CFD Letters 11, Issue 12 (2019) 53-65

Penerbit Akademia Baru

CFD Letters

Journal homepage: www.akademiabaru.com/cfdl.html
ISSN: 2180-1363

Boundary-Layer Flow and Heat Transfer of Blasius and Sakiadis Problems in Nanofluids with Partial Slip and Thermal Convection

Open Access

Nor Hathirah Abd Rahman[1*], Norfifah Bachok[1,2], Haliza Rosali[1]

[1] Department of Mathematics, Faculty of Science, Universiti Putra Malaysia, UPM Serdang, Selangor 43400, Malaysia
[2] Institute for Mathematical Research, Universiti Putra Malaysia, UPM Serdang, Selangor 43400, Malaysia

ARTICLE INFO

Article history:
Received 21 October 2019
Received in revised form 17 December 2019
Accepted 20 December 2019
Available online 30 December 2019

Keywords:
Heat transfer; Blasius; Sakiadis; Partial Slip; Nanofluids

ABSTRACT

This study aims to investigate the steady two-dimensional laminar boundary layer flow past a fixed (Blasius) or past a moving (Sakiadis) semi-infinite flat plate in water-based nanofluids with partial slip and thermal convective boundary condition. The similarity equations are solved numerically for three types of metallic or non-metallic nanoparticles such as copper (Cu), alumina (Al_2O_3), and Titania (TiO_2) in the base fluid of water with the Prandtl number Pr = 6.2 to investigate the effect of the solid volume fraction parameter φ of the nanofluids. The governing partial differential equations are transformed into a system nonlinear ordinary differential equation using a similarity transformation which is then solved numerically using a shooting method in Maple software. The numerical results are presented in tables and graphs for the skin friction coefficient C_f and local Nusselt number Nu which represents the heat transfer rate at the surface as well as the velocity and temperature profile for a range of various parameters such as nanoparticles volume fraction, slip parameter and Biot number. The results indicate that the solid volume fraction affects the fluid flow and heat transfer characteristics.

1. Introduction

In 1908, Blasius [1] was among the first who studied the problems of boundary layer flow past a static semi-infinite flat plate, without considering the heat transfer aspect. Blasius himself gave the matching inner and outer series solutions and the Blasius equation was never yielded to the exact analytical solution. While, in 1961, Sakiadis [2] had been investigated the boundary layer flow over a continuous solid surface moving with constant velocity. He found the exact equation as Blasius, but in different boundary conditions. Pantokratoras [3] had presented a theoretical study of the effect of variable fluid properties on the classical Blasius and Sakiadis flow. These investigation concerns about engine oil, water and air taking into account the variation of their physical properties with temperature. The result of numerical simulation of the governing equations and cover large

* *Corresponding author.*
E-mail address: norhathirah95@gmail.com (Nor Hathirah Abd Rahman)

temperature differences are obtained. Ishak *et al.,* [4] have studied the extended classical Blasius and Sakiadis equations, by considering a uniform free stream parallel to a fixed or moving flat plate, which has more practical significance. It is assumed that the plate is subjected to constant heat flux, and moves in the same or opposite direction to the free stream.

However, the above problems have not dealt with the nanofluid. In this field, there are two types of model that have been found which are Buongiorno [5] model and Tiwari and Das [6] model. Tiwari and Das [6] model analyse the behaviour of nanofluids into the nanoparticle volume fraction φ parameter. Ahmad *et al.,* [7] had investigated the classical problems of forced convection boundary layer flow and heat transfer past a semi-infinite static flat plate (Blasius problem) and past a moving semi-infinite slat plate (Sakiadis problem) using nanofluids. This problem solved by using the model proposed by Tiwari and Das [6] by the shooting method. It is found that the inclusion of nanoparticles into the base water fluid has produced the increasing of skin friction, heat transfer coefficients and volume fraction. Then, Bachok *et al.,* [8] had extended the Blasius and Sakiadis problems in nanofluids, by considering a uniform free stream parallel to a fixed or moving flat plate, which has more practical significance. The plate was assumed moves in the same or opposite direction to the free stream. The effect of the nanoparticle volume fraction parameter of the nanofluids on the heat transfer characteristics is investigated. The results indicate that dual solutions exist when the plate and the free stream move in opposite directions. Anuar and Bachok [9] have studied the classical problems of boundary layer flow and heat transfer characteristics past a semi-infinite static flat plate (Blasius problem) and past a moving semi-infinite slat plate (Sakiadis problem) in a water-based nanofluids with Prandtl number (Pr = 6.2) by using the effects of Brownian motion Nb, thermophoresis Nt and nanoparticle volume fraction φ parameters. Then, Devi and Suriyakumar [10] have presented a theoretical study on the effect of magnetic field on the classical Blasius and Sakiadis flow of nanofluids over an inclined plate. They choose two types of nanoparticles such as copper and alumina in the base fluid of water with Prandtl number, Pr = 6.2. The investigations on nanofluids were continued by Chan *et al.,* [11] and Zulkifli *et al.,* [12] who analyses effect on the moving surface with convective boundary condition and viscous dissipation, respectively.

