



Exact Analysis on Heat and Mass Flow Rates Computations over a Ramping Wall Boundary with C_8H_{18}/Al_2O_3 Based MHD Nanofluid Flow Past a Vertical Surface

Kayalvizhi Joshua ¹, and Vijaya Kuma Avula Golla ^{1,*}

¹ Department of Mathematics, School of Advanced Sciences Vellore Institute of Technology, Vellore-632014, India

* Correspondence: vijayakumarag@vit.ac.in

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ABSTRACT

An exact solution is derived for parabolic equations with partial differentials for the analysis of the ramping wall boundary of a nanofluid flow, suspended aluminium oxide nanoparticle in the Gasoline influenced by magnetic and gravitational forces in a semi-infinite flow region using integral transform method. Rosseland's approximation is used for radiative heat flow in the energy equation, whereas Bousinessq's approach is used in the momentum equation. Fluid temperature, species concentration, and transport are solved using Heaviside, exponential and complementary error functions; friction drag, heat and mass transfer rates are solved using Gaussian error functions. Numerical calculations have been carried out for rate of heat transmission and Sherwood number is swotted to put in the form of tables while the temperature, transport and species concentration are graphically exhibited. Higher radiation parameters lead to an increase in fluid temperature. The velocity boundary layer is decreased by the magnetic field and the opposite behaviour observed in porous media parameters. The heat transfer rate drops as Prandtl number and radiation parameter grows for both ramping and isothermal situations, whereas increases when time, volume fraction and heat source parameter increase. While increasing the schimidt number, mass transfer increase magnitudely for both cases.

Keywords:

Ramped wall conditions; gasoline; aluminium oxide; method of integral transform; radiative heat flux; magnetic induction

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1. Introduction

Nanotechnology is gaining popularity as a result of its important function of improving the heat capacity of liquids by boosting heat and mass transfer rates. Due to its wide variety of applications in biomedicine, heat exchangers, electronic device cooling, double windowpane, food, transportation

and other fields, the idea of nanofluids has become a more expansive topic for the research community in recent years. In order to improve the heat capacity of common fluids like water, kerosene, and motor oils etc., we must add various types of nanoparticles to the base fluids, such as graphene, silica, silver, gold, copper, alumina, carbon nanotubes, and so on. These nanofluids were introduced by Choi and Eastman [1]. A large number of research papers have been published in the literature that deals with improving the thermal conductivity of base fluids by adding various types of nanoparticles. Makinde has investigated the modelling of nanofluid flow [2]. Eastman *et al.*, [3,4] reported that when CuO nanoparticles with a volume fraction of 5 % were introduced to the base fluid (water), the heat conduction of the considered base fluid increases at most 0.6. Whereas 1 % of copper nanoparticles was introduced into ethylene glycol or oil, 40 % thermal conductivity was increased. This is due to metals have three times the thermal conductivity of general fluids, this one is permissible to carry out heat transmission with mix up of two constituents that act to be a fluid matter but possesses the thermal conductivity of metals. Nanofluids (water-Al₂O₃) in a two-dimensional horizontal pipe have been studied by Elfaghi *et al.*, [5] to increase heat transfer. Nura Muaz Muhammad *et al.*, [6] did research on a numerical investigation on the combined effect of aluminum-nitride/water nanofluid with different mini-scale geometries for passive hydrothermal augmentation

Many researchers are interested in discovering MHD in the combination of heat and mass transport with radiation impact due to its vast variety of applications. It is used in astrophysics and geophysics to investigate stellar and solar structures, as well as radio transmission via the ionosphere. It has uses in engineering, such as MHD pumps and MHD bearings. Mass transfer is a well-known phenomenon in stellar structure theory, and observable consequences may be seen on the surface of the sun. The thermal physics of hydromagnetic issues with mass transport has a lot of applications in power engineering. Thermal radiation plays an immense role in surface heat transfer, it is used in manufacturing, the design of dependable equipment, nuclear power plants, gas turbines, aeroplanes, missiles, satellites, space vehicles, space technologies, and procedures involving high temperatures. In the medical field, lukewarm radiation is in high demand. The impact of thermal radiation with doubling diffusion has been an important research issue for many experimenters because of its use in medical therapy. Infrared radiation is a common kind of heat treatment that is used in many different parts of the human body it is produced by oscillations in the electromagnetic field. Radiative heat transfer allows electromagnetic waves to carry vitality. It's a link between visible brightness and microwaving. It can help with a variety of skin issues. The wavelength of radiation penetrating the skin was determined by its radiating structure, vascularity, and pigmentation. Heat treatment is aided by infrared radiation, which warms the afflicted area's blood capillaries directly. It improves the body's blood flow, which helps to prevent infection in superficial wounds. It also draws attention to white blood cells while removing waste products.

