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A Short Review on RANS Turbulence Models

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ABSTRACT

Reynolds-Averaged Navier-Stokes (RANS) are such model equations and are used to simulate numerous fluid flow problem. This article focuses on the most well-known of RANS turbulence modelling and its application to industrial flows. Among all the RANS models, low Reynold number (LRN) $k-\varepsilon$ turbulence model is more accurate that the standard $k-\varepsilon$ turbulence model. This paper intends to provide a brief review of researches on RANS turbulence modelling for the fundamental understanding in solving fluid flow problem and identifies opportunities for future research.

1. Introduction

Turbulent is known as an unsteady flow field and occurred in an irregular motion, and three-dimensional velocity in fluid field. Turbulent flows exit in nature. Turbulence plays an important role in engineering applications as most flows in industrial equipment and surroundings. Reynolds [1] determined a non-dimensional number that can be used to demonstrate when transition to turbulence will usually occurred. The Reynolds number is determined as $\mathrm{Re}=ul/v$, where u and l are velocity of fluid and length scales of the flow. Turbulent flows always occur at high Reynolds number and also dissipative [2-3]. Another study was done by Richardson [4], he highlighted that turbulent flow field can be seen as a superposition of vortices modes, called eddies of different scales.

The fundamental basic for many flows of engineering interest requires the solution of the general equations of viscous fluid motion which include the continuity and the Navier-Stokes equations. Thus, the continuity and Navier-Stokes equations can be expressed as

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i\right)=0 \tag{1}$$

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$$\frac{\partial}{\partial t}\left(\rho u_i\right)+\frac{\partial}{\partial x_j}\left(\rho u_i u_j\right)=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\rho\mu\frac{\partial ui}{\partial x_j}\right) \quad (2)$$

where ρ is the density, p is the pressure. Eqs. (1) and (2) are applicable for both laminar and turbulent flow. However, for the turbulent flow every velocity and pressure term in Eqs. (1) and (2) varying in time due to the turbulent fluctuations.

In the last three decades, many numerical methodologies have been used commonly to numerically analyses turbulent flow such as Reynolds-Averaged Navier-Stokes (RANS) model [5-10], Large Eddy Simulation (LES) model [11-14] and Direct-Numerical Simulation (DNS) [15-17] and also other flow problems [18-21]. DNS pursues a thorough three-dimensional resolution all the turbulent scales in time and space by solving the Navier-Stokes equations and this is the most accurate approach for simulating turbulent flow [14,16-17]. However, DNS is very expensive and currently only can be applicable for low Reynold number flows over simple geometry. LES resolved only the large-scale motions (large eddies) of turbulent flow [11-13]. Model is less expensive than DNS, however the amount of computational resources and efforts are still too large for most practical applications. An alternative approach to simulate turbulent flow is RANS model. All turbulent length scales are modeled in RANS. It has been the backbone for the last few decades in modern CFD method for simulating the turbulent flow due to its less costing computing requirement and affordable to use [5-9,22-24].

The lack of a sufficient understanding of turbulence presents one of the significant remaining fundamental challenges to young researchers, scientists, students, and engineers as well since the actual flows are turbulent. The RANS models capture research interest on many researchers in recent years. There are lots of RANS models. Thus, the purpose of the present effort is to provide a comprehensive review of RANS models. The relevant material is certainly too much to be reviewed in a single paper. For this reason, the authors confine attention to the most well-known of RANS model that have been widely employed in simulating turbulent flow.