Noghrehabadi *et al.,* [13] have analysed the development of the slip effects on the boundary layer flow and heat transfer over a stretching surface in the presence of nanoparticle fractions. In the modeling of nanofluid the dynamic effects including the Brownian motion and thermophoresis are considered. A similarity solution is presented in the case of constant wall temperature. The solution depends on a Prandtl number, slip factor, Brownian motion number, Lewis number, and thermophoresis number. These five parameters are numerically investigated for the dependency of the local Nusselt and local Sherwood numbers. But, they did not investigate the effects of a slip boundary condition in the presence of dynamic effects of nanoparticles yet. The slip parameter had strongly influenced the flow velocity and the surface shear stress on the stretching sheet and also reduced Nusselt number and reduced Sherwood number. The stability of unsteady boundary layer flow and heat transfer over stretching/shrinking sheet immersed in Cu-water nanofluid with the presence of partial slip, Soret and Dufour effects have been studied by Dzulkifli *et al.,* [14]. Najib *et al.,* [15] investigated on boundary layer flow and mass transfer near stagnation point past a stretching or shrinking cylinder in copper water nanofluid under consideration of chemical reaction and slip effect. The numerical results indicate that with examination of slip at the boundary causes to decrease the skin friction coefficient but increased the mass transfer rate. Then, the problem of the flow of H_2O-$C_2H_6O_2$ (50:50) based boehmite alumina nanofluid over a Blasius and Sakiadis in the presence of slip effects and viscous dissipation effects were analysed by Ganesh *et al.,* [16]. The problems show that as the slip conditions enhanced, the velocity profile is increase for Blasius case and decrease for Sakiadis case.

Bataller [17] had analysed the effects of thermal radiation on the laminar boundary layer about a flat-plate in a steady stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Ishak *et al.,* [18] had studied the steady laminar boundary layer flow over a moving plate in a moving fluid with convective surface boundary condition and in the presence of thermal radiation. They investigated under certain conditions, the present problem reduces to the classical Blasius and Sakiadis problems. They found that the heat transfer rate at the surface decreases in the presence of thermal radiation and convective boundary condition. Azam *et al.,* [19] have been studied the partial slip and convective boundary condition of the Carreau nanofluid with the presence of magnetic field. They found that by increasing the Biot number in shear thickenings and shear thinning fluids will increase the temperature and nanoparticle concentration. Then, Yasin *et al.,* [20] have been study the effect of thermal radiation and Newtonian heating on the stagnation point flow past a flat surface with the presence of magnetic field.

Therefore, in this paper, we extend the work of Ahmad *et al.,* [7] by including the effect of partial slip and thermal convective boundary condition on the steady two-dimensional laminar boundary layer flow past a Blasius and Sakiadis semi-infinite plate in a nanofluid by using Tiwari and Das [6] model. The effects of the partial slip, thermal convective boundary condition, solid volume fraction and the types of nanoparticles on characteristics of energy flow will be studied numerically and discussed further. For some particular cases of the present study, the results are compared with Ahmad *et al.,* [7] to support their validity.

2. Methodology

Consider the steady two-dimensional boundary layer flow past a fixed (Blasius) or past a moving (Sakiadis) semi-infinite flat plate in a water-based nanofluid containing different types of nanoparticles: Cu, Al_2O_3 and TiO_2. It is assumed that the nanofluid is incompressible, laminar flow and the viscous dissipation and radiation effects are neglected. Let u and v be the velocity components along x and y directions (Figure 1). The flow takes places at $y \geq 0$ where y is the coordinate measured normal to the surface. Further, we assume that the uniform temperature and the uniform nanofluid volume fraction at the surface of the plate are T_w and C_w, while the uniform temperature and the uniform nanofluid volume fraction far from the surface of the plate are T_∞ and C_∞, respectively.

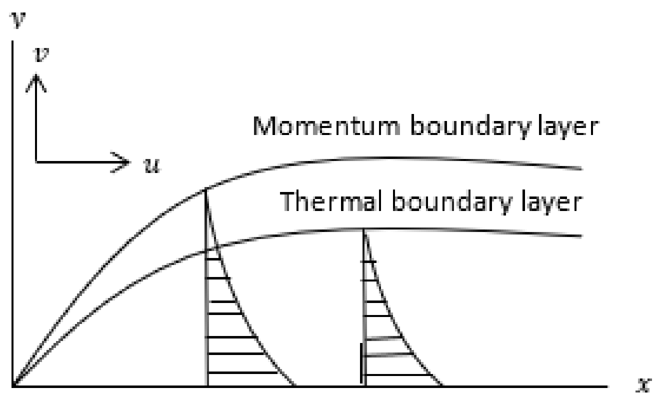


Fig. 1. Physical flow model of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

We assume that these equations are subject to the boundary conditions

i) Blasius problem

$$v=0, u=L\left(\frac{\partial u}{\partial y}\right) \text{ at } y=0$$

$$u=U \text{ as } y\longrightarrow\infty \tag{4}$$

ii) Sakiadis problem

$$v=0, u=U+L\left(\frac{\partial u}{\partial y}\right) \text{ at } y=0$$

$$u=0 \text{ as } y\longrightarrow\infty \tag{5}$$

Where u and v are the velocity components along the axes x and y. U is the constant velocity of the free stream or that of a moving flat plate and L denotes the slip length, T is the temperature of a nanofluid. The boundary conditions for the energy equation are

$$-k\left(\frac{\partial T}{\partial y}\right)=h_f(T_w-T) \text{ at } y=0, T=T_\infty \text{ as } y\longrightarrow\infty \tag{6}$$

It is that ρ is the pressure of nanofluid, μ_{nf} is the dynamic viscosity of the nanofluid, ρ_{nf} is the density of the nanofluid and α_{nf} is the thermal diffusivity of the nanofluid, which are given by Oztop and Abu-Nada [21].