Makinde *et al.*, [7] investigated the flow and heat transmission properties of an MHD nanofluid. Aly and Chamkha [8] investigated the numerical analysis of a nanofluid flow by a mode of natural convection in a steady state containing nanoparticles through upward porous plate under the action of magnetism. Thermal production or shrink, influenced by the magnetic force of action the thermal radiating processes identified by Nayak *et al.*, [9] were used to study 3D natural convection magnetohydrodynamic nanofluid flow through porous linear stretching surfaces. An analysis of radiation impacts on magnetohydrodynamics on the convective flow of nanofluids traversing a vertical stretching sheet was carried out by Turkyilmazoglu and Pop [10]. To investigate the effect of the radiative flux of heat on the thermal process and the flow of magnetic nanofluid, Sheikholeslami *et al.*, [11] developed a two-phase model. A study conducted by Chamkha [12] investigated the fluid flow by a convective mechanism at a sloping surface nearer to the porous zone. In addition, Chamkha

[13] explored the source of heat or sink impact in a magnetic fluid along with a magnetohydrodynamic flow over an accelerated porosity surface. Unsteadiness, magnetic fluid flow was seen on a motion of an up straight plate with nanofluid by M. Veera Krishna *et al.* [14], and a heat source or shrink effects were discovered. J. Prakash *et al.*, [15] explored the role of MHD free convection flow using a transversely applied magnetic field combined with a temperature gradient and mass diffusion in a porous medium. Non-Darcian natural convection flow of viscoelastic fluids between vertical plates has been addressed by Ewis *et al.*, [16]. Adnan Ashgar *et al.*, [17] computes a three-dimensional hybrid nanofluid flow across a stretching/shrinking sheet. Nurul Shahirah Mohd Adnan *et al.*, [18] examines the stability of a dual solution to the stagnation-point slip flow issue across a stretching or shrinking cylinder. Ansys Fluent software is used by Azraf Azman *et al.*, [19] to simulate the flow of hybrid nanofluids in a straight tube. Fluid flow and heat transfer toward a moving flat plate are studied by Mohammad Ferdows *et al.*, [20] in a stable free convective boundary layer. Shervin Sharafatmandjoor *et al.*, [21] tested the Influence of Imposition of Viscous and Thermal Forces on Dynamical Features of Microorganism Swimming in Nanofluids. Waqar Ahmed *et al.*, [22] evaluated Metal Oxide and Ethylene Glycol Based Well Stable Nanofluids for Mass Flow in a Covered Pipe.

With the magnetic field consideration, Hamad *et al.*, [23] inspected the features of spontaneously the mechanism of convective nanofluid flow along with the bounded vertical plate. The transport of nanofluid across an endless upward-directed plate, studied by Das and Jana [24]. Unsteady flow through an accelerated vertical plate embedded in pores material was studied by Hussain *et al.*, [25]. Sheikholeslami *et al.*, [26] examined convective MHD. $Al_2O_3-H_2O$ nanofluid transport. Das *et al.*, [27] demonstrated nanofluid transport and heat transport utilising magnetic Nano-fluid flow over a vertical stretching sheet. Chamkha and Aly reported on MHD convective nanofluid flow across a vertical plate in the presence of a heat source/sink [28]. The stagnation point flow of nanofluids was investigated studied by Soomroet *et al.*, [29] for heat generation/absorption and radiation impacts. For mixed free and forced convection flow, Hayat *et al.*, [30] studied the ability of an Oldroyd-B fluid in view of thermal source/shrink. Casson-type nanofluid, researched by Khan *et al.*, [31] within the presence of magnetic force and source and sink of heat radiative effects. E Kumaresan *et al.*, [32] researched to examine the effects of absorption of radiation which is chemically reacting in a nanofluid flow of with $CuO-H_2O$ and $MgO-H_2O$ nanoparticle combinations across a stretched sheet in pores material with source or shrink of heat. Rahimah Mahat *et al.*, [33] calculate the effect of viscous dissipation by a sodium carboxymethyl cellulose (CMC-water) nanofluid containing copper nanoparticles at ambient temperature with convective boundary conditions. Mohamad Hafzan Mohamad Jowsey *et al.*, [34] examine the heat and flow profile of nanofluid flow within a multilayer microchannel heat sink. T. W. Akaje *et al.*, [35] studied the impact of nonlinear radiative heat on the species heat transfer of an MHD Casson nanofluid flow with a Thompson and Troian boundary condition. By using pure water and $Fe_3O_4-H_2O$ as working fluids, Nor Azwadi Che Sidik [36] performed numerical analysis on the three-dimensional rectangular silicon microchannel heat sink (MCHS). According to Abdolbaqi Mohammed Khdher [37], three different nanofluids in a fully developed turbulent flow inside a long horizontal duct are being studied for pressure drop as well as secondary flow and convection heat transfer phenomena