2. RANS Turbulence Model

The turbulent motion inflow causes significant fluctuation of flow properties (i.e. velocity, pressure, temperature and even density (if compressible flow)). By decomposing the flow properties such as velocity component u into an average value and a fluctuation component, the equation for turbulence fluctuation is obtained

$$u=\bar{u}+u' \quad (3)$$

where $\bar{u}$ is the average velocity and u' is the fluctuating velocity. The time-average of the fluctuating component is zero $\overline{u'}$=0 and the average value is expresses as

$$\overline{u(x)}=\lim_{\Delta t-\infty}\frac{1}{\Delta t}\int_{t_1}^{t_1+\Delta t}u(x,t)dt \quad (4)$$

Note that variables with symbols (-) represent Reynolds averaging (ensemble time averaging) variables and variables with (~) represent Favre averaging variables. By substituting Eq. (3) into Eqs. (1) and (2), the following equation is obtained

$$\frac{\partial \rho \bar{u}_j}{\partial x_j} = 0 \tag{5}$$

$$\frac{\partial \rho \overline{u_i}}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{\rho u_i}\,\bar{u}_j\right) = -\frac{\partial \bar{p}}{\partial x_i} + \mu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\left(\overline{\rho u_i^{'} u_j^{'}}\right) \tag{6}$$

where ρ is the fluid density and μ is the dynamic viscosity. Note that Eq. (6) is a Reynolds-Averaged Navier-Stokes (RANS) equation and is identical to the laminar flow Eq. (1), however, the different only due to the additional term, $\overline{u_i^{'} u_j^{'}}$. This term is known as the Reynolds stress and it expressed as

$$\overline{u_i^{'} u_j^{'}} = \begin{bmatrix} \overline{u^{'2}} & \overline{u^{'} v^{'}} & \overline{u^{'} w^{'}} \\ \overline{v^{'} u^{'}} & \overline{v^{'2}} & \overline{v^{'} w^{'}} \\ \overline{w^{'} u^{'}} & \overline{v^{'} w^{'}} & \overline{v^{'2}} \end{bmatrix} \tag{7}$$

The diagonal terms denoted as normal stress, whereas symmetric upper and lower diagonal denoted as shear stress. Because of this, Reynold averaging has created six independent elements. The six independent elements are the Reynold normal stresses ($\overline{u^{'2}}, \overline{v^{'2}}, \overline{w^{'2}}$) and Reynold shear stresses ($\overline{u'v'}, \overline{v'w'}, \overline{u'w'}$), and it is called as a closure problem. Thus, to close this problem, modeling the Reynold-stresses in terms of mean flow quantities is needed.

In 1877, the first of the turbulent-viscosity approximation was proposed by Boussinesq [25]. The approximation is based on analogy with the kinematic viscosity in Newton's law for the laminar flow. Thus, according to a Boussinesq's approximation, a linear relationship between turbulent or Reynolds stresses and mean strain rate are expressed as

$$\tau_{ij} = \rho \overline{u_i^{'} u_j^{'}} = \frac{2}{3}\rho k \delta_{ij} - \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{8}$$

The turbulence kinetic energy is expressed as

$$k = \frac{1}{2}\overline{u_i^{'} u_i^{'}} = \frac{1}{2}\left(\overline{u_1^{'2}} + \overline{u_2^{'2}} + \overline{u_3^{'2}}\right) \tag{9}$$

where k is expressed as the turbulence kinetic energy, μ_t is denoted as the turbulent viscosity and δ_{ij} is the Kronecker delta. By substituting Eq. (7) into Eq. (6), the equation become as

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right) = -\frac{1}{\rho}\frac{\partial \overline{p^*}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\mu + \mu_t)\frac{\partial \bar{u}_i}{\partial x_j}\right] \tag{10}$$

The isentropic part of the Reynolds-stress tensor is blended into the pressure term as $\overline{p^*} = \overline{p} + \frac{2k}{3}$.

The next sub-chapter will review only on the most familiar models and widely used equations based on the RANS equation to solve the turbulent flow problems in industrial applications.

2.1 One-Equation Models

One equation RANS models were specifically developed to solve one turbulent transport equation, like turbulent kinetic energy. One equation most well-known and widely used in the nuclear and aerospace applications [26-30]. The most recognised and extensively used one-equation models are Baldwin – Barth [26] and Spalart – Allmaras (SA) [31]. The SA model was developed and optimised for flows past wings and airfoils. The advantages of this model are produced very desirable outcomes for adverse pressure gradient and boundary layers and only require one additional equation to solve.