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}, \rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s, \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}}$$

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)}, (\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s$$

Where φ is the nanoparticle volume fraction parameter of the nanofluid, k_{nf} is the thermal conductivity of the fluid fraction, k_s is the thermal conductivity of the nanoparticle volume fraction, ρ_f is the reference density of solid fraction, μ_f is the viscosity of the fluid fraction and $(\rho C_p)_{nf}$ is the heat capacitance of the nanofluids, where C_p is the specific heat at constant pressure. The viscosity

μ_{nf} of the nanofluid given by Brinkman [22] can be approximated as the viscosity of the base fluid μ_f containing a dilute suspension of fine spherical particles.
To obtain a similarity solution for Eqs. (1) – (6), the similarity transformation is introduced

$$\psi = \left(U\upsilon_f x\right)^{\frac{1}{2}} f(\eta), \theta(\eta) = \frac{(T-T_\infty)}{(T_w-T_\infty)}, \eta = (U/\upsilon_f x)^{\frac{1}{2}} y \quad (7)$$

Where υ_f is the kinematic viscosity of the fluid fraction and ψ is the stream function that is defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, which satisfy Eq. (1). By substituting variables Eq. (7) into Eqs. (2) and (3), the transformed ordinary differential equations are obtained

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)} f''' + \frac{1}{2} f f'' = 0 \quad (8)$$

$$\frac{1}{\Pr} \frac{k_{nf}/k_f}{\left[(1-\varphi)+\varphi(\rho C_p)_s/(\rho C_p)_f\right]} \theta'' + \frac{1}{2} f\theta' = 0 \quad (9)$$

The corresponding initial and boundary conditions (4), (5) and (6) are

$$f(0) = 0, f'(0) = \sigma f''(0), f'(\infty) \longrightarrow 1 \quad (10)$$

$$f(0) = 0, f'(0) = 1 + \sigma f''(0), f'(\infty) \longrightarrow 0 \quad (11)$$

$$\theta'(0) = Bi(\theta(0) - 1), \theta(\infty) \longrightarrow 0 \quad (12)$$

where primes denote differentiation with respect to η. $\Pr = \upsilon_f/\alpha_f$ is the Prandtl number,

$\sigma = L\left(\frac{U}{\upsilon_f x}\right)^{1/2}$ is the slip parameter and

$Bi = \frac{c}{k}\left(\frac{\upsilon_f}{U}\right)^{1/2}$ is the Biot number.

Quantities of pratical interest which are the skin friction coefficient C_f and the local Nusselt number Nu are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2}, Nu = \frac{x q_w}{k_f(T_w-T_\infty)} \quad (13)$$

Where $\tau_w$is the surface shear stress and q_w is the surface heat flux which can be expressed as

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (14)$$

With μ_{nf} is the dynamic viscosity of nanofluids and k_{nf} is the thermal conductivity of nanofluids. Using the variables of similarity (7), we obtained

$$Re_x^{1/2} C_f = \frac{1}{(1-\varphi)^{2.5}} f''(0), Re_x^{-1/2} Nu = -\frac{k_{nf}}{k_f} \theta'(0) \quad (15)$$

3. Numerical Scheme

The nonlinear ordinary differential Eqs. (8) and (9) subject to the boundary conditions (10) and (11) form a two-point boundary value problem (BVP) and has been solved numerically using the shooting method. It is an iterative algorithm technique implemented in Maple program which attempts to identify the appropriate initial conditions for a related initial value problem (IVP). Bhattacharyya and Layek [23] and Bhattacharyya *et al.,* [24] have been described this method in their papers. In this method, the suitable finite values of η, say η_∞, which depend on the values of the parameters were considered. First, high-order Eqs. (8) and (9) should be reduced to the first order equations by introducing some variables as follows

$$f' = p, p' = q, \frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)} q' + \frac{1}{2} f q = 0 \quad (16)$$

$$\theta' = r, \frac{1}{\Pr} \frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]} r' + \frac{1}{2} f r = 0 \quad (17)$$

With the boundary conditions

$$f(0) = 0, p(0) = \sigma q(0), p(\eta_\infty) = 1 \quad (18)$$

$$f(0) = 0, p(0) = 1 + \sigma q(0), p(\eta_\infty) = 0 \quad (19)$$

$$r(0) = Bi(\theta(0) - 1), \theta(\eta_\infty) = 0 \quad (20)$$

Now we have a set of 'partial' initial conditions

$$f(0) = 0, p(0) = \sigma q(0), q(0) = ?, \theta(0) = (r(0)/Bi) + 1, r(0) = ?. \quad (21)$$

As we notice, we do not have the values of $q(0)$ and $r(0)$, i.e., $f''(0)$ and $-\theta'(0)$. Thus, we are setting the different initial guesses for the values and apply the Runge-Kutta-Fehlberg method to see if this guess matches the boundary conditions at the very end. Differing the initial slopes gives rise to a range of profiles that indicate from the initial point the trajectory of a projectile 'shot'. That initial slope is sought which results in the trajectory 'hitting' the target, that is, the final value (Bailey *et al.,* [25]).

The velocity and the temperature profiles are needed to check, to determine the solution obtained is valid or not. The correct profiles must satisfy the boundary conditions at $\eta = \eta_\infty$ asymptotically. This process is repeated for other guessing values of $q(0)$ and $r(0)$ for same values of parameters.

4. Results and Discussion

The effect of the nanoparticle volume fraction φ, slip parameter σ and Biot number Bi are analysed for three different nanofluids: Cu, Al_2O_3 and TiO_2 as the working fluids and water as the base working fluid. The Prandtl number is taken to be Pr = 6.2 and nanoparticles value fraction is considered from 0 to 0.2 ($0 \leq \varphi \leq 0.2$), where $\varphi = 0$ is corresponding to the regular fluid. The thermophysical properties of the base fluid and nanoparticles are listed in Table 1.