Many studies are exploring ramped temperature profiles. The term "ramped temperature" refers to the fact that, for a period of time, the rate of change in temperature. But it should be remembered that the time domain for ramped profiles alter from substance to substance based on the material's specified heat capacity. Varying ramped curves of temperature may be found in real-world applications such as building air conditioning systems, thermal management of nuclear operations, processes of warming and freezing, designing of devices, material processing of phase transition,

thin-film photovoltaic device manufacturing, and heat exchangers. MHD natural convection flows with mass and heat transfer past an impulsively moving plate with ramping temperature was studied by Siva Reddy Sheri *et al.*, [38]. Convective mode of heat transfer besides mass transfer has been studied by Seth *et al.*, [39] in which Hall currents, rotation, radiation, and heat absorption are all considered. Magnetic convective flow transport on a moving normal plate was scrutinized by Seth *et al.* [40] to notice the impact of Hall currents.

Unsteady Newtonian fluid flow across a suddenly started infinitely long upright plate was originally addressed by Ahmed and Dutta [41] using ramping wall velocity and temperature. Chang *et al.* performed a study to reveal the ramping boundary temperatures on convective viscous fluid transport [42]. According to Kataria *et al.*, [43] analysed the importance of dufour and parabolic motion impacts over an infinitely upright surface, observed the unsteadiness, through the mode of convection, a second-grade fluid, in a porosity region with ramping wall temperatures and concentrations are studied for heat production and absorption. A computational study of convective magnetohydrodynamic heat flow by radiation over an impulsive plate with ramping fluid temperature was undertaken by Shri Siva Reddy *et al.*, [44], and Seth *et al.*, [45] developed an even more detailed study of ramping wall temperature. The influence of fluid temperature due to ramp over a plate boundary, the heat transmission is due to convection is investigated by Narahari *et al.*, [46]. In their work, Seth *et al.*, [47-48] investigated the mass and heat transfer properties of materials under a variety of physical conditions such as chemical reaction, Darcy's law and Hall current. While taking Hall current into consideration, M. VeeraKrishna *et al.*, [49] examined the thermo-diffusion and heat injection/suction impact in an unstable magnetic convective flow with radiation influence by chemical reaction of a type of second-grade fluid near an endless vertically projected plate. Analysis of the unstable magnetic convective flow of Casson fluid along the surface of the upright plate moving with exponential acceleration influence by a first-order chemical reaction with heat source/sink via embedded porosity zone was explored by Kataria *et al.*, [50,52]. Very recently, under ramping temperature and velocity circumstances at the wall, researchers V. Talha Anwar *et al.*, [51] evaluated the unstable Casson MHD fluid flow through an endless vertical plate. Chamkha and Veera Krishna and [53] examined, a second-grade fluid, in a permeable region using ramped thermal conditions. Under chemical reaction and heat source impact, Ansab Azam Khan *et al.*, [54] analyze the heat and mass transfer of MHD micropolar fluid flow including twofold stratification via a stretching/shrinking vertical sheet. This CFD research of convective heat transfer in a micro-pipe with mixed constant wall temperature and heat flux wall boundary conditions was carried out by Amjad Ali Pasha *et al.*, [55]. Free convection flow of Jeffrey nanofluid across a horizontal circular cylinder with viscous dissipation effect, as shown by Syazwani Mohd Zokri *et al.*, [56].

