

Comparative Study Of Topsis And Fuzzy Topsis For The Determination Of Water Allocation In An Urbanized River Basin

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ABSTRACT

The water allocation decision-making model developed by Lee et al. (2023a, 2023b) provides a robust framework that integrates economic, social, and environmental factors with basin water availability to prioritize water allocation. However, the model's reliance on extensive data collection poses significant challenges in data-scarce regions. To mitigate this issue, this study proposes an alternative using a fuzzy TOPSIS algorithm, designed to yield similar outcomes while requiring less data. A comparative analysis between the traditional TOPSIS and fuzzy TOPSIS methods demonstrates analogous trends, with both identifying Option 1 as optimal despite minor ranking differences. Fuzzy TOPSIS stands out for its ability to handle imprecise data, simplifying the evaluation process in uncertain contexts. Nonetheless, its success depends on the strategic perspective of decision-makers. This research underscores the necessity for ongoing refinement and evaluation of the models to effectively tackle uncertainties in water resource management. The enhanced model significantly advances water governance by promoting transparency, stakeholder inclusivity, and informed decision-making. By incorporating multiple criteria and stakeholder perspectives, it fosters equitable and efficient water resource utilization. Ultimately, the model contributes to a more sustainable and resilient future for societies and the environment.

Keywords:

Water Allocation; TOPSIS; Fuzzy Set Theory

1. Introduction

Water stands as a critical foundational, economic, and communal resource (Wang *et al.*, 2022) [1], with only a minuscule fraction, 0.007%, of global water resources directly available for human use (Loucks and Van Beek, 2017) [2]. As socio-economic advancement and population growth persist, global water demand is expected to rise (Sone *et al.*, 2022) [3], potentially leading to conflicts among regions and water users when resources fall short (Sohrabi *et al.*, 2022) [4]. Optimal water resource allocation emerges as a crucial challenge in water management, necessitating efficient utilization

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while ensuring equitable distribution among various stakeholders and preserving ecological health (Li *et al.*, 2020 [5]; Li *et al.*, 2022 [6]; Yuan *et al.*, 2022 [7]).

Water allocation presents a complex, multi-dimensional, constrained optimization problem (Deng *et al.*, 2022) [8]. In recent years, multi-objective decision-making methods, including multi-objective linear and nonlinear programming and evolutionary algorithms, have been employed to address water resource optimization challenges (Zhuang *et al.*, 2015 [9]; Avarideh *et al.*, 2017 [10]; Fang *et al.*, 2018 [11]; Hatamkhani *et al.*, 2022 [12]). However, existing approaches often overlook the interests of different levels of water resource managers and may lack sufficient participation from lower-level managers, leading to implementation challenges (Yao *et al.*, 2019) [13].

Multi-Criteria Decision Making (MCDM) stands out as a widely embraced method for mathematical optimization across various domains including water allocation, land allocation, forest management, energy production, project management and environmental protection, and so on (Kangas & Kangas, 2005 [14]; Estrella *et al.*, 2014 [15]; Veintimilla-Reyes *et al.*, 2019 [16]; Mardani *et al.*, 2015 [17]; Sahabuddin & Khan, 2021 [18]). The application has remained an active area of operational research for decades, proving to be an effective approach for tackling complex and conflicting decision problems by accommodating both quantitative and qualitative evaluation factors (Sitorus *et al.*, 2019) [19].