Table 1
Thermophysical properties of the fluid phase (water) and nanoparticles Oztop and Abu-Nada [21]

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
C_p (J/kgK)	4179	385	765	686.2
ρ (kg/m^3)	997.1	8933	3970	4250
k (W/mK)	0.613	400	40	8.9538

Figures 2(a) until 3(b) illustrate the variations of the skin friction $f''(0)$ and local Nusselt number $-\theta'(0)$ for different nanoparticles, Biot number, nanoparticle volume fraction and also slip parameter for both Blasius and Sakiadis problem. These reduced skin frictions and reduced Nusselt number are shown graphically for several values of slip parameter σ and Biot number *Bi*. Figures 2 (a) and (b) illustrate the variations of the skin friction coefficient given by Eq. (15) with parameter φ for three different nanoparticles: Copper Cu, Alumina Al_2O_3 and Titania TiO_2, respectively when Pr = 6.2. It is interesting to observe that the skin friction coefficient increases almost monotonically with increasing φ but the reverse effect for slip parameter for both Blasius and Sakiadis problem. These figures show that Al_2O_3 has the lowest skin friction coefficient and the difference between the values for TiO_2 and Al_2O_3 is very small, as can be seen from Tables 2 and 3 and Figures 2 (a) and (b).

However, this behaviour is the same as the reported by Ahmad *et al.,* [7]. This is explained by looking at Eq. (11) where the ordinary differential equation shows that the nanoparticle volume fraction ' parameter and slip parameter σ give the effect to the result. Other than that, Tables 2 and 3 also shows the comparison of skin friction coefficient with Ahmad *et al.,* [7], which show a favourable agreement, thus give confidence that the numerical results obtained are accurate.

However, we are more interested to know the influenced of the nanoparticle volume fraction φ parameter and slip σ parameter towards the heat transfer rate. The variation of Nusselt number with φ for Blasius and Sakiadis problem are presented in Figures 3 (a) and (b) considering the various value of σ parameter. It is seen that Nusselt number increase when parameter φ increase and it is also increased when parameter σ increasing for Blasius problem but not in Sakiadis problem. From Figure 3 (a), Cu has the highest heat transfer rate, while TiO_2 has the lowest heat transfer rate compared to Cu due to domination of conduction mode of heat transfer. Table 1 clearly shows that TiO_2 has the lowest value of thermal conductivity compared with Cu and Al_2O_3. In contrast, for the Sakiadis problem, Al_2O_3 has the highest heat transfer rate. The thermal conductivity of Al_2O_3 is approximately one-tenth of Cu, as given in Table 1. However, it has its unique property which is low thermal diffusivity. A decrease in thermal diffusivity leads to a higher temperature gradient and thus will increase enhancement in heat transfers. However, Cu nanoparticle has a high value of thermal diffusivity and therefore, this automatically will reduce the temperature gradient and will affect the performance of Cu-water working fluid. In summation, it is noted that the lowest heat transfer rate obtained to TiO_2 nanoparticles.

Further, Figures 4 - 7 presents the velocity and temperature for both Blasius and Sakiadis problems for some values of nanoparticle volume fraction φ parameter ($0 \leq \varphi \leq 0.2$) when Pr = 6.2 , *Bi* = 0.1 and Cu-water as a working fluid. Figures 4 and 6 show that the momentum boundary layer increases with nanoparticle volume fraction φ for the Blasius problem, while decrease with nanoparticle volume fraction φ for the Sakiadis problem. However, Figures 5 and 7 show that the thermal boundary layer thickness increases with an increase in the parameter φ because of the increase in the local Nusselt number, as can be seen from Figures 3. Therefore, nanofluids are capable of changing the velocity and temperature profiles within the boundary layer.

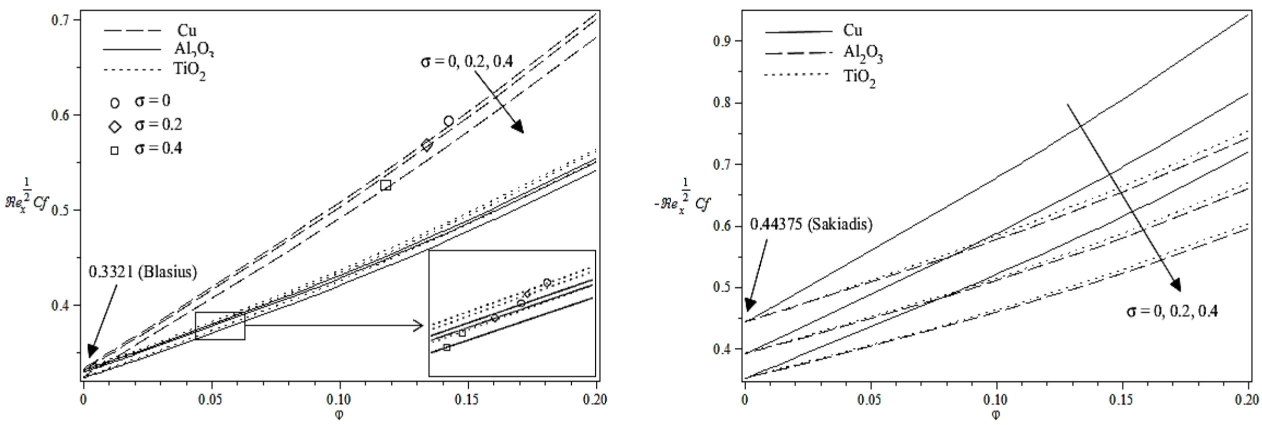
However, for Figures 8 - 11 presents the velocity and temperature for both Blasius and Sakiadis problems for different values of slip parameter σ, $(\sigma = 0, 0.2, 0.4)$ when Pr = 6, 2, *Bi* = 0.1, φ = 0.1 and Cu-water as working fluid. Figures 8 and 10 show that the momentum boundary layer increases with slip σ parameter for the Blasius problem but decreases for Sakiadis problem. It is same goes for the thermal boundary layer thickness in Figures 9 - 11 which is decreases for Blasius problem but increases for Sakiadis problems as the slip σ parameter increase. It can be seen that all these profiles are asymptotically satisfied all boundary conditions Eqs. (10) - (12). Hence, the numerical results we obtained are valid.