To the best knowledge of the author's, an unstable incompressible MHD nanofluid flow suspended boron and aluminium oxide nanoparticles in the base fluid that is exposed to ramped velocity, ramped temperature, and ramped wall concentration conditions at a vertical edge in a porous zone field has not yet been addressed. In addition, the radiation and heat generation/absorption are included in heat transfer in order to assess their relevance in the heat transfer process. Integral transformation techniques are applied to execute the results of modelled concentration, energy, and momentum equations, and they are implemented. Finally, with the use of tables and graphs, the physical characteristics of the necessary parameters are investigated.

2. Geometry and Formulation of the Problem

The rectilinear system of coordinates is cogitated. The plate is located at the origin along the y -axis and filled with the fluid in plane quadrant. An infinite plate is surrounded by the x' -axis in an

upward direction against to axis y' and the semi-region of $x'y'$ – plane filled with nanofluid contains boron and aluminium oxide nanoparticles in a pores zone. In the initial state ($t' \leq 0$) the plate surface and nanofluid overall the region is in static condition and after elapsing certain time ($t' > 0$), the plate is given a ramped motion $u' = u_0 t' / t_0$ at that instant, the plate temperature ramped $T' = T'_0 + (T'_w - T'_0) t' / t_0$ with increasing levels of concentration $C' = C'_0 + (C'_w - C'_0) t' / t_0$ all within the range of time $0 < t' \leq t_0$, after the certain characteristic time (t_0) plate velocity, temperature and concentration reaches to the isothermal state of condition. The effect of Lorentz force $\bar{J} \times \bar{B}$ is acting in x' – momentum, such that the magnetic force of lines impact developed by the momentum of electrified conductive fluid is ignorable. This assumption gives rise to as the magnetic Reynolds number is too tiny which is generally occurs in the case of aerodynamic applications. Since no outside electrical field is introduced and the effect of polarisation of the ionised field is neglected it is also assumed that the field of electricity $\bar{E} = 0$. Under the above assumption the components of electromagnetic induction are given by $B_x = 0, B_y = B(x)$ the ponderomotive $\bar{F} = \bar{J} \times \bar{B} / \sigma$ reduces to $F_x = (-\sigma B_0^2(x) / \rho) u, F_i = 0$ while radiative energy flux and a heat source are encountered in the energy equation. The buoyant force is the essential mechanism in driving the fluid through gravity is pulling down. Since the plate is taken infinite length, the boundary layer thickness is considerably small when compare, hence all the flow fields only function of one special coordinate and with respect to time. The fluid is supposed to be grey in colour and capable of both absorbing and emitting radiation, although it is not considered to be a scattering medium. The governing equations of momentum, energy, and concentration describing the fluid flow phenomena are formulated accordingly to simplify pressure gradient and body force in the Navier-Stokes equation, while the approximation of Rosseland was applied to approximate the thermal equation in radiative terms and presented in dimensionless form as follows:

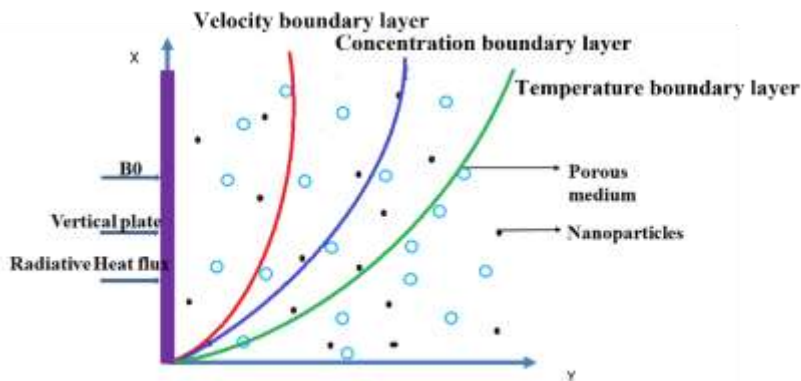


Fig. 1. Physical representation of the problem

As a result of the above analysis, the flow is governed by [53] such as,

$$\rho_{nf} \frac{\partial u'}{\partial t'} = \mu_{nf} \frac{\partial^2 u'}{\partial y'^2} + g(\rho\beta)_{nf} (T' - T'_0) + g(\rho\beta)_{nf} (C' - C'_0) - \sigma_{nf} B^2 u' - \frac{\mu_{nf} \Psi}{k} u \quad (1)$$

$$(\rho cp)_{nf} \frac{\partial T'}{\partial t'} = k_{nf} \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q'_r}{\partial y'} - Q'(T' - T_0') \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