Recently, Karabelas and Markatos [32] solved heat and mass transfer for multiphase processes involving water vapor condensation in forced convection flow over an airfoil. The turbulent model used is the SA model. In this study, the condensation is studied based on a mixture two-phases model. The results of the numerical analysis demonstrate the flow is influenced by the mass transfer between two phases which affects significantly the momentum of both phases. Crivellini and Alessandro [27] solved Laminar Separation Bubble (LSB) problems on airfoils at low Reynolds numbers using the SA model.

However, the disadvantage of this method that it is far restrained to flow fields with transition precipitated by means of separated flow and it cannot be applied for prediction of a natural transition inside an attached boundary layer [29].

2.2 Two-Equation Models

Generally, the purposed of the two-equation models are to derive two transport equations for two turbulence properties, the turbulence kinetic energy (k) and one of any others from these turbulence properties which are either the dissipation rate of turbulence kinetic energy (ε) or the specific dissipation rate (ω) or so on [33]. In this section, only the most recognised and extensively been used two – equation models are provided. The two models are the $k-\omega$ [34] and the standard $k-\varepsilon$ model [35].

The $k-\omega$ model uses the turbulence frequency of the large eddies ω, to model the turbulence. This model was first proposed by Kolmogorov, and then by Saffmann [36]. Since then lots of development and improvement have been done by many researchers, and the modified model from Wilcox [25] finally demonstrated its accuracy for a wide range of turbulent flows. Thus, the $k-\omega$ model from Wilcox [25] is specified as follows;

Turbulent viscosity equation is calculated using k and ω as follows

$$\mu_t = \frac{\rho k}{\omega}$$

$$\omega = \max\left\{\omega, C_{\lim}\sqrt{\frac{2\bar{S}_{ij}\bar{S}_{ij}}{\beta^*}}\right\}, \qquad \bar{S}_{ij} = S_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ki}$$
$$C_{\lim} = \frac{7}{8} \tag{11}$$

The turbulent kinetic energy (k) is expressed as

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = \rho\tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta^*\rho k\omega + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma^*\frac{\rho k}{\omega}\right)\frac{\partial k}{\partial x_j}\right] \tag{12}$$

The specific dissipation rate (ω) is expressed as

$$\frac{\partial}{\partial t}(\rho\omega) + \frac{\partial}{\partial x_j}(\rho u_j\omega) = \alpha\frac{\omega}{k}\rho\tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta^*\rho\omega^2$$
$$+\sigma_d\frac{\rho}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma^*\frac{\rho k}{\omega}\right)\frac{\partial \omega}{\partial x_j}\right] \tag{13}$$

The various closure coefficients of the $k-\omega$ model is given as

$$\alpha = 0.52, \quad \beta = \beta_0 f_\beta, \quad \beta^* = 0.09,$$
$$\sigma = 0.5, \quad \sigma^* = 0.6, \quad \sigma_{d0} = 0.125, \tag{14}$$

$$\sigma_d = \begin{cases} 0, & \frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} \le 0 \\ \sigma_{d0}, & \frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} > 0, \end{cases} \qquad f_\beta = \frac{1+85\chi_\omega}{1+100\chi_\omega},$$
$$\chi_\omega = \left|\frac{\Omega_{ij}\Omega_{jk}S_{ki}}{(\beta^*\omega)^3}\right|, \qquad S_{ki} = S_{ki} - \frac{1}{2}\frac{\partial u_m}{\partial x_m}\delta_{ki} \tag{15}$$
$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i}\right),$$

where $C_{\lim}$ is expressed as the stress-limiter strength and it was introduced by Coakley [37], τ_{ij} is the Reynolds stress tensor, f_β is the vortex-stretching function, χ_ω is the dimensionless vortex-stretching parameter, whereas the function of S_{ki} or S_{ki} to yields undesired effects in two-dimensional compressible flow and Ω_{ij} is the mean-rotation tensor. The advantages of the $k-\omega$ model is near the wall treatment where the model is more powerful and accurate. Apart from that, under the influence of adverse pressure gradients it achieves higher accuracy for boundary layers.