Table 2
Values of $Re_x^{1/2}C_f$ for the Blasius problem

σ	φ	Ahmad *et al.*, [7]			Present		
		Cu-water	Al_2O_3-water	TiO_2-water	Cu-water	Al_2O_3-water	TiO_2-water
0	0	0.3321	0.3321	0.3321	0.3321	0.3321	0.3321
	0.002	0.3355	0.3339	0.3340	0.3355	0.3339	0.3340
	0.004	0.3390	0.3357	0.3359	0.3390	0.3357	0.3359
	0.008	0.3459	0.3394	0.3398	0.3459	0.3394	0.3398
	0.01	0.3494	0.3412	0.3417	0.3494	0.3412	0.3417
	0.012	0.3528	0.3431	0.3436	0.3528	0.3431	0.3436
	0.014	0.3563	0.3449	0.3456	0.3563	0.3449	0.3456
	0.016	0.3597	0.3468	0.3476	0.3597	0.3468	0.3476
	0.018	0.3632	0.3487	0.3495	0.3632	0.3487	0.3495
	0.02	0.3667	0.3551	0.3515	0.3667	0.3551	0.3515
	0.1	0.5076	0.4316	0.4362	0.5076	0.4316	0.4362
	0.2	0.7066	0.5545	0.5642	0.7066	0.5545	0.5642
0.2	0				0.3298	0.3298	0.3298
	0.002				0.3332	0.3316	0.3317
	0.004				0.3367	0.3334	0.3336
	0.008				0.3435	0.3371	0.3375
	0.01				0.3469	0.3389	0.3394
	0.012				0.3503	0.3408	0.3413
	0.014				0.3537	0.3426	0.3432
	0.016				0.3571	0.3445	0.3452
	0.018				0.3605	0.3463	0.3471
	0.02				0.3639	0.3482	0.3491
	0.1				0.5030	0.4287	0.4332
	0.2				0.6996	0.5511	0.5606
0.4	0				0.3238	0.3238	0.3238
	0.002				0.3271	0.3256	0.3257
	0.004				0.3304	0.3274	0.3275
	0.008				0.3370	0.3309	0.3313
	0.01				0.3403	0.3327	0.3332
	0.012				0.3436	0.3345	0.3351
	0.014				0.3469	0.3364	0.3370
	0.016				0.3502	0.3382	0.3389
	0.018				0.3534	0.3400	0.3408
	0.02				0.3567	0.3418	0.3427
	0.1				0.4908	0.4209	0.4252
	0.2				0.6816	0.5419	0.5509

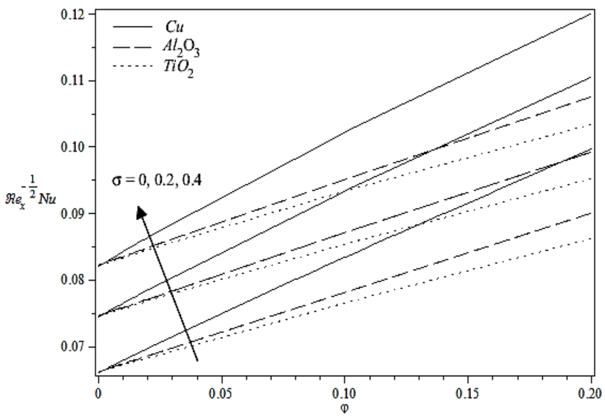
Table 3
Values of $-Re_x^{1/2}C_f$ for the Sakiadis problem

σ	φ	Ahmad *et al.*, [7]			Present		
		Cu-water	Al_2O_3-water	TiO_2-water	Cu-water	Al_2O_3-water	TiO_2-water
0	0	0.4446	0.4446	0.4446	0.4438	0.4438	0.4438
	0.002	0.4492	0.4470	0.4471	0.4484	0.4454	0.4455
	0.004	0.4538	0.4494	0.4497	0.4530	0.4462	0.4463
	0.008	0.4630	0.4544	0.4548	0.4530	0.4486	0.4489
	0.01	0.4676	0.4568	0.4574	0.4623	0.4535	0.4540
	0.012	0.4722	0.4593	0.4600	0.4669	0.4560	0.4566
	0.014	0.4768	0.4618	0.4626	0.4715	0.4585	0.4592
	0.016	0.4814	0.4643	0.4653	0.4808	0.4635	0.4645
	0.018	0.4860	0.4668	0.4679	0.4854	0.4660	0.4671
	0.02	0.4906	0.4693	0.4705	0.4900	0.4685	0.4697
	0.1	0.6788	0.5778	0.5840	0.6784	0.5768	0.5830
	0.2	0.9446	0.7428	0.7556	0.9442	0.7410	0.7540
0.2	0				0.3925	0.3925	0.3925
	0.002				0.3964	0.3925	0.3948
	0.004				0.4003	0.3968	0.3970
	0.008				0.4080	0.4011	0.4015
	0.01				0.4118	0.4033	0.4038
	0.012				0.4156	0.4055	0.4060
	0.014				0.4195	0.4076	0.4083
	0.016				0.4233	0.4098	0.4106
	0.018				0.4272	0.4120	0.4129
	0.02				0.4310	0.4142	0.4152
	0.1				0.5885	0.5103	0.5151
	0.2				0.8152	0.6588	0.6690
0.4	0				0.3531	0.3531	0.3531
	0.002				0.3564	0.3550	0.3551
	0.004				0.3598	0.3570	0.3571
	0.008				0.3664	0.3608	0.3611
	0.01				0.3697	0.3627	0.3631
	0.012				0.3729	0.3647	0.3652
	0.014				0.3762	0.3666	0.3672
	0.016				0.3795	0.3686	0.3692
	0.018				0.3828	0.3706	0.3713
	0.02				0.3861	0.3725	0.3733
	0.1				0.5219	0.4591	0.4631
	0.2				0.7205	0.5949	0.6033