The adequate corresponding initial and boundary conditions are given by [53,57]

$$T' = T_0', \quad C' = C_0', \quad u' = 0 \quad \text{for } 0 \leq y', t' > 0 \quad (4)$$

$$u' = \begin{cases} u_0' \frac{t'}{t_0'} & \text{if } 0 < t' < t_0 \\ u_0' & \text{if } t' \geq t_0 \end{cases} \quad \text{at } t' \geq 0 \text{ and } y' = 0 \quad (5)$$

$$T' = \begin{cases} T_0' - (T_0' - T_w') \frac{t'}{t_0'} & \text{if } 0 < t' < t_0 \\ T_w' & \text{if } t' \geq t_0 \end{cases} \quad \text{at } y' = 0, t' \geq 0 \quad (6)$$

$$C' = \begin{cases} (C_w' - C_0') \frac{t'}{t_0'} + C_0' & \text{if } 0 < t' < t_0 \\ C_w' & \text{if } t' \geq t_0 \end{cases} \quad \text{at } y' = 0, t' \geq 0 \quad (7)$$

$$\text{as } y' \rightarrow \infty \text{ when } t' \geq 0 \quad u' \rightarrow 0, \quad T' \rightarrow T_0', \quad C' \rightarrow C_0' \quad (8)$$

Famous astrophysicist Svein Rosseland provided an approximation of radiation flux for a boundary layer that is optically thick as the following:

$$\frac{\partial q'_r}{\partial y'} = -4a^1 \sigma^1 (T_\infty - T^4) \quad (9)$$

When expanding Taylor's series T^4 on the point T_∞ and omitting the terms from the second degree onwards, it has the form of:

$$T^4 \cong (4T - 3T_\infty)T_\infty^3 \quad (10)$$

Plugging Eq. (9) along with Eq. (10) in Eq. (3) yields

$$(\rho cp)_{nf} \frac{\partial T'}{\partial t'} = \left(k_{nf} + \frac{16\sigma_1 T_\infty^3}{3k_1} \right) \frac{\partial^2 T'}{\partial y'^2} - Q'(T' - T_0') \quad (11)$$

The physical properties of nanofluid provided by [38] as follows:

$$\begin{aligned} \mu_{nf} &= (1-\phi)^{-2.5} \mu_f, \quad \rho_{nf} = \phi \rho_s + \rho_f (1-\phi), \quad (\rho c p)_{nf} = (\rho c p)_s \phi + (1-\phi)(\rho c p)_f \\ (\rho \beta)_{nf} &= \phi(\rho \beta)_s + (1-\phi)(\rho \beta)_f, \quad \sigma_{nf} / \sigma_f = \left[1 + \frac{3(\sigma-1)\phi}{(\sigma+2)-\phi(\sigma-1)} \right], \quad \sigma = \frac{\sigma_s}{\sigma_f} \\ knf &= k_f \left[\frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \right] \end{aligned}$$

The base fluid and nanoparticles are denoted by the subscripts f and s, respectively, while the nanoparticles volume concentration is denoted by ϕ

Together with the corresponding parameters

$$\begin{aligned} y &= \frac{u_0 y'}{\gamma_f}, \quad t = \frac{u_0^2 t'}{\gamma_f}, \quad u = \frac{u'}{u_0}, \quad \theta = \frac{T' - T_w'}{T_w' - T_0'}, \quad C = \frac{C' - C_0'}{C_w' - C_0'}, \quad t_0 = \frac{\gamma_f}{u_0^2} \\ pr &= \frac{\mu c p}{k}, \quad M^2 = \frac{\sigma B_0^2}{\rho u_0^2} t_0, \quad \frac{1}{K} = \frac{\gamma_f^2 \psi}{k u_0^2}, \quad Nr = \frac{16 \sigma_1 T_\infty^3}{3 k_f k_1} \\ Gr &= \frac{\beta g \gamma (T_w' - T_0')}{u_0^3}, \quad Gm = \frac{\beta g \gamma (C_w' - C_0')}{u_0^3}, \quad sc = \frac{\gamma}{D} \end{aligned} \tag{12}$$

