , & \text{if } \bar{\tau} < \bar{\tau}_y, \end{cases} \tag{4}$$

where $\bar{\tau}_y$ is the yield stress and $\bar{\mu}_B$ viscosity coefficient of Bingham model. The boundary condition for Eq. (4) is,

$$\bar{w} = 0 \text{ at } \bar{r} = \bar{R}\left(\bar{z}\right), \tag{5}$$

where $\bar{R}(\bar{z})$ is the stenosis size expressed by Jaafar *et al.,* [14] as,

$$\bar{R}\left(\bar{z}\right) = \begin{cases} \bar{R} & \text{otherwise} \\ \bar{R} - \dfrac{\bar{\delta}}{2}\left[1 + \cos\left(\dfrac{2\pi}{\bar{l}_0}\left(\bar{z} - \bar{d} - \dfrac{\bar{l}_0}{2}\right)\right)\right] & \text{when } \bar{d} \le \bar{z} \le \bar{l}_0 + \bar{d}, \end{cases} \tag{6}$$

where $\bar{\delta}$ is stenosis height, $\bar{l}_0$ is stenosis length and $\bar{d}$ is stenosis location. The simplified mass transport equation of unsteady convective-diffusion equation with the respective initial and boundary condition for the dispersion of solute is given as,

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{w}\frac{\partial \bar{C}}{\partial \bar{z}} = \bar{D}_m\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial}{\partial \bar{r}}\right) + \frac{\partial^2}{\partial \bar{z}^2}\right)\bar{C}, \tag{7}$$

$$\begin{aligned} &\bar{C}\left(\bar{r},\bar{z},0\right) = \bar{C}_0 \quad \text{if} \quad \left|\bar{z}\right| \le \bar{z}_s/2, \\ &\bar{C}\left(\bar{r},\bar{z},0\right) = 0 \quad \text{if} \quad \left|\bar{z}\right| > \bar{z}_s/2, \\ &\bar{C}\left(\bar{r},\infty,\bar{t}\right) = 0, \\ &\frac{\partial \bar{C}}{\partial \bar{r}}\left(0,\bar{z},\bar{t}\right) = \frac{\partial \bar{C}}{\partial \bar{r}}\left(\bar{R}\left(\bar{z}\right),\bar{z},\bar{t}\right) = 0, \end{aligned} \tag{8}$$

where $\overline{C}$ is the solute concentration, $\bar{t}$ is the time, $\overline{D}_m$ is the molecular diffusivity, $\overline{C}_0$ is the reference solute concentration and $\bar{z}_s$ is the solute length.

2.2 Method of Solution

A set of non-dimensional variables are proposed as below,

$$r=\frac{\bar{r}}{\overline{R}_0},\quad w=\frac{\bar{w}}{\bar{w}_s},\quad t=\frac{\bar{t}\bar{w}_s}{\overline{R}_0},\quad \tau=\frac{\bar{\tau}\overline{R}_0}{\bar{\mu}\bar{w}_s},\quad z=\frac{\bar{z}}{\overline{R}_0},\quad p=\frac{\bar{p}\overline{R}_0}{\bar{\mu}\bar{w}_s}$$
$$\alpha=\frac{\overline{R}_0\bar{w}_s\bar{\rho}}{\bar{\mu}},\quad C=\frac{\overline{C}}{\overline{C}_o},\quad D_m=\frac{\overline{D}_m}{\bar{w}_s\overline{R}_0}, \tag{9}$$

where α is the Reynolds number. Substituting the non-dimensional variables into the momentum and constitutive equations of Bingham model in Eq. (2) and (4), their dimensionless forms are obtained as,

$$\alpha\frac{\partial w}{\partial t}=-\frac{dp}{dz}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau\right)+\frac{\sin\theta}{Fr}, \tag{10}$$

where Fr is the Froude number and,

$$\frac{dw}{dr}=\begin{cases}-\left(\tau-\tau_y\right), & \text{if} \quad \tau\geq\tau_y,\\ \quad 0 \quad, & \text{if} \quad \tau<\tau_y.\end{cases} \tag{11}$$

The series expansion of the perturbation method is derived using the small parameter of Reynolds number α (where $\alpha \ll 1$). Expanding the velocity w and shear stress τ in perturbation series, the following sets of expression are obtained,

$$w\left(r,z,t\right)=w_0\left(r,z,t\right)+\alpha w_1\left(r,z,t\right)+..... \tag{12}$$

and,

$$\tau\left(r,z,t\right)=\tau_0\left(r,z,t\right)+\alpha\tau_1\left(r,z,t\right)+..... \;. \tag{13}$$

The perturbation series expansion of w in Eq. (12) and τ in Eq. (13) are substituted into Eq. (10) and Eq. (11) respectively; and the coefficient of constant and α term are equated to obtain the following sets of equations of,

$$\frac{\partial}{\partial r}r\tau_0=r\left(-\frac{dp}{dz}+\frac{\sin\theta}{Fr}\right), \tag{14}$$

$$\frac{\partial w_0}{\partial t}=-\frac{1}{r}\frac{\partial}{\partial r}r\tau_1, \tag{15}$$

$$\frac{\partial w_0}{\partial r} = -\tau_0 + \tau_y, \tag{16}$$

$$\frac{\partial w_1}{\partial r} = -\tau_1. \tag{17}$$

The dimensionless boundary conditions are,

$$\tau_0 \text{ and } \tau_1 \text{ are finite at } r = 0 \tag{18}$$

For Eq. (14) and (15),

$$w_0 = w_1 = 0 \text{ at } r = R(z) \tag{19}$$

For Eq. (16) and (17). Eq. (14) is integrated with respect to *r* and subjected to the boundary condition in Eq. (18). The first term of the perturbation series of shear stress τ_0 is obtained as