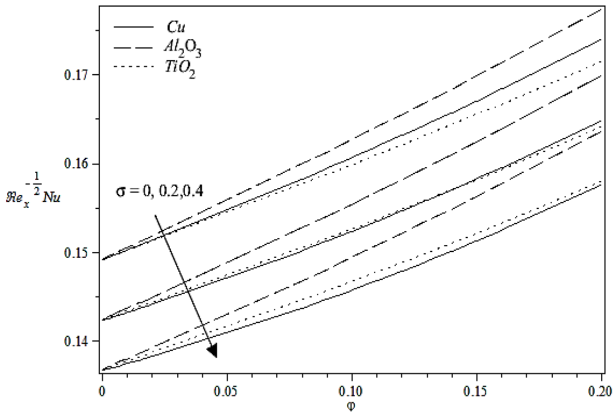


(a) (b)

Fig. 2. Variation of the skin friction coefficient with φ for (a) Blasius problem and (b) Sakiadis problem



(a)



(b)

Fig. 3. Variation of the Nusselt number coefficient with φ for (a) Blasius problem and (b) Sakiadis problem

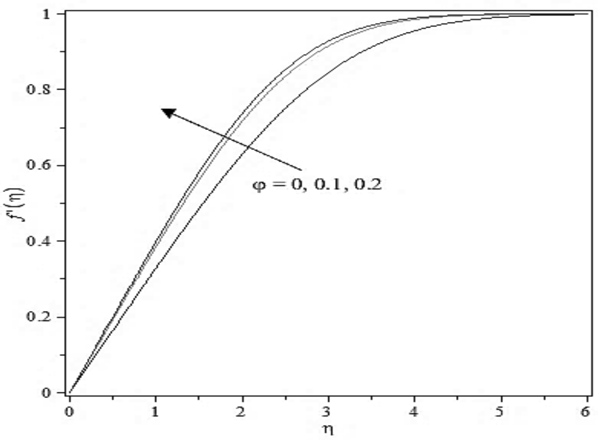


Fig. 4. Velocity profiles for various φ for the Blasius problem with Cu-water as working fluid with $\sigma =$ 0.2 and Pr = 6.2

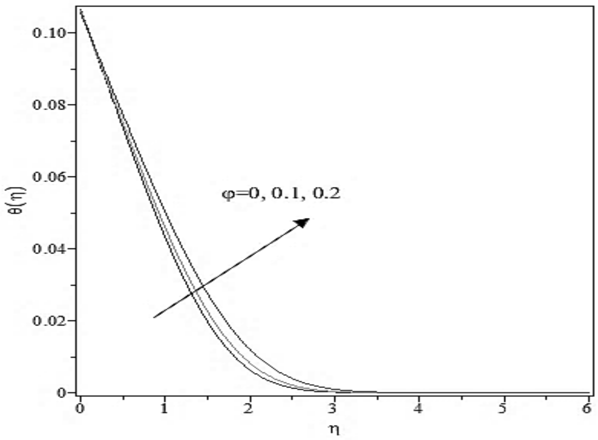


Fig. 5. Temperature profiles for various φ for the Blasius problem with Cu-water as working fluid with $\sigma = 0.2, Bi = 0.1$ and Pr = 6.2

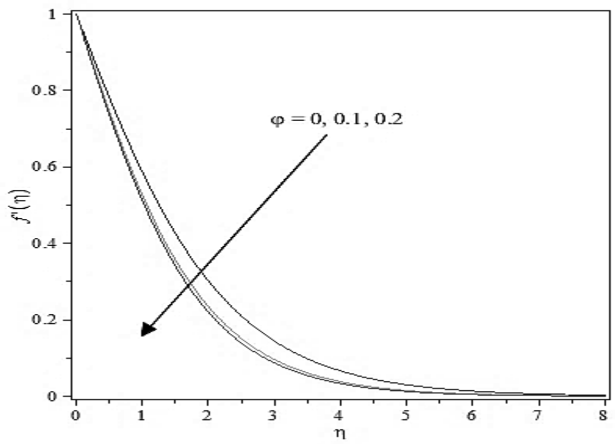


Fig. 6. Velocity profiles for various φ for the Sakiadis problem with Cu-water as working fluid with $\sigma =$ 0.2 and Pr = 6.2

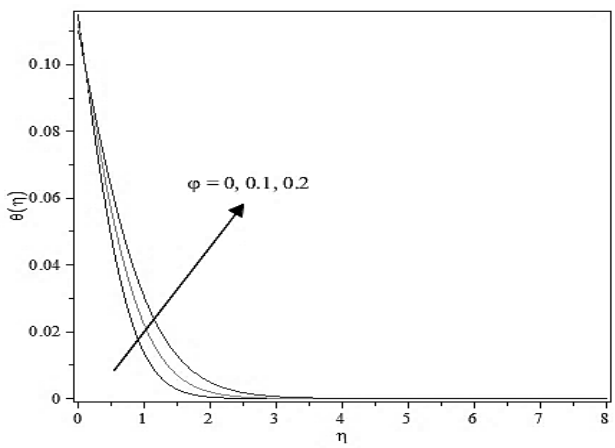


Fig. 7. Temperature profiles for various φ for the Sakiadis problem with Cu-water as working fluid with $\sigma = 0.2, Bi = 0.1$ and Pr = 6.2