MCDM encompasses two main categories: Multi-Attribute Decision Making (MADM) and Multiple-Objective Decision Making (MODM) (Leake & Malczewski, 2000 [20]; Zimmermann & Gutsche, 1991 [21]). MADM is apt for selecting from a finite set of alternatives and preference ranking based on predetermined attributes, while MODM is more suitable for continuous optimization problems where alternatives are not pre-defined but optimized subject to constraints (Chen *et al.*, 2018) [22]. MADM methods include Value/Utility function (Churchman & Ackoff, 1954 [23]; Pokehar & Ramachandran, 2004 [24]; Bisht & Pal, 2024 [25]), pairwise comparison (Saaty, 1980 [26]; Strantzali & Aravossis, 2016 [27]; Kuo & Chen, 2023 [28]), distance-based (eg., Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Tzeng & Huang, 2011 [29]; Garg *et al.*, 2020 [30]; Lam *et al.*, 2023 [31]), and outranking methods (Brans & Vincke, 1985 [32]; Rouyendegh & Erol, 2012 [33]; Hwang & Yoon, 1981 [34]; Liu *et al.*, 2023 [35]), whereas MODM encompass mathematical programming models like linear programming and heuristic algorithms such as genetic algorithm and simulated annealing (De Meyer *et al.*, 2014 [36]; Belton & Stewart, 2002 [37]; Deb *et al.*, 2002 [38]; Castillo-Villar, 2014 [39]; Soltanifar, 2021 [40]).

The application of the water allocation decision-making model holds great promise for improving water governance and resource management practices. This study aims to compare two distance-based methods, TOPSIS and Fuzzy TOPSIS, to determine which method is more significant in enhancing water allocation decisions. The research question guiding this study is: How does Fuzzy TOPSIS improve upon TOPSIS in terms of practical utility and effectiveness? The practical utility of the model lies in its ability to provide decision makers with a structured framework for evaluating alternative water allocation options and identifying the most optimal solutions. Moreover, by involving stakeholders and fostering participatory decision-making processes, this model promotes transparency, accountability, and inclusivity in water management.

Lee *et al.* (2023a) [41] presented a multi-criteria framework which combines the triple bottom line, i.e. economy, social and environment with the basin water availability to evaluate the basin water priority. The method is extended to determine the water allocation strategy which yields optimum basin-wide benefit (Lee *et al.*, 2023b) [42]. The indices are derived using GIS-based zonal statistics and are subjected to TOPSIS analysis. The methods proposed require extensive data collection to provide accurate representation of the triple-bottom line and a method of evaluating the water availability using a novel water resources index (WRI) was proposed. Furthermore, the

benefit of water allocation is also evaluated numerically using a proposed benefit function curve (see Ishak et al., 2024) [43].

The main drawback of the methodology presented in Lee et al. (2023a [41], 2023b [42]) is its intensive data requirements. This method can be particularly laborious and impractical for large river basins and those with sparse data availability. These challenges highlight the need for an alternative approach. In response, this paper proposes the use of the fuzzy TOPSIS algorithm, which reduces the data burden while maintaining robust decision-making capabilities. The fuzzy TOPSIS algorithm addresses the limitations of the previous methodologies by providing a more feasible and efficient solution for data-scarce regions. This study aims to compare the effectiveness of the fuzzy TOPSIS algorithm against the traditional TOPSIS method, offering insights into their respective strengths and contributing to an improved decision-making process in water allocation.

2. Methodology

2.1 TOPSIS Method

TOPSIS stands as a valuable technique in Multi-Attribute Decision Making, prized for its simplicity and ease of implementation, making it a preferred choice for users seeking a straightforward weighting approach. Conversely, the Analytic Hierarchy Process (AHP) entails establishing a decision hierarchy and conducting pairwise comparisons among criteria. Initially proposed by Hwang & Yoon in 1981 [34], the TOPSIS method determines the best alternative based on its proximity to the positive ideal solution and distance from the negative ideal solution. The positive ideal solution maximizes benefit criteria and minimizes cost criteria, while the negative ideal solution maximizes cost criteria and minimizes benefit criteria. Put simply, the positive ideal solution comprises the best attainable values of criteria, whereas the negative ideal solution comprises the worst attainable values of criteria.