$$\tau_0 = -\frac{r}{2}\left(\frac{dp}{dz} - \frac{\sin\theta}{Fr}\right), \tag{20}$$

where Eq. (20) is then substituted into Eq. (16) and integrated with respect to *r* subject to the boundary conditions in Eq. (19) to gain the first term of the perturbation series of velocity w_0 as,

$$w_0 = \frac{1}{4}\left(\frac{dp}{dz} - \frac{\sin\theta}{Fr}\right)\left(r^2 - R(z)^2\right) + \tau_y\left(r - R(z)\right). \tag{21}$$

Eq. (21) is substituted into Eq. (15) and integrated with respect to *r* according to its boundary condition in Eq. (18) to solve for the second term of the perturbation series of shear stress τ_1 as,

$$\tau_1 = \frac{1}{t}\left(-\frac{1}{4}\left(\frac{dp}{dz} - \frac{\sin\theta}{Fr}\right)\left(\frac{r^3}{4} - R(z)^2\frac{r}{2}\right) - \tau_y\left(\frac{r^2}{3} - R(z)\frac{r}{2}\right)\right). \tag{22}$$

Substituting τ_0 from Eq. (20) and τ_1 from Eq. (22) into Eq. (17) and integrating with respect to *r*, the second term of the perturbation series of velocity w_1 is obtained as,

$$w_1 = \frac{1}{t}\left(\frac{1}{4}\left(\frac{dp}{dz} - \frac{\sin\theta}{Fr}\right)\left(\frac{r^4}{16} - R(z)^2\frac{r^2}{4} + \frac{3R(z)^4}{16}\right) + \tau_y\left(-\frac{r^3}{9} - R(z)\frac{r^2}{4} + \frac{5R(z)^3}{36}\right)\right) \tag{23}$$

Substituting w_0 and w_1 in Eq. (21) and Eq. (23) respectively into the perturbation series of velocity in Eq. (12), the unsteady velocity in the outer flow region is obtained as,

$$w_o=\frac{1}{4}\left(\frac{dp}{dz}-\frac{\sin\theta}{Fr}\right)\left(r^2-R(z)^2\right)+\tau_y\left(r-R(z)\right)$$
$$+\alpha\left(\frac{1}{t}\left(\frac{1}{4}\left(\frac{dp}{dz}-\frac{\sin\theta}{Fr}\right)\left(\frac{r^4}{16}-R(z)^2\frac{r^2}{4}+\frac{3R(z)^4}{16}\right)+\tau_y\left(-\frac{r^3}{9}-R(z)\frac{r^2}{4}+\frac{5R(z)^3}{36}\right)\right)\right). \tag{24}$$

The expression of velocity in the core flow region can be obtained by evaluating $r = r_c$ in the w_o to obtain w_c as follows,

$$w_c=\frac{1}{4}\left(\frac{dp}{dz}-\frac{\sin\theta}{Fr}\right)\left(r_c^2-R(z)^2\right)+\tau_y\left(r_c-R(z)\right)$$
$$+\alpha\left(\frac{1}{t}\left(\frac{1}{4}\left(\frac{dp}{dz}-\frac{\sin\theta}{Fr}\right)\left(\frac{r_c^4}{16}-R(z)^2\frac{r_c^2}{4}+\frac{3R(z)^4}{16}\right)+\tau_y\left(-\frac{r_c^3}{9}-R(z)\frac{r_c^2}{4}+\frac{5R(z)^3}{36}\right)\right)\right). \tag{25}$$

To find the steady dispersion function, Gill and Sankarasubramanian [15] proposed a dimensionless solution to Eq. (7), which takes the form of a series expansion of,

$$C(r,z,t)=C_m(z,t)+\sum_{j=1}^{\infty} f_j(r,t)\frac{\partial^j C_m(z,t)}{\partial z_1^j}, \tag{26}$$

where f_j is the dispersion function and C_m is the mean solute concentration defined by,

$$C_m(z,t)=\frac{2}{\left(R(z)^2\right)}\int_0^{R(z)} C(r,z,t)r\,dr. \tag{27}$$

By adhering to the process of resolving the dispersion function using the GDM method, we can derive the initial and boundary conditions for the coefficient f_j as outlined below:

$$f_j(r,0)=0 \quad \text{for} \quad j=1,2,3,..., \tag{28}$$

$$\frac{\partial f_{1s}}{\partial r}(r=0)=\frac{\partial f_{1s}}{\partial r}(r=R(z))=0, \quad \text{for} \quad j=1,2,3,..., \tag{29}$$

$$\frac{\partial f_{1t}}{\partial r}(0,t)=\frac{\partial f_{1t}}{\partial r}(R(z),t)=0, \quad \text{for} \quad j=1,2,3,... \tag{30}$$

The function f_1, which is the dispersion function of solute can be solved from the obtained equation of,

$$\frac{\partial f_1}{\partial t}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_1}{\partial r}\right)+\left(w-w_m\right)=0. \tag{31}$$

The dispersion function f_1 in Eq. (31) can be expressed as $f_1(r,t) = f_{1s}(r) + f_{1t}(r,t)$ where f_{1s} as the steady dispersion function solution and f_{1t} as the unsteady dispersion function component of the solution. By substituting the f_1 expression into Eq. (31), the differential equation needed to obtain the steady dispersion function is derived, which can be represented as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_1}{\partial r}\right)=\left(w-w_m\right). \tag{32}$$