The model equations can be easily integrated into the viscous sub-layer due to this model does not involve dumping function. However, the disadvantage of this model is it exhibits poor performance in free shear flows due to a severe sensitivity of the results to the freestream values specified for ω outside boundary and shear layers [38].

Another acknowledged and widely used model is $k-\varepsilon$ model. Jones and Launder [39] was proposed the first model of $k-\varepsilon$ model. The turbulent viscosity μ_t is calculated using k and ε is expressed as

$$\mu_t = c_\mu \frac{k^2}{\varepsilon} \tag{16}$$

The transport equations for the standard $k-\varepsilon$ is expressed as

$$\begin{aligned} \frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) &= \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \rho\overline{u_i^{'} u_j^{'}}\frac{\partial u_i}{\partial x_j} - \rho\varepsilon \\ \frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_j}(\rho u_j \varepsilon) &= \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - c_{1\varepsilon}\frac{\varepsilon}{k}\rho\overline{u_i^{'} u_j^{'}}\frac{\partial u_j}{\partial x_{ji}} - c_{2\varepsilon}\rho\frac{\varepsilon^2}{k} \end{aligned} \tag{17}$$

The model coefficients are constants

$$c_{1\varepsilon} = 1.44, \quad c_{2\varepsilon} = 1.92, \quad c_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3 \tag{18}$$

Both of the two equations models are applicable to solve the turbulent flows at high Reynolds numbers Re only. Still, there is a limitation which is inaccurate to predict the flow in the vicinity of the wall where viscous forces dominate the flow. Thus, in order to overcome this limitation, many researchers have gained an interest to propose new models with near-wall modifications. These models are known as Low-Reynolds number (LRN) models.

2.3 Low-Reynolds Number Models

The advantages of this model over standard models ($k-\omega$ and $k-\varepsilon$) are that it requires less mesh points, introduces the quite well establish near-wall distribution [40] and reducing mesh sensitivity [41, 42]. In the last three decades, a lot of suggestions have been made by many researchers for the extension of turbulence closure models to ensure it can be applied at LRN models and to elucidate the flow near the wall. The relevant equations for two-dimensional boundary layers can be expressed as

$$-\overline{\rho u_i ' u_j '} = \mu_t \left\{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right\} - \frac{2}{3}\delta_{ij}\,\overline{p}\,\tilde{k} \tag{19}$$

The $k-\varepsilon$ model

$$\frac{\partial \bar{\rho} u k}{\partial x}+\frac{1}{r}\frac{\partial \bar{\rho} r \tilde{v} k}{\partial r}=\frac{\partial}{\partial x}\left\{\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x}\right\}+\frac{1}{r}\frac{\partial}{\partial r}\left\{\left(\mu+\frac{\mu_t}{\sigma_k}\right)r\frac{\partial k}{\partial r}\right\}$$
$$+(Pro.)_k+(Dis.)_k \tag{20}$$

$$\frac{\partial \bar{\rho} u \varepsilon}{\partial x}+\frac{1}{r}\frac{\partial \bar{\rho} r \tilde{v} \varepsilon}{\partial r}=\frac{\partial}{\partial x}\left\{\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x}\right\}+\frac{1}{r}\frac{\partial}{\partial r}\left\{\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)r\frac{\partial \varepsilon}{\partial r}\right\}$$
$$+(Pro.)_\varepsilon+(Dis.)_\varepsilon+(vis)_\varepsilon \tag{21}$$

The terms and coefficients of the low-Reynolds number functions for the $k-\varepsilon$ group of models are summarized in Table 1 to Table 5.