A Multi-Attribute Decision Making (MADM) scenario involving m alternatives (A_1, A_2, \dots, A_m) assessed across n attributes (C_1, C_2, \dots, C_n) can be conceptualized as a geometric configuration featuring m points within an n -dimensional space. Here, each element x_{ij} within the matrix denotes the performance evaluation of the i th alternative, A_i , concerning the j th attribute, C_j , as illustrated in Eq. (1).

$$D = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & \cdot & \cdot & \cdot & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{matrix} & \left| \begin{array}{ccccccc} x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn} \end{array} \right| \end{matrix} \quad (1)$$

Alternatives, also referred to as 'options' or 'candidates' ($A_i, i = 1, 2, \dots, m$), are distinct from each other and considered in a mutually exclusive manner. Attribute weights (w_j) indicate the relative significance of each attribute in relation to the others, forming the set $W = \{w_j, j = 1, 2, \dots, n\}$. Normalization is employed to achieve consistent scales for attribute comparison. Utilizing a vector normalization method, the rating of each attribute is divided by its norm to determine the normalized value of x_{ij} , as outlined in Eq. (2).

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=2}^m x_{ij}^2}}, i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (2)$$

With the above concepts in mind, the formal procedure for TOPSIS is outlined as follows; step 1 involves constructing a normalized decision matrix, converting various attribute dimensions into non-dimensional attributes to enable comparisons across criteria. In Step 2, the weighted normalized decision matrix is created by assigning a set of weights to each criterion w_j for $j = 1, \dots, n$. Each column of the normalized decision matrix is multiplied by its corresponding weight, yielding elements of the new matrix as:

$$v_{ij} = w_j r_{ij}, \text{ where } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n \quad (3)$$

Step 3 entails determining the positive ideal (A^*) and negative ideal (A^-) solutions, defined in terms of the weighted normalized values, as depicted in Eqs. (4) and (5), respectively:

Positive Ideal Solution:

$$A^* = \{v_1, \dots, v_n\}, \text{ where } v = \{\max(v_{ij}) \text{ if } j \in J; \max(v_{ij}) \text{ if } j \in J'\} \quad (4)$$

Negative Ideal Solution:

$$A^- = \{v_1, \dots, v_n\}, \text{ where } v' = \{\max(v_{ij}) \text{ if } j \in J; \min(v_{ij}) \text{ if } j \in J'\} \quad (5)$$

In this context, J represents a collection of benefit attributes (larger-the-better type), while J' denotes a collection of cost attributes (smaller-the-better type). Moving to Step 4, the procedure involves computing separation measures for each alternative, representing the distance of each alternative from the positive ideal alternative as follows:

$$S_i^* = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^*)^2}, i = 1, \dots, m \quad (6)$$

Likewise, the distance of each alternative from the negative ideal alternative is calculated as:

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, \dots, m \quad (7)$$

Step 5 involves determining the relative proximity to the ideal solution or the resemblance to the ideal solution CC_i^* .

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, 0 < C_i^* < 1 \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, \dots, m \quad (8)$$

Note that $0 \leq C_i^* \leq 1$, where $C_i^* = 0$, when $A_i = A^-$, when $C_i^* = 1$, when $A_i = A^*$.

Step 6 entails evaluating the alternatives based on their C_i values to establish their ranking. Select the alternative with the highest C_i value or arrange the alternatives in descending order according to C_i^*

2.2 Fuzzy TOPSIS Model

For decision-makers, accurately assigning performance ratings to alternatives for the attributes under consideration can pose challenges. The advantage of employing a fuzzy approach lies in using fuzzy numbers to denote the relative importance of attributes, rather than precise values. This section expands the TOPSIS method to accommodate a fuzzy environment (Yang & Hung, 2007) [44], making it particularly applicable for addressing group decision-making issues within a fuzzy setting. Prior to the development of fuzzy TOPSIS, the theoretical basis of fuzzy theory was examined, drawing on mathematical concepts from works such as those by Ashtiani *et al.* (2009) [45], Buyukozkan *et al.* (2007) [46], Wang & Chang (2007) [47], Kabir *et al.* (2011) [48], and Bahram & Asghari (2011) [49].

In Definition 1, a fuzzy set M within a domain of discourse X is described by a membership function $\mu_M(x)$, assigning a real number between 0 and 1 to each element x in X , denoted as the membership grade of x in M . This study employs triangular fuzzy numbers, which are defined by a triplet (a_1, b_1, c_1) . Their conceptual framework and mathematical representation are illustrated by Eq. (9).