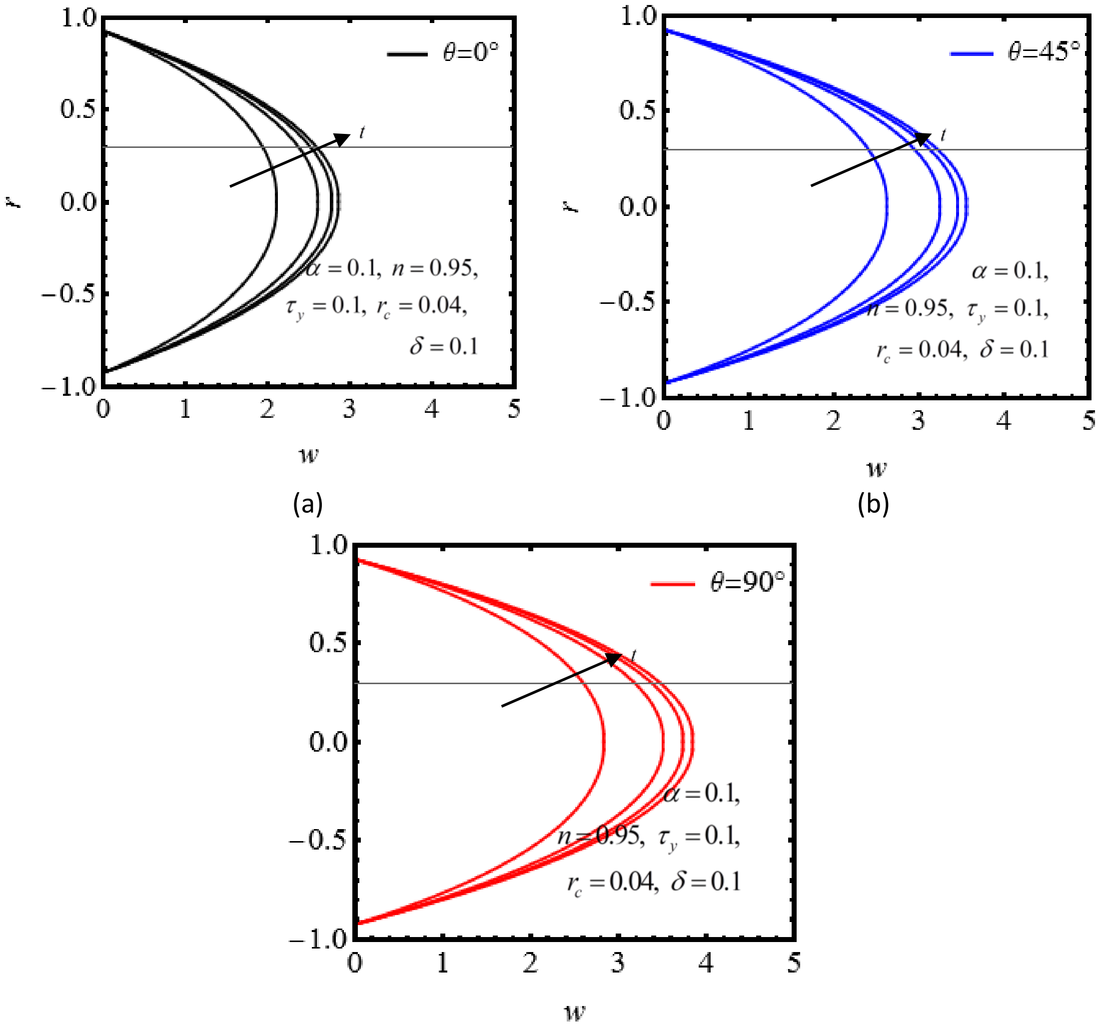
A numerical approach is utilized to solve Eq. (32) by integrating it with respect to to *r*. The integration adheres to the boundary conditions defined in Eq. (29) and relies on the Simpson's 3/8 rule to obtain the solution for the steady dispersion function f_{1s}.

3. Results

The primary aim of this research is to examine the blood flow and solute dispersion within an inclined stenosed artery, with a focus on the effect of different angles of inclination on those phenomena. The results and discussions are presented graphically. The Herschel-Bulkley value for the power-law index used in this study is $n = 0.95$. The gravitational acceleration value of $g = 9.81$ is used, as suggested by Mandin *et al.,* [16] which is rounded up to for this study to $g = 10$. The parameters that are varied are the angle of inclination, time, stenosis height and yield stress for the investigation on their influence on the blood velocity and steady dispersion function.

3.1 Velocity Profile

Figure 2 (a) to (c) illustrating the relation of blood flow velocity concerning time and inclination angles are investigated in a stenosed artery at $\delta = 0.1$. The graphical plotting reveals the blood flow velocity steadily rises as the time progresses. Initially, there is a rapid and substantial increases in velocity when the time increases from 0.05 to 0.1. However, as time continues to progress, the rate of increase diminishes, resulting in a more gradual rise approaching a large-time blood velocity. These observations provide valuable insights into the interplay of time and inclination on blood flow dynamics in a stenosed artery. Notably, this temporal behavior is observed uniformly across all angle of inclination. Furthermore, the graphical analysis shows a positive correlation between inclination angle and blood flow velocity. As the inclination angle increases, so does the blood flow velocity; suggesting that adopting inclined positions may provide a practical and effective means to enhance blood circulation. The increase in blood velocity as the arterial inclination increases can be attributed to the aid from the gravitational acceleration acting on the blood flow axially.