Table 1
Terms and coefficient of the $k-\varepsilon$ group of models

Model	$(Pro.)_k$	$(Dis.)_k$	$(Pro.)_\varepsilon$	$(Dis.)_\varepsilon$
LB (Lam & Bremhorst) model [40]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
LB1 (Lam & Bremhorst) model + Sarkar [2,40]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon(1+M_t^2)$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
Abe (Abe & Kondoh) model [43]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
MK (Myong & Kasagi) model [44]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
SN (Nagano & Shimada) model [45]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
HR+Sarkar (Mehta) [46]	$\left\{\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)\right\}\times\left(1+0.4M_t^2\right)$	$-\bar{\rho}\varepsilon(1+0.7M_t^2)$	$c_1 f_1 \frac{\varepsilon}{k}\left\{\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)\right\}$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
HR (Launder & Spalding) model [35]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$
LS (Launder & Sharma) model [47]	$\mu_t\phi-\frac{2}{3}\bar{\rho}k\left(\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right)$	$-\bar{\rho}\varepsilon$	$c_1 f_1 \frac{\varepsilon}{k}(\text{Pro.})_k$	$-c_2 f_2 \bar{\rho}\frac{\varepsilon^2}{k}$

Table 2
Boundary condition and viscosity for the $k-\varepsilon$ group of models

Model	Boundary condition for ε	$(vis)_{\varepsilon}$
LB (Lam & Bremhorst) model [40]	$\varepsilon_w = v\left(\frac{\partial^2 k}{\partial n^2}\right)$	0
LB1 (Lam & Bremhorst) model + Sarkar [2,40]	$\frac{\partial \varepsilon}{\partial n} = 0$	0
Abe (Abe & Kondoh) model [43]	$\varepsilon_w = 2v\left(\frac{\partial \sqrt{k}}{\partial n}\right)^2$ or $2v\frac{k_\ell}{n_\ell^2}$	0
MK (Myong & Kasagi) model [44]	$\frac{\partial \varepsilon}{\partial n} = 0$	0
SN (Nagano & Shimada) model [45]	$\frac{\partial \varepsilon}{\partial n} = 0$	0
HR+Sarkar (Mehta) [46]	Wall function	$f_w \frac{\mu\mu_t}{\bar{\rho}}\left(\frac{\partial^2 u_i}{\partial x_j \partial x_k}\right)^2 = f_w \frac{\mu\mu_t}{\bar{\rho}}\left\{\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 \tilde{v}}{\partial x^2} + 2\frac{\partial^2 \tilde{v}}{\partial x \partial y} + \frac{\partial^2 \tilde{v}}{\partial y^2}\right\}^2$ $= f_w \frac{\mu\mu_t}{\bar{\rho}}\left\{\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial}{\partial x}\left(\frac{1}{r}\frac{\partial ru}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial ru}{\partial r}\right) + \frac{\partial^2 \tilde{v}}{\partial x^2} + 2\frac{\partial}{\partial x}\left(\frac{1}{r}\frac{\partial r\tilde{v}}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial r\tilde{v}}{\partial r}\right)\right\}^2$
HR (Launder & Spalding) model [35]	Wall function	$-\bar{\rho}\varepsilon$
LS (Launder & Sharma) model [47]	0	$-\bar{\rho}\varepsilon$