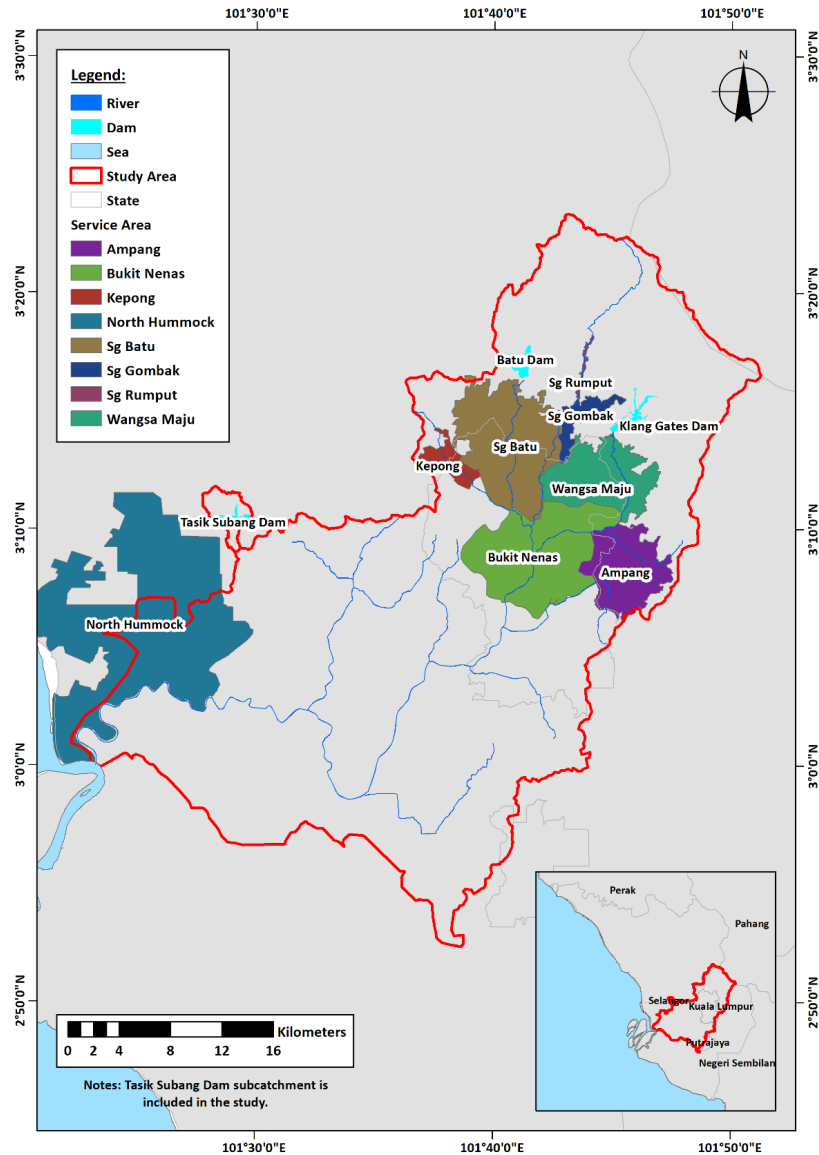


Fig. 1. Klang River Basin and Service Area of the Respective WTPs

Considering a 50-y drought, several water allocation alternatives to the respective WTPs are proposed and compared to validate the model by testing the propositions that were developed. The proposed water allocations WA are shown in Table 1, where WA ranges between 0 (no supply) to 1 (full supply).