(a) (b) (c)

Fig. 2. Effect of time increment on the blood velocity flowing within a stenosed artery of $\delta = 0.1$ with different arterial inclination at a) $\theta = 0°$, b) $\theta = 45°$ and c) $\theta = 90°$

The stenosis height is increased to $\delta = 0.2$ and the graphical plotting is shown in Figure 3 (a) to (c). The relationship between blood flow velocity, time, and inclination angles are investigated when the arterial flow region is further narrowed. The graphical representations indicate a consistent upward trend in blood flow velocity as time advances. A trend of rapid increase in velocity initially and eventually reaches a constant blood velocity as the time progresses can be observed. Although the pattern of the blood velocity increment is similar to when the stenosis height is $\delta = 0.1$, the overall blood velocity is lower when the stenosis height is increased to $\delta = 0.2$ for all angle of inclination. It can be clarified that the increase in stenosis height reduces the blood velocity. This is due to the reduction in the arterial flow region as the stenosis height increases. The reduced space within the artery exert an impedance to the blood flow. Thus, making the blood flow slower. Moreover, the graphical analysis highlights a positive correlation between inclination angle and blood flow velocity. As the inclination angle increases, so does the blood flow velocity. The increase in blood velocity with elevated arterial inclination can be attributed to the assistance provided by gravitational acceleration, which acts on the axial blood flow.

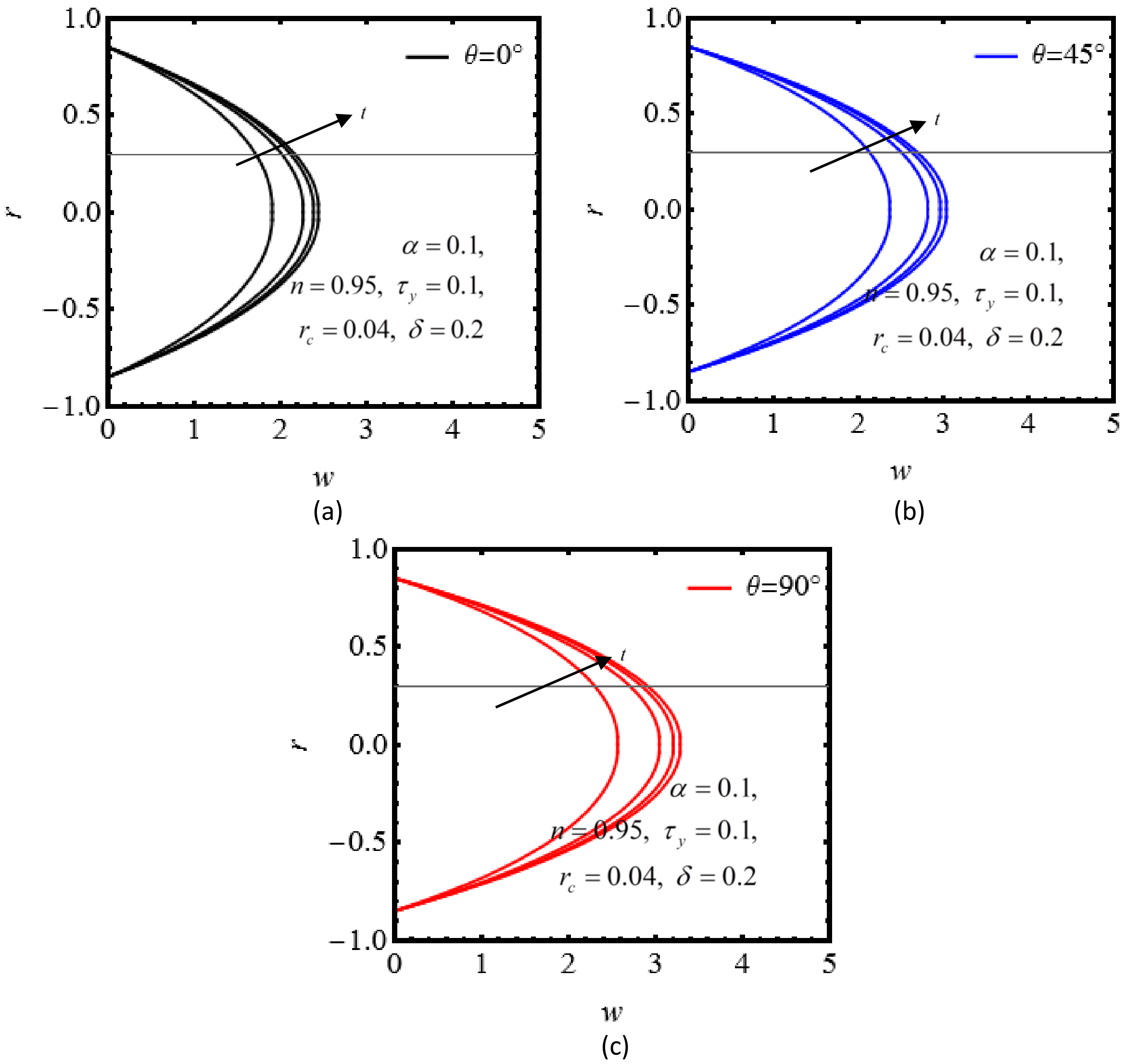


Fig. 3. Effect of time increment on the blood velocity flowing within a stenosed artery of $\delta = 0.2$ with different arterial inclination at a) $\theta = 0°$, b) $\theta = 45°$ and c) $\theta = 90°$