Eqs. (1), (2), and (11) are transformed into

$$\frac{\partial u}{\partial t} = a_1 \frac{\partial^2 u}{\partial y^2} + a_2 (Gr \theta + Gm C) - \left(a_3 M^2 + \frac{a_1}{K} \right) u \tag{13}$$

$$\frac{\partial \theta}{\partial t} = a_4 \frac{\partial^2 \theta}{\partial y^2} - a_5 \theta \tag{14}$$

$$\frac{\partial C}{\partial t} = \frac{1}{sc} \frac{\partial^2 C}{\partial y^2} \tag{15}$$

where

$$\begin{aligned} z_1 &= \left[1 - \phi + \phi \frac{(\rho)_s}{(\rho)_f} \right], \quad z_2 = \left[1 - \phi + \phi \frac{(\rho \beta)_s}{(\rho \beta)_f} \right], \\ z_3 &= \left[1 - \phi + \phi \frac{(\rho c p)_s}{(\rho c p)_f} \right], \quad z_4 = \left[1 + \frac{k_s - 2\phi(k_f - k_s) + 2k_f}{k_s + \phi(k_f - k_s) + 2k_f} \right], \end{aligned}$$

$$z_5 = \left[1 + \frac{3(\sigma - 1)}{(\sigma + 2) - (\sigma - 1)} \right]$$

$$a_1 = \frac{\mu_f}{(1 - \phi)^{2.5} z_1}, \quad a_2 = \frac{z_2}{z_1}, \quad a_3 = \frac{z_5}{z_1}, \quad a_4 = \frac{1}{z_3 pr} (z_4 + Nr), \quad a_5 = \frac{Q}{z_3}$$

The boundary conditions are taken into consideration

$$u = T = C = 0 \quad y \geq 0, \quad t < 0 \tag{16}$$

$$u = \begin{cases} t & \text{if } 0 < t \leq 1 \\ 1 & \text{if } t > 1 \end{cases} \quad \text{at } y = 0, t > 0 \tag{17}$$

$$T = \begin{cases} t & \text{if } 0 < t \leq 1 \\ 1 & \text{if } t > 1 \end{cases} \quad \text{at } y = 0, t > 0 \tag{18}$$

$$C = \begin{cases} t & \text{if } 0 < t \leq 1 \\ 1 & \text{if } t > 1 \end{cases} \quad \text{at } y = 0, t > 0 \tag{19}$$

$$\text{for } t > 0, \text{ as } y \rightarrow \infty, \text{ the fluid components leads to } u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \tag{20}$$

3. Solution of the Problem

The terminology specifies the physical factors that have shown themselves. The equations governed by the flow in non-dimensional form, listed from (13) to (15), solved with associated initial condition, along with the conditions defined at boundaries, from (16) to (20), by the usual Integral transform technique, and the resultant solutions are derived in form of exponential function, and Gaussian probability density function and unit step functions.

$$\bar{u}(y, s) = \left(\frac{1 - e^{-s}}{s^2} \right) e^{-y\sqrt{\beta s + \beta a^*}} + (1 - e^{-s}) \left[\frac{b}{ds^2} - \frac{b}{d^2s} + \frac{b}{d^2(s+d)} \right] \left(e^{-y\sqrt{\beta(s+a^*)}} - e^{-y\sqrt{\alpha(s+a_5)}} \right) + \left(1 - e^{-s} \right) \left[-\frac{b1}{d1s^2} - \frac{b1}{d1^2s} + \frac{b1}{d1^2(s+d1)} \right] \left(e^{-y\sqrt{\beta(s+a^*)}} - e^{-y\sqrt{ssc}} \right) \tag{21}$$

$$\bar{\theta}(y, s) = \exp(-y\sqrt{\alpha s + \alpha a_5}) \left(\frac{1 + (-e^{-s})}{s^2} \right) \tag{22}$$

$$\bar{C}(y, s) = \exp(-y\sqrt{ssc}) \left(\frac{1 - e^{-s}}{s^2} \right) \tag{23}$$

where

$$\alpha = \frac{1}{a_4}, \quad \beta = \frac{1}{a_1}, \quad a^* = a_3 M^2 + \frac{a_1}{K}, \quad a_6 = \frac{a_2}{a_1}, \quad b = \frac{Gra_2}{(a_1 \alpha - 1)}, \quad b_1 = \frac{Gma_2}{(a_1 sc - 1)}, \quad d = \frac{\alpha a_5 - a^*}{\alpha - 1},$$