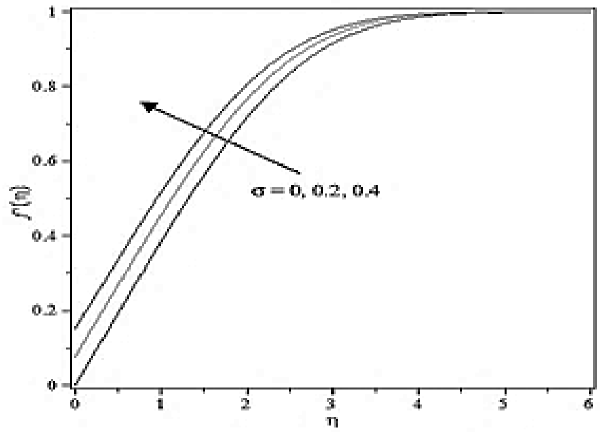


Fig. 8. Velocity profiles for various σ for the Blasius problem with Cu-water as working fluid with $\varphi = 0.1$ and Pr = 6.2

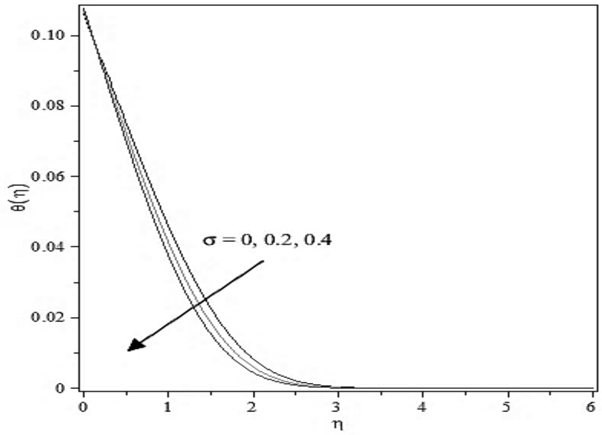


Fig. 9. Temperature profiles for various σ for the Blasius problem with Cu-water as working fluid with $\varphi = 0.1, Bi = 0.1$ and Pr = 6.2

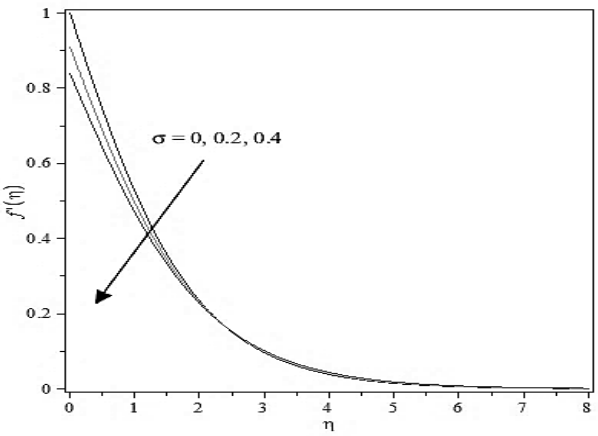


Fig. 10. Velocity profiles for various σ for the Sakiadis problem with Cu-water as working fluid with $\varphi = 0.1$ and Pr = 6.2

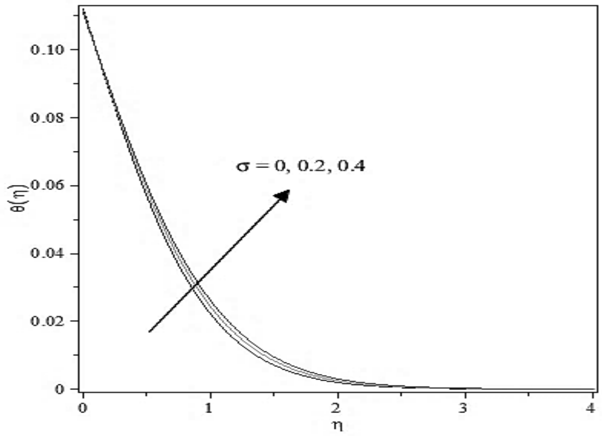


Fig. 11. Temperature profiles for various σ for the Sakiadis problem with Cu-water as working fluid with $\varphi = 0.1, Bi = 0.1$ and Pr = 6.2

5. Conclusions

We have studied theoretically and analysed the effects of partial slip and thermal convective boundary condition on the flow and the heat transfer characteristics for both Blasius and Sakiadis problems in the presence of nanofluids, and considered using the model proposed by Tiwari and Das [6]. The effects of nanoparticle volume fraction φ, slip parameter σ, as well as Biot number on the skin friction coefficient and heat transfer rate at the surface, were investigated and discussed. The problem was solved using a shooting method by Maple software. The results indicate that

i) As the slip parameter σ increase, the skin friction will decrease for both Blasius and Sakiadis problems, while the Nusselt number increase for Blasius problem but decrease for Sakiadis problem.
ii) As the Biot number increase, the thickness of the thermal boundary layer also increase for both Blasius and Sakiadis problems.
iii) The presence of nanoparticles into the base water fluid has produced an increase in the skin friction and the heat transfer (Nusselt number) coefficients, which increases significantly with

an increase in nanoparticle volume fraction. The addition of nanoparticles showed an improvement in the heat transfer rate.

iv) Nanofluids are capable of changing the velocity and temperature profiles in the boundary layer.

v) Cu has the highest values for both skin friction coefficient and Nusselt number in the base fluid of water with the Prandtl number Pr = 6.2 compared with TiO_2 and Al_2O_3.

Acknowledgement
The authors gratefully acknowledge the financial support received in the form of a fundamental research grant scheme (FRGS/1/2018/STG06/UPM/02/4/5540155). Lastly, a humble yet sincere gratitude to all the reviewers for their honest feedback and suggestions.