Table 1 Proposed Water Allocation Options

Service area	Water Allocation				
	Option 1	Option 2	Option 3	Option 4	Option 5
Sg Batu	1.0	1.0	1.0	1.0	1.0
Kepong	0.5	0.4	0.3	0.3	0.3
Sg Gombak	1.0	0.9	0.8	0.8	0.8
Sg Rumput	1.0	0.9	0.8	0.8	0.8
Wangsa Maju	1.0	0.9	0.8	0.7	0.6
Bukit Nanas	0.6	0.5	0.4	0.3	0.3
Ampang	0.8	0.7	0.6	0.6	0.6
North Hummock	0.6	0.5	0.4	0.3	0.3

For every proposed water allocation, the benefits to the social, economy, and environmental dimensions are evaluated. The tradeoff is the water availability, which reduces as a result of continued water abstraction to fulfill the sectorial user demand and the environmental need. The multicriteria framework is used to determine the optimal abstraction usage while preserving the amount of water in the basin should the duration of the drought be prolonged beyond expectations. The weightage assigned to each of the dimensions considered are as follows: social (0.4), economy (0.3), water availability (0.2) and environment (0.1).

3.1 TOPSIS Method

The benefit matrix for the options considered are determined using zonal statistics as detailed in Lee *et al.* (2023b) [35]. Each element in the matrix is then normalized using Eq. (2). The resulting normalized decision matrix for the TOPSIS analysis is shown in Table 2.

Table 2 Normalized Benefit Matrix

Dimension	Social	Economy	Environmental	Water Availability
Weight	0.4	0.3	0.1	0.2
Option	Normalized Benefit Matrix			
1	0.84	0.77	0.61	0.10
2	0.70	0.56	0.63	0.19
3	0.55	0.36	0.65	0.27
4	0.46	0.23	0.66	0.30
5	0.44	0.22	0.66	0.27

Positive and negative ideal solutions are identified by taking the maximum and minimum values for each criterion using Eqs. (4) and (5). The distance of each alternative from the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each criterion is then calculated using Eqs. (6) and (7). Table 4 presents the separation measures of each alternative from the PIS and NIS. The closeness coefficient for each logistics service provider is computed using Eq. (8), and the alternatives are ranked based on these values as shown in Table 3.

Table 3 Closeness Coefficient, CC_i / Benefit for Each Alternative

Dimension	Social	Economy	Environmental	Water Availability	Distance to Ideal Solution		Benefit
Weight	0.4	0.3	0.1	0.2	Ideal	Non-ideal	
Option	Normalized Benefit Matrix				D+	D-	CC_i
1	0.84	0.77	0.142	0.10	0.077	0.142	0.649
2	0.70	0.56	0.080	0.19	0.081	0.080	0.495
3	0.55	0.36	0.063	0.27	0.142	0.063	0.309
4	0.46	0.23	0.089	0.30	0.187	0.089	0.323
5	0.44	0.22	0.082	0.27	0.191	0.082	0.299

3.1 Fuzzy TOPSIS Method

The numeric performance ratings from Table 1 are used again for the Fuzzy TOPSIS analysis. To convert these performance ratings into fuzzy linguistic variables, the ratings in Table 1 are normalized to the range [0,1] as shown in Table 4.

Table 4 Normalized Fuzzy Benefit Matrix

Weightage	7	9	9	5	7	9	1	3	5	3	5	7
Alternative/ Criteria	Social			Economy			Environment			Water availability		
Option 1	0.778	1.000	1.000	0.556	0.778	1.000	0.556	0.778	1.000	0.111	0.111	0.333
Option 2	0.556	0.778	1.000	0.333	0.556	0.778	0.556	0.778	1.000	0.111	0.111	0.333
Option 3	0.333	0.556	0.778	0.111	0.333	0.556	0.556	0.778	1.000	0.111	0.333	0.556
Option 4	0.333	0.556	0.778	0.111	0.333	0.556	0.556	0.778	1.000	0.111	0.333	0.556
Option 5	0.333	0.556	0.778	0.111	0.333	0.556	0.556	0.778	1.000	0.111	0.333	0.556

The weighted fuzzy decision matrix is next determined. By applying Eq (17) and the fuzzy multiplication rules from Eq (13), the resulting fuzzy weighted benefit matrix is presented in Table 5.