In Figure 4 (a) to (c), The yield stress is varied in an increasing manner of $\tau_y = 0.1{,}0.2{,}0.3$ at the stenosis height of $\delta = 0{,}0.1{,}0.2$ to observe the effect of yield stress on the blood flow velocity. Referring to Figure 4 (a) when the stenosis is $\delta = 0$, the blood velocity is observed to be decreasing as the yield stress increases. Nevertheless, a difference in the blood velocity for each arterial inclination can be noted. As the artery is inclined more, the blood velocity increases. However, it is interesting to note that the blood velocity decreases at a fixed rate as the yield stress increases for all angle of inclination. This is also true for when the stenosis is further increased to $\delta = 0.1$ and $\delta = 0.2$ as shown in Figure 4 (a) and (b) respectively. The increment in yield stress reduces the blood velocity for all angle of inclination. However, in addition to the reduction of blood velocity due to increased yield stress, the blood velocity reduction is amplified by the increased stenosis height. The smaller flow region hinders the blood flow within the artery due blood cells having lack of space to flow efficiently.

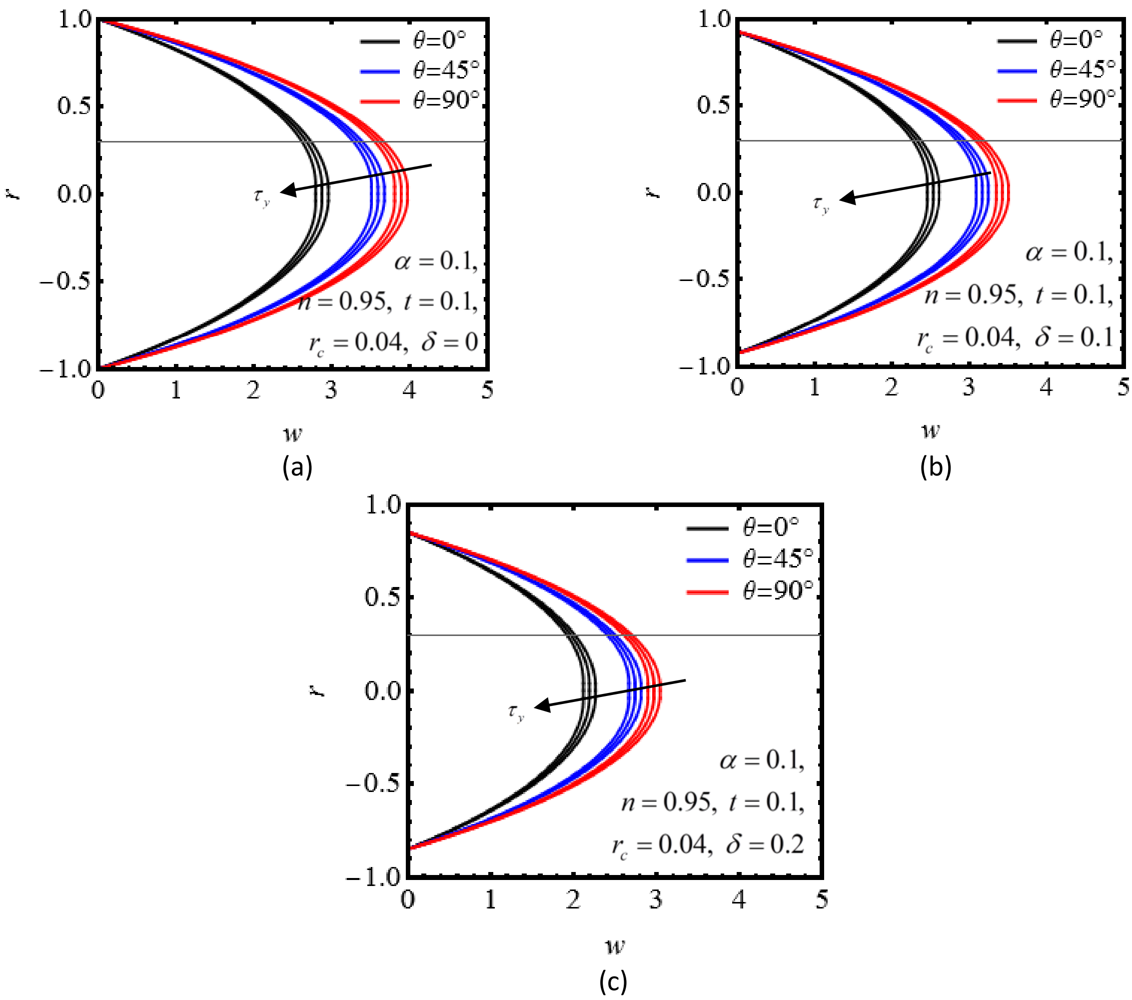


Fig. 4. Effect of yield stress increment on the blood velocity flowing within a stenosed artery with different arterial inclination and stenosis height at a) $\delta = 0$, b) $\delta = 0.1$ and c) $\delta = 0.2$

3.2 Steady Dispersion Function

Figure 5 (a) to (c) investigate the relationship between steady dispersion function, time and arterial inclination within an inclined stenosed artery at $\delta = 0.1$. It can be seen that as the time increases, the steady dispersion function increases. However, a rapid increase in steady dispersion function is noted at the beginning of the dispersion. This can be attributed to the fact that there are more solutes at the beginning of the dispersion. As time progresses from 0.05 to 0.1, the rate of increment in steady dispersion function diminishes, leading to a large-time dispersion. This is due to the less solutes presented at the stenosed location since most of the solutes have already been dispersed. Not to mention, this temporal pattern remains consistent for all angles of inclination. The graphical analysis also shows an increase in steady dispersion function as the inclination is increased as observed in Figure 5 (b) and (c). As the inclination angle increases, gravitational acceleration add force onto the solute dispersion happening within the artery axially. This increases the efficiency of the dispersion as seen in the graphical plotting.