Table 3
Constants and functions of the $k-\varepsilon$ group of models

Model	μ_t	c_μ	f_μ	f_1
LB (Lam & Bremhorst) model [40]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	$\left[1-\exp\left(-0.0165 R_y\right)\right]^2 \times \left\{1+\frac{20.5}{R_T+10^{-10}}\right\}$	$1+\left[\frac{0.05}{f_\mu+10^{-3}}\right]^3$
LB1 (Lam & Bremhorst) model + Sarkar [2,40]	$\frac{c_\mu f_\mu \bar{\rho} k^2}{\varepsilon\left(1+M_t^2\right)}$	0.09	$\left[1-\exp\left(-0.0165 R_y\right)\right]^2 \times \left\{1+\frac{20.5}{R_T+10^{-10}}\right\}$	$1+\left[\frac{0.05}{f_\mu+10^{-3}}\right]^3$
Abe (Abe & Kondoh) model [43]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	$\left\{1-\exp\left(\frac{-y^+}{14}\right)\right\}^2 \times \left\{1+\frac{5}{R_T^{3/4}}\exp\left[-\left(\frac{R_T}{200}\right)^2\right]\right\}$	1
MK (Myong & Kasagi) model [44]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	$\left\{1-\exp\left(\frac{-y^+}{70}\right)\right\}^2 \times \left\{1+\frac{3.45}{\sqrt{\left(R_T+10^{-10}\right)}}\right\}$	1
SN (Nagano & Shimada) model [45]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	$(1-f_w)\times\left\{1+\frac{50}{R_T}\exp\left[-\left(\frac{R_T}{400}\right)^2\right]\right\}$ where $fw = \exp\left(\frac{-R^{3/4}}{4500}\right)$	1
HR+Sarkar (Mehta) [46]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	1	1
HR (Launder & Spalding) model [35]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	1	1
LS (Launder & Sharma) model [47]	$c_\mu f_\mu \bar{\rho} k^2/\varepsilon$	0.09	$\exp\left\{\frac{-3.4}{\left(1+R_T/50\right)^2}\right\}$	1

Table 4
Constants and functions of the $k-\varepsilon$ group of models

Model	f_2	R_T	R_y or y^+
LB (Lam & Bremhorst) model [40]	$1-\exp(-R_T^2)$	$\frac{\rho k^2}{\varepsilon\mu}$	$R_y=\frac{k^{1/2}y}{v}$
LB1 (Lam & Bremhorst) model + Sarkar [2,40]	$1-\exp(-R_T^2)$	$\frac{\rho k^2}{\varepsilon\mu}$	$R_y=\frac{k^{1/2}y}{v}$
Abe (Abe & Kondoh) model [43]	$\left[1-0.3\exp\left\{-\left(\frac{R_T}{6.5}\right)^2\right\}\right]\times\left\{1-\exp\left(\frac{-y^+}{3.1}\right)\right\}^2$	$\frac{\rho k^2}{\varepsilon\mu}$	$y^+=\frac{yu_\varepsilon}{v}$ where $u_\varepsilon=(v\varepsilon)^{1/4}$
MK (Myong & Kasagi) model [44]	$\left[1-\frac{2}{9}\exp\left\{-\left(\frac{R_T}{6}\right)^2\right\}\right]\times\left\{1-\exp\left(\frac{-y^+}{5}\right)\right\}^2$	$\frac{\rho k^2}{\varepsilon\mu}$	$y^+=\frac{yu_\tau}{v_w}$ where $u_\tau=\sqrt{\frac{\tau_w}{\rho}}$
SN (Nagano & Shimada) model [45]	$\left[1-0.3\exp\left\{-\left(\frac{R_T}{6.5}\right)^2\right\}\right]\times\left\{1-\exp\left(-\frac{2}{3}R^{1/4}\right)\right\}^2$	$\frac{\rho k^2}{\varepsilon\mu}$	$R_u=\frac{k/\varepsilon}{v/u_iu_i}$, $R=R_u+\gamma R_T\,(*)$
HR+Sarkar (Mehta) [46]	1	-	-
HR (Launder & Spalding) model [35]	1	-	-
LS (Launder & Sharma) model [47]	$1-0.3\exp(-R_T^2)$	$\frac{\rho k^2}{\varepsilon\mu}$	-