References
[1] Blasius, Heinrich. *Grenzschichten in Flüssigkeiten mit kleiner Reibung*. Druck von BG Teubner, 1907.
[2] Sakiadis, Byron C. "Boundary‐layer behavior on continuous solid surfaces: I. Boundary‐layer equations for two‐dimensional and axisymmetric flow." *AIChE Journal* 7, no. 1 (1961): 26-28.
[3] Pantokratoras, Asterios. "The Blasius and Sakiadis flow with variable fluid properties." *Heat and Mass Transfer* 44, no. 10 (2008): 1187-1198.
[4] Ishak, Anuar, Roslinda Nazar, and Ioan Pop. "Flow and heat transfer characteristics on a moving flat plate in a parallel stream with constant surface heat flux." *Heat and mass transfer* 45, no. 5 (2009): 563-567.
[5] Buongiorno, Jacopo. "Convective transport in nanofluids." *Journal of heat transfer* 128, no. 3 (2006): 240-250.
[6] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids." *International Journal of heat and Mass transfer* 50, no. 9-10 (2007): 2002-2018.
[7] Ahmad, Syakila, Azizah Mohd Rohni, and Ioan Pop. "Blasius and Sakiadis problems in nanofluids." *Acta Mechanica* 218, no. 3-4 (2011): 195-204.
[8] Bachok, Norfifah, Anuar Ishak, and Ioan Pop. "Boundary layer flow over a moving surface in a nanofluid with suction or injection." *Acta Mechanica Sinica* 28, no. 1 (2012): 34-40.
[9] Anuar, Nur Syazana, and Norfifah Bachok. "Blasius and Sakiadis problems in nano-fluids using Buongiorno model and thermo-physical properties of nano-liquids." *European International Journal of Science and Technology* 5, no. 4 (2016): 65-81.
[10] Anjali Devi, S. P., and P. Suriyakumar. "Effect of magnetic field on Blasius and Sakiadis flow of nanofluids past an inclined plate." *Journal of Taibah University for Science* 11, no. 6 (2017): 1275-1288.
[11] Chan, Sze Qi, Fazlina Aman, and Syahira Mansur. "Bionanofluid Flow Through a Moving Surface Adapting Convective Boundary Condition: Sensitivity Analysis." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 54, no. 1 (2019): 57-69.
[12] Zulkifli, Siti Norfatihah, Norhafizah Md Sarif, and Mohd Zuki Salleh. "Numerical Solution of Boundary Layer Flow over a Moving Plate in a Nanofluid with Viscous Dissipation: A Revised Model." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 56, no. 2 (2019): 287-295.
[13] Noghrehabadi, A., R. Pourrajab, and M. Ghalambaz. "Flow and heat transfer of nanofluids over stretching sheet taking into account partial slip and thermal convective boundary conditions." *Heat and Mass Transfer* 49, no. 9 (2013): 1357-1366.
[14] Dzulkifli, N. F., N. Bachok, I. Pop, N. A. Yacob, N. M. Arifin, and H. Rosali. "Stability of Partial slip, Soret and Dufour effects on unsteady boundary layer flow and heat transfer in Copper-water nanofluid over a stretching/shrinking sheet." In *Journal of Physics: Conference Series*, vol. 890, no. 1, p. 012031. IOP Publishing, 2017.
[15] Najib, N., N. Bachok, N. M. Arifin, F. M. Ali, and I. Pop. "Stability solutions on stagnation point flow in Cu-water nanofluid on stretching/shrinking cylinder with chemical reaction and slip effect." In *Journal of Physics: Conference Series*, vol. 890, no. 1, p. 012030. IOP Publishing, 2017.
[16] Ganesh, N. Vishnu, Qasem M. Al-Mdallal, K. Reena, and Sidra Aman. "Blasius and Sakiadis slip flow of H2O–C2H6O2 (50: 50) based nanoliquid with different geometry of boehmite alumina nanoparticles." *Case Studies in Thermal Engineering* 16 (2019): 100546.
[17] Bataller, Rafael Cortell. "Radiation effects for the Blasius and Sakiadis flows with a convective surface boundary condition." *Applied Mathematics and Computation* 206, no. 2 (2008): 832-840.

[18] Ishak, Anuar, Nor Azizah Yacob, and Norfifah Bachok. "Radiation effects on the thermal boundary layer flow over a moving plate with convective boundary condition." *Meccanica* 46, no. 4 (2011): 795-801.
[19] Azam, M., M. Khan, and A. S. Alshomrani. "Effects of magnetic field and partial slip on unsteady axisymmetric flow of Carreau nanofluid over a radially stretching surface." *Results in physics* 7 (2017): 2671-2682.
[20] Siti Hanani Mat Yasin, Muhammad Khairul Anuar Mohamed, Zulkhibri Ismail@Mustofa, Basuki Widodo, and Mohd Zuki Salleh."Numerical Solution on MHD Stagnation Point Flow in Ferrofluid with Newtonian Heating and Thermal Radiation Effect." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences.* 57, no. 1 (2019): 12-22.
[21] Oztop, Hakan F., and Eiyad Abu-Nada. "Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids." *International journal of heat and fluid flow* 29, no. 5 (2008): 1326-1336.
[22] Brinkman, H. C. "The viscosity of concentrated suspensions and solutions." *The Journal of Chemical Physics* 20, no. 4 (1952): 571-571.
[23] Bhattacharyya, Krishnendu, and G. C. Layek. "Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation." *International Journal of Heat and Mass Transfer* 54, no. 1-3 (2011): 302-307.
[24] Bhattacharyya, Krishnendu, Swati Mukhopadhyay, and G. C. Layek. "Slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet." *International Journal of Heat and Mass Transfer* 54, no. 1-3 (2011): 308-313.
[25] Bailey, Paul B., Lawrence F. Shampine, and Paul E. Waltman. *Nonlinear two point boundary value problems*. Academic press, 1968.