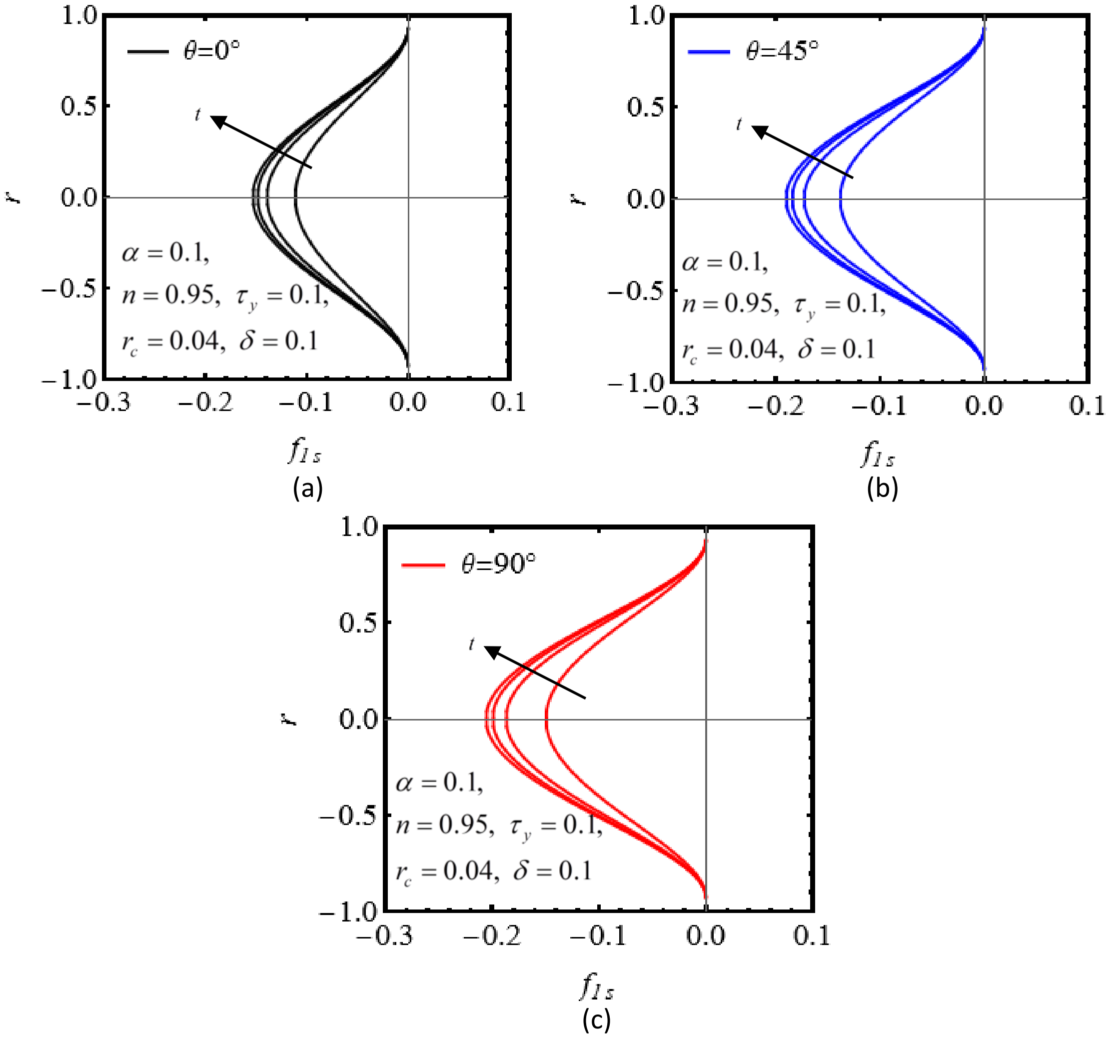


Fig. 5. Effect of time increment on the steady dispersion function within a stenosed artery of $\delta = 0.1$ with different arterial inclination at a) $\theta = 0°$, b) $\theta = 45°$ and c) $\theta = 90°$

Figure 6 (a) to (c) depicts the steady dispersion function when the stenosis height is increased to $\delta = 0.2$. These figures explore the interaction of steady dispersion function, time and inclination angles interact with the reduced arterial flow region due to the increased stenosis height. The graphical representations reveal a consistent trend of increasing steady dispersion function as time progresses. However, the overall steady dispersion function decreased for all angle of inclination when compared to graphical result in Figures 5 (a) to (c). This reduction in steady dispersion function with higher stenosis height can be attributed to the narrowing of the arterial flow region, resulting in increased resistance to the dispersion process due to the reduced available space within the artery. This constrained space causes the solutes within the artery to accumulate and unable to disperse smoothly. Nevertheless, the graphical analysis shows an increase in steady dispersion function as the inclination angle increases from Figures 6 (a) to (c). Although the increased in stenosis height hinders the efficiency of the steady dispersion function, gravitational acceleration can counteract this effect by inclining the artery.