$$d_1 = \frac{a^*}{(sc - 1)}$$

For inverse Laplace transform of Eqs. (21) – (23) we get

$$u(y, t) = \left[\begin{aligned} & \left(1 + \frac{b}{d} - \frac{b_1}{d_1} \right) F - \left(1 + \frac{b}{d} - \frac{b_1}{d_1} \right) F_1 H(t-1) + \left(\frac{b}{d^2} \right) G - \left(\frac{b}{d^2} \right) G_2 H(t-1) - \left(\frac{b}{d^2} \right) G_1 + \left(\frac{b}{d^2} \right) G_3 H(t-1) \\ & + \left(\frac{b_1}{d_1^2} \right) I - \left(\frac{b_1}{d_1^2} \right) I_2 H(t-1) - \left(\frac{b_1}{d_1^2} \right) I_1 + \left(\frac{b_1}{d_1^2} \right) I_3 H(t-1) - \left(\frac{b}{d^2} \right) R + \left(\frac{b}{d^2} \right) R_3 H(t-1) + \left(\frac{b}{d^2} \right) R_1 \\ & - \left(\frac{b}{d^2} \right) R_4 H(t-1) - \left(\frac{b}{d} \right) R_2 + \left(\frac{b}{d} \right) R_5 H(t-1) - \left(\frac{b_1}{d_1^2} \right) J + \left(\frac{b_1}{d_1^2} \right) J_3 H(t-1) + \left(\frac{b_1}{d_1^2} \right) J_1 \\ & - \left(\frac{b_1}{d_1^2} \right) J_4 H(t-1) + \left(\frac{b_1}{d_1} \right) J_2 - H(t-1) \left(\frac{b_1}{d_1} \right) J_5 \end{aligned} \right] \quad (24)$$

$$\theta(y, t) = R_2(y, t) - H(t-1)R_5 \quad (25)$$

$$C(y, t) = J_2(y, t) - H(t-1)J_5$$

4. Solution in the Case of Isothermal Plate with Uniform Boundary

These solutions may be beneficial to compare fluid transport with uniform temperature along an upright surface with uniform transport and concentration to emphasise the influence of ramping temperature, velocity and species concentration on flow transport phenomena. The results of MHD convective fluid flow for isothermal temperature, transport and concentration across a straight-up surface infinitely long enough with uniform boundary was found using the assumptions stated in this work and is expressed as follows:

The temperature, transport and concentration solution for isothermal condition from Eqs. (13) to (15)

$$u(y, t) = \left(1 + \frac{b}{d} - \frac{b_1}{d_1} \right) G_1 - \left(\frac{b}{d} \right) G - \left(\frac{b}{d} \right) R_4 + \left(\frac{b}{d} \right) R + \left(\frac{b_1}{d_1} \right) I - \left(\frac{b_1}{d_1} \right) J + \left(\frac{b_1}{d_1} \right) J_1 \quad (27)$$