Table 5 Fuzzy Weighted Benefit Matrix

Weightage	7	9	9	5	7	9	1	3	5	3	5	7
Alternative/ Criteria	Social			Economy			Environment			Water availability		
Option 1	5.444	9.000	9.000	2.778	5.444	9.000	0.556	2.333	5.000	0.333	0.556	2.333
Option 2	3.889	7.000	9.000	1.667	3.889	7.000	0.556	2.333	5.000	0.333	0.556	2.333
Option 3	2.333	5.000	7.000	0.556	2.333	5.000	0.556	2.333	5.000	0.333	1.667	3.889
Option 4	2.333	5.000	7.000	0.556	2.333	5.000	0.556	2.333	5.000	0.333	1.667	3.889
Option 5	2.333	5.000	7.000	0.556	2.333	5.000	0.556	2.333	5.000	0.333	1.667	3.889
A*	5.444	9.000	9.000	2.778	5.444	9.000	0.556	2.333	5.000	0.333	1.667	3.889
A-	2.333	5.000	7.000	0.556	2.333	5.000	0.556	2.333	5.000	0.333	0.556	2.333

The distance of each alternative from A^* and A^- can now be calculated using Eqs. (20) and (21) as shown in Table 6 and Table 7, respectively. Next, the similarities to the ideal solution are determined using Eq. (22). The outcomes of the fuzzy TOPSIS are summarized in Table 8.

Table 6 Fuzzy Positive to Ideal Solution (FPIS)

Alternative/ Criteria	Social	Economy	Environment	Water availability	d_i^*
Option 1	0.00	0.00	0.00	1.10	1.10
Option 2	1.46	1.60	0.00	1.10	4.16
Option 3	3.15	3.19	0.00	0.00	6.34
Option 4	3.15	3.19	0.00	0.00	6.34
Option 5	3.15	3.19	0.00	0.00	6.34

Table 7 Fuzzy Negative to Ideal Solution (NPIS)

Alternative/ Criteria	Social	Economy	Environment	Water availability	d_i^-
Option 1	3.15	3.19	0.00	0.00	6.34
Option 2	1.86	1.60	0.00	0.00	3.46
Option 3	0.00	0.00	0.00	1.10	1.10
Option 4	0.00	0.00	0.00	1.10	1.10
Option 5	0.00	0.00	0.00	1.10	1.10

Table 8 Closeness Coefficient
 CC_i for Each Alternative

Alternative/ Criteria	CC_i	Rank
Option 1	0.85	1
Option 2	0.45	2
Option 3	0.15	3
Option 4	0.15	3
Option 5	0.15	3

4. Discussions

Figure 2 compares the closeness coefficient derived from the TOPSIS and fuzzy TOPSIS methods. Overall, the values obtained from both methods follow similar trend. TOPSIS gives higher estimates for Option 2 to 5, whereas fuzzy TOPSIS gives higher estimates for Option 1. The correlation of the values obtained using both methods is excellent, where $R = 0.992$.