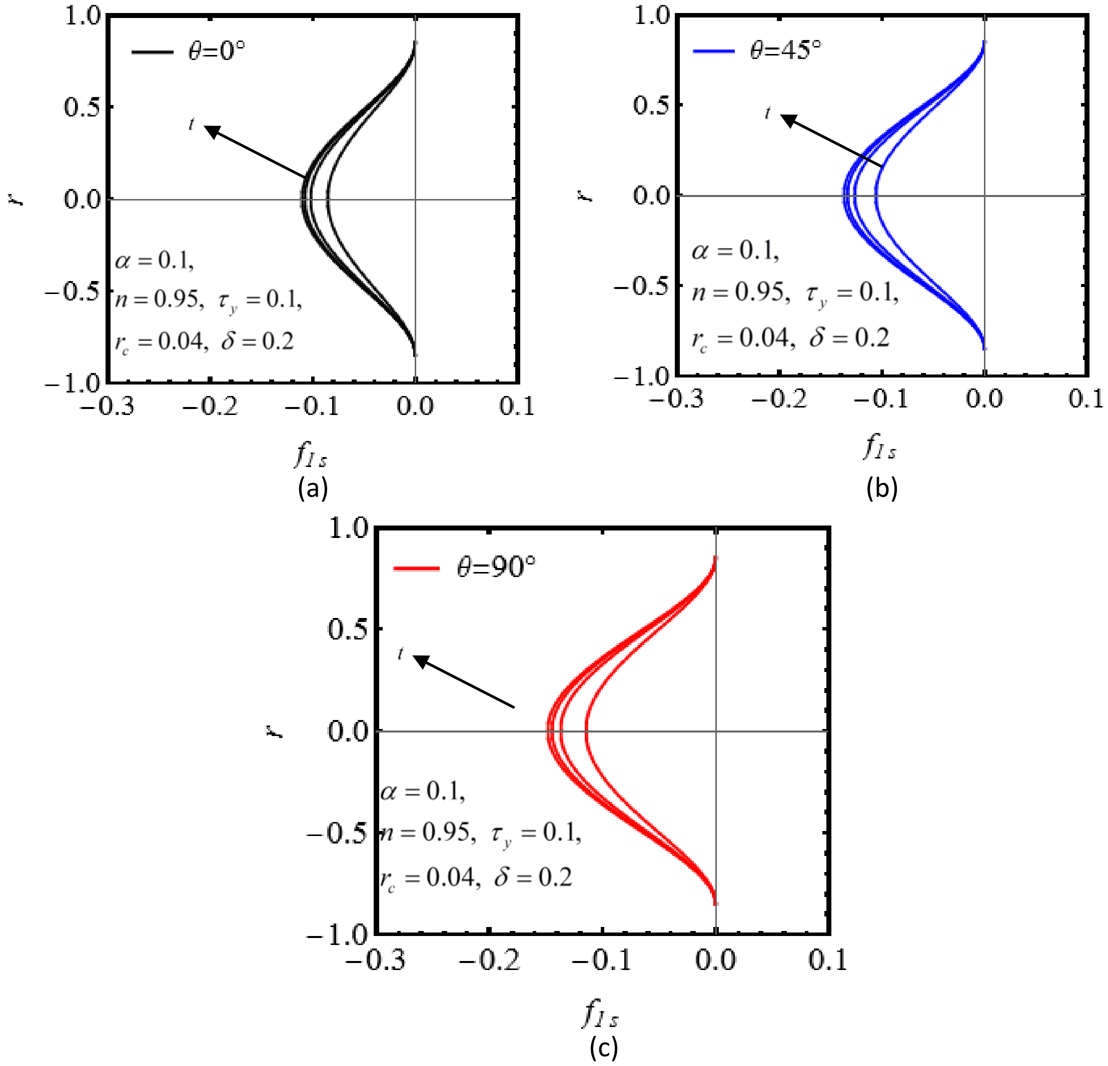


Fig. 6. Effect of time increment on the steady dispersion function within a stenosed artery of $\delta = 0.2$ with different arterial inclination at a) $\theta = 0°$, b) $\theta = 45°$ and c) $\theta = 90°$

In Figure 7 (a) to (c), the yield stress is varied within a value range of $\tau_y = 0.1,0.2,0.3$ for different stenosis heights of $\delta = 0,0.1,0.2$ to investigate the impact of yield stress on steady dispersion function. Figure 7 (a) illustrates the steady dispersion function when there is no presence of stenosis. It is observed that as the yield stress increases, the steady dispersion function decreases for all angle of inclination. Additionally, the rate of decrease in steady dispersion function due to increasing yield stress is constant and this is true for all angle of inclination. As the artery inclination increases, the steady dispersion function increases. This pattern remains consistent when the stenosis is increased to $\delta = 0.1$ and $\delta = 0.2$, as depicted in Figure 7 (b) and (c) respectively. Nevertheless, the increase in stenosis height decreases the overall steady dispersion function. Even with the presence of stenosis, the increase in yield stress reduces the steady dispersion function for all angles of inclination. It can be concluded that the increased stenosis height exacerbates the decline in steady dispersion function resulting from higher yield stress.