Table 5
Constants and functions of the $k-\varepsilon$ group of models

Model	c_1	c_2	σ_k	σ_ε	σ_T
LB (Lam & Bremhorst) model [40]	1.44	1.92	1.0	1.3	0.9
LB1 (Lam & Bremhorst) model + Sarkar [2,40]	1.44	1.92	1.0	1.3	0.9
Abe (Abe & Kondoh) model [43]	1.5	1.9	1.4	1.4	(0.9)
MK (Myong & Kasagi) model [44]	1.4	1.8	1.4	1.3	0.9
SN (Nagano & Shimada) model [45]	1.45	1.9	$\frac{1.2}{f_t}$	$1.3/f_t$ where $f_t=1.20\exp(-R_T/30)$	1
HR+Sarkar (Mehta) [46]	1.44	1.92	1.0	1.3	0.9
HR (Launder & Spalding) model [35]	1.44	1.92	1.0	1.3	0.9
LS (Launder & Sharma) model [47]	1.44	1.92	1.0	1.3	0.9

3. Past Studies on Low-Reynolds Number Models

The low Reynolds number turbulence characteristics affect relatively increased molecular viscosity to flow. It not only affects the mean flow of transport but also, directly and indirectly, affects various turbulent processes. The most common low Reynolds turbulence number occurs in the near-wall region and therefore focuses on the low Reynolds turbulence number following the near-wall region.

Mathur and He [48] employed the LS (Launder & Sharma) model to predict the near-wall flow behaviour. In their study, they found that their results had a good agreement with experimental and DNS data for a range of turbulent flow problems, performing better than many other LRN k–e models. Xin and Lie [49] calculated near-wall shear flow using the six LRN k − ε turbulence model which are AB model, Abe model, CHC model, LB model, LS model, YS model [40, 43, 47, 50-52], and standard k − ε turbulence model [35] in FLUENT14.0. Based on their studied, they highlighted that the LRN k−ε turbulence model is more accurate than the standard k−ε turbulence model. In the case of LRN k−ε turbulence model, Lam & Bremhorst low Reynolds number (LB1) model [40] and Yang (YS model) calculated value and actual value deviation are more significant compared to another model. In their study, the LRN k−ε turbulence models in the wall of the calculation require more mesh nodes and more computing time. Therefore, in the case of calculating insufficient resources, the standard k − ε turbulence model [35] is applicable.

Huang *et al.,* [53] gave a detailed literature review on the topic of impingement heat transfer in both experimental and numerical aspects. They pointed out that the standard k- ε model with different wall functions fails to predict the stagnation heat transfer correctly. That it is suggested to test the low Reynolds number k- ε models, as well as advanced turbulent models for jet impingement, flows characterised by high curvature of streamline, pressure gradients, and recirculation zones.

Recently, Yusof *et al.,* [54-57] employed the Lam & Bremhorst low Reynolds number (LB1) model [10,38] modified for compressible flows by Sarkar and Balakrishnan [2] to analyze the irreversible processes in a piston-cylinder system. They used this model in their study since this model was widely used and is very stable [58, 59]. Zhang *et al.,* [59] predicted the drag and lift forces, pressure, and velocity field on a full-scale passenger vehicle with two different front-end configurations using four RANS models. The models were the realisable k − ε two-layer, ABE model, SST k − ω, and V2F model [60]. They found that the realisable k − ε two-layer performed better than the other three RANS models for the baseline case. However, the RANS model may still be the right choice for predicting drag values due to its reasonable accuracy, low calculation cost, and fast recovery time.

4. Conclusions

All turbulent length scales are modelled in RANS. It has been the backbone for the last few decades in modern CFD method for simulating the turbulent flow due to its less costing computing requirement and affordable to use. This review is primarily concerned with the most well-known low Reynolds number k−ε turbulence models for such computer prediction in solving turbulence flows. The advantages of low Reynolds number k−ε turbulence models over standard models ($k-\omega$ and $k-\varepsilon$) are that it requires less mesh points, introduces the quite well establish near-wall distribution and reducing mesh sensitivity. The challenge posed in this statement is known. Still, it is also recognised that substantial improvements in the capabilities of RANS turbulence modelling have been made in the recent past, and it is hoped that this article will encourage researchers to concentrate their efforts and continue to make progress in future years.

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