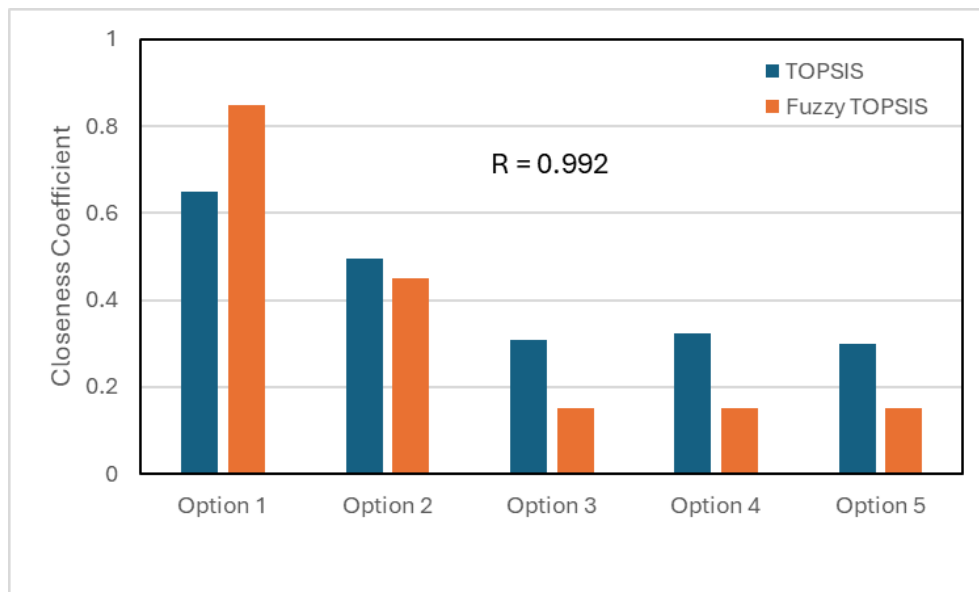


Fig. 2. Comparison and Correlation of the Closeness Coefficient Derived

Based on the TOPSIS and fuzzy TOPSIS methods, the ranking of the alternatives is summarised in Table 9. It is clear that both methods favor Option 1 as the preferred choice, indicating that it demonstrates the best water allocation practice. Other than Option 1, the preferences vary between methods. The TOPSIS method concludes with the order ranking Option 1 > Option 2 > Option 4 > Option 3 > Option 5, whereas Fuzzy TOPSIS concludes order of ranking Option 1 > Option 2 > Option 3, 4 and 5. It should be noted that the closeness coefficients obtained for Option 3 to 5 in both methods yield similar results in very close range. Fuzzy TOPSIS does not distinguish the 3 because they fall under the same rank.

Table 9 The Order of Ranking of the Alternatives for Different Methods

Preference Order	1	2	3	4	5
TOPSIS	Option 1	Option 2	Option 4	Option 3	Option 5
Fuzzy TOPSIS	Option 1	Option 2	Option 3,4,5	NA	NA

It is essential to acknowledge the limitations of this model. While it aims to capture the complexities of water allocation decision making, it is not without uncertainties. The accuracy and reliability of input data, as well as the assumptions made during the modelling process, can influence the outcomes. Therefore, continuous monitoring, evaluation, and refinement of the model are necessary to enhance its performance and adapt it to evolving water resource challenges.

In summary, systematic evaluation of the MADM problem can minimize the risk of selecting poor service quality. When precise performance ratings are accessible, the TOPSIS method is regarded as a suitable approach for addressing a water allocation problem. Fuzzy TOPSIS is the preferred choice when dealing with imprecise or vague performance ratings in resolving the service quality problem at hand. However, the limitations of fuzzy TOPSIS is that the membership function of natural-language expressions depends on the managerial perspective of the decision-maker. Therefore, the decision-maker needs to be at a strategic level of the basin or state to evaluate the importance and trends of all aspects, such as economy, social, environment and water availability.

5. Conclusions

The water allocation decision-making model presented in this paper represents a significant step forward in addressing the complexities of water resource management. By comparing multi-criteria analysis using both TOPSIS and fuzzy TOPSIS, we demonstrate that both methods yield similar outcomes, reinforcing the applicability of the identified options. The study shows that the fuzzy TOPSIS method offers a simplified way to evaluate options with minimal basin data input, except for the proposed water allocation at the WTPs. In contrast, while the TOPSIS method requires extensive basin data related to social, economic, and environmental factors, it provides a detailed representation of the benefits and impacts across various dimensions, offering decision-makers rich information to substantiate their final choices.

Our findings highlight the practical utility of fuzzy TOPSIS in data-scarce regions, emphasizing its potential to streamline the evaluation process. However, the comprehensive insights provided by the traditional TOPSIS method underscore the importance of having detailed data for more informed decision-making. This comparative analysis underscores the necessity for continuous model refinement to address uncertainties inherent in water resource management.

In conclusion, by providing a structured approach that incorporates multiple criteria and stakeholder perspectives, this model offers decision-makers a valuable tool for making informed and sustainable water allocation decisions. Its application has the potential to contribute to the equitable and efficient utilization of water resources, ensuring a more secure and resilient future for both human societies and the environment. The study's contributions significantly enhance the decision-making processes and address uncertainties, paving the way for improved water governance and resource management practices.

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