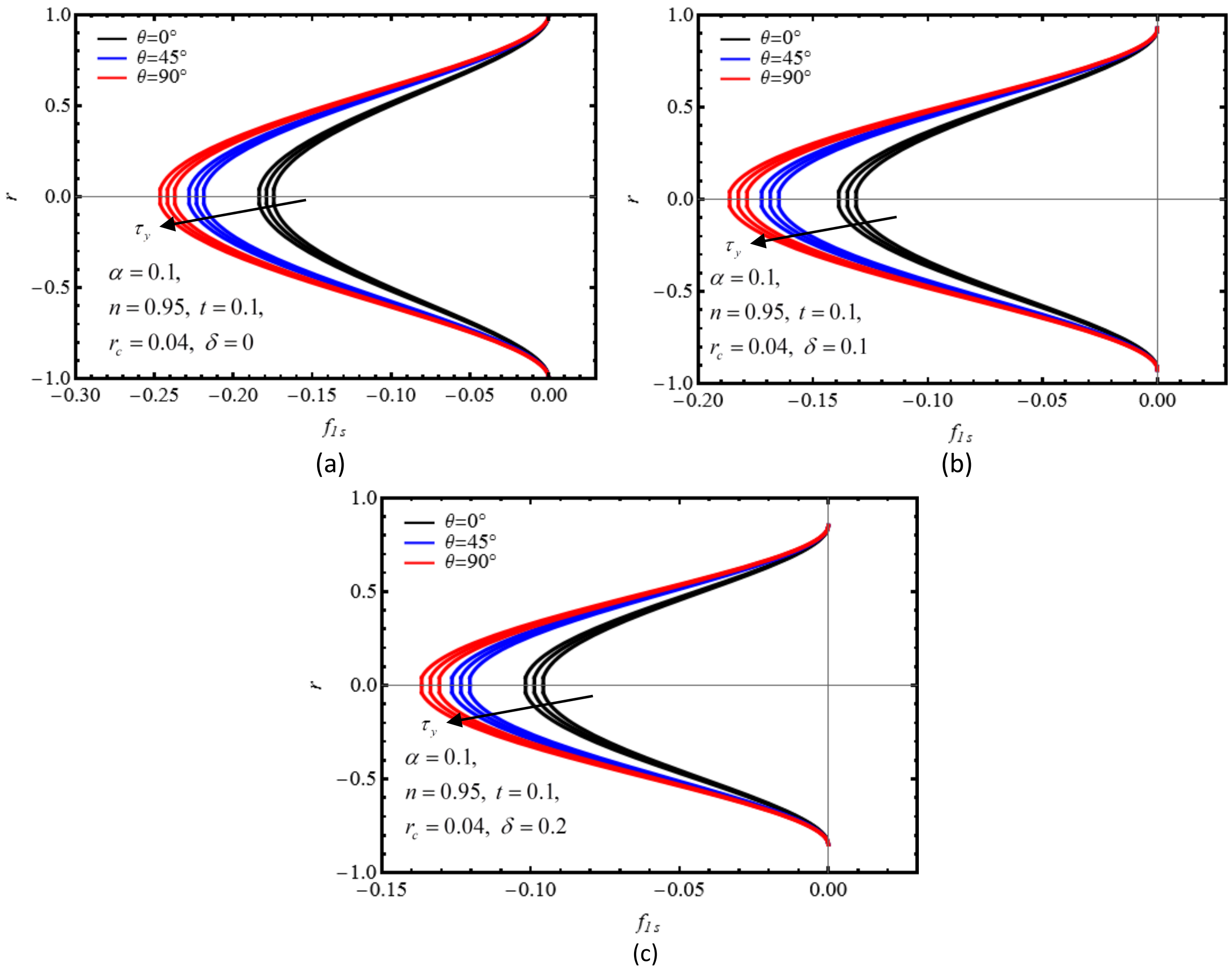


Fig. 7. Effect of yield stress increment on the steady dispersion function within a stenosed artery with different arterial inclination and stenosis height at a) $\delta = 0$, b) $\delta = 0.1$ and c) $\delta = 0.2$

4. Conclusions

The present study has examined the key aspects of the rheological behavior of blood flow and solute dispersion within an inclined stenosed artery modelled by the Bingham model and adopting the perturbation method in solving the blood velocity and GDM method in solving the steady dispersion function. The theoretical results obtained and the graphical plotting analysed yields several noteworthy conclusions:

i. Elevating the arterial inclination leads to an increase in axial blood velocity and steady dispersion function due to the aid from the gravitational acceleration acting on the blood flow and solute dispersion. However, as the arterial inclination approaches the $\vartheta = 90°$, the rate in increment of blood velocity and steady dispersion function slows down.
ii. Both axial blood velocity and steady dispersion function decreases at a constant rate as the yield stress is increased.
iii. Both axial blood velocity and steady dispersion function increases as the time parameter increases. Nevertheless, the increment is more significant initially compared to later on; and reaches a steady state after a certain time.

The theoretical results from this present research can advanced the understanding of blood flow and solute dispersion in an inclined stenosed artery from the perspective of using a Bingham model and GDM method incorporated into the solution; further adding knowledge to the area of mathematical fluid dynamics. Additionally, the results explaining the blood flow and solute dispersion behavior in an inclined stenosed artery can potentially offer insights in a general and clinical sense to improve the diagnosis, treatment, or management of atherosclerosis in patients i.e. optimise stent designs and treatment strategies. Thus, atherosclerosis patients can benefit from this study through more accurate atherosclerosis diagnosis, improved treatment outcomes and reduced complications. In addition to this present research contribution in understanding the blood flow and solute dispersion behavior in an inclined stenosed artery, this present research can also serve as an initial starting point for future investigations by extending this present research using different non-Newtonian models, adding more arterial condition and using hybrid blood blow such as hybrid nano-blood flow.

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