$$\theta(y, t) = R_1(y, t) \quad (28)$$

$$C(y, t) = J_1(y, t) \quad (29)$$

where

$$\begin{aligned}
 F(y,t) &= \left[e^{y\sqrt{\beta a^*}} \left(t \left(\frac{1}{2} \right) + \frac{y\sqrt{\beta}}{4\sqrt{a^*}} \right) \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} + \sqrt{ta^*} \right) \right] + \left[\operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} - \sqrt{ta^*} \right) \left(t \left(\frac{1}{2} \right) - \frac{y\sqrt{\beta}}{4\sqrt{a^*}} \right) e^{-y\sqrt{\beta a^*}} \right] \\
 F_1(y,t-1) &= \left[e^{y\sqrt{\beta a^*}} \left((t-1)/2 + \frac{y\sqrt{\beta}}{4\sqrt{a^*}} \right) \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{-(1-t)}} + \sqrt{a^* t - a^*} \right) \right] + \left[e^{-y\sqrt{\beta a^*}} \left((t-1)/2 - \frac{y\sqrt{\beta}}{4\sqrt{a^*}} \right) \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{-(1-t)}} - \sqrt{a^* t - a^*} \right) \right] \\
 G(y,t) &= 0.5 e^{-dt} \left[e^{y\sqrt{\beta(a^*-d)}} \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} + \sqrt{(a^*-d)t} \right) + e^{-y\sqrt{\beta(a^*-d)}} \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} - \sqrt{(a^*-d)t} \right) \right] \\
 G_1(y,t) &= 0.5 \left[e^{-y\sqrt{\beta a^*}} \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} - \sqrt{ta^*} \right) + e^{y\sqrt{\beta a^*}} \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} + \sqrt{ta^*} \right) \right] \\
 G_2(y,t-1) &= 0.5 e^{-d(t-1)} \left[\operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{(t-1)}} + \sqrt{(a^* t - dt) - (a^* - d)} \right) e^{y\sqrt{\beta(a^*-d)}} + \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{(t-1)}} - \sqrt{(a^* t - dt) - (a^* - d)} \right) e^{-y\sqrt{\beta(a^*-d)}} \right] \\
 G_3(y,t-1) &= 0.5 \left[\operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{-(1-t)}} + \sqrt{(a^* t - a^*)} \right) e^{y\sqrt{\beta a^*}} + \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{-(1-t)}} - \sqrt{(a^* t - a^*)} \right) e^{-y\sqrt{\beta a^*}} \right] \\
 I(y,t) &= 0.5 e^{d_1 t} \left[\operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} + \sqrt{(a^* + d_1)t} \right) e^{y\sqrt{\beta(a^* + d_1)}} + \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} - \sqrt{(a^* + d_1)t} \right) e^{-y\sqrt{\beta(a^* + d_1)}} \right] \\
 I_1(y,t) &= 0.5 \left[\operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} + \sqrt{\beta t} \right) e^{y\sqrt{\beta a^*}} + \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{t}} - \sqrt{\beta t} \right) e^{-y\sqrt{\beta a^*}} \right] \\
 I_2(y,t-1) &= 0.5 e^{d_1(t-1)} \left[\operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{(t-1)}} + \sqrt{(a^* + d_1)(t-1)} \right) e^{y\sqrt{\beta(a^* + d_1)}} + \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{(t-1)}} - \sqrt{(a^* + d_1)(t-1)} \right) e^{-y\sqrt{\beta(a^* + d_1)}} \right] \\
 I_3(y,(t-1)) &= 0.5 \left[e^{y\sqrt{\beta a^*}} \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{(t-1)}} + \sqrt{\beta t - \beta} \right) + e^{-y\sqrt{\beta a^*}} \operatorname{erfc} \left(\frac{y\sqrt{\beta}}{2\sqrt{(t-1)}} - \sqrt{\beta t - \beta} \right) \right] \\
 R(y,t) &= 0.5 e^{-dt} \left[\operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{t}} + \sqrt{(a_5 - d)t} \right) e^{y\sqrt{\alpha(a_5 - d)}} + \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{t}} - \sqrt{(a_5 - d)t} \right) e^{-y\sqrt{\alpha(a_5 - d)}} \right] \\
 R_1(y,t) &= 0.5 \left[\operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{t}} + \sqrt{a_5 t} \right) e^{(\sqrt{\alpha a_5})y} + \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{t}} - \sqrt{a_5 t} \right) e^{-(\sqrt{\alpha a_5})y} \right] \\
 R_2(y,t) &= \left[\left(\frac{t}{2} + \frac{y\sqrt{\alpha}}{4\sqrt{a_5}} \right) e^{y\sqrt{\alpha a_5}} \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{t}} + \sqrt{a_5 t} \right) + \left(\frac{t}{2} - \frac{y\sqrt{\alpha}}{4\sqrt{a_5}} \right) e^{-y\sqrt{\alpha a_5}} \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{t}} - \sqrt{a_5 t} \right) \right] \\
 R_3(y,t-1) &= 0.5 e^{-d(t-1)} \left[\operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{-(1-t)}} + \sqrt{(t-1)(a_5 - d)} \right) e^{\sqrt{\alpha(a_5 - d)}y} + \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{-(1-t)}} - \sqrt{(t-1)(a_5 - d)} \right) e^{-\sqrt{\alpha(a_5 - d)}y} \right] \\
 R_4(y,t-1) &= 0.5 \left[\operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{-(1-t)}} + \sqrt{a_5 t - a_5} \right) e^{\sqrt{\alpha a_5}y} + \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{-(1-t)}} - \sqrt{a_5 t - a_5} \right) e^{-\sqrt{\alpha a_5}y} \right] \\
 R_5(y,t-1) &= \left[\left((t-1)/2 + \frac{y\sqrt{\alpha}}{4\sqrt{a_5}} \right) e^{y\sqrt{\alpha a_5}} \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{-(1-t)}} + \sqrt{a_5 t - a_5} \right) + \left((t-1)/2 - \frac{y\sqrt{\alpha}}{4\sqrt{a_5}} \right) e^{-y\sqrt{\alpha a_5}} \operatorname{erfc} \left(\frac{y\sqrt{\alpha}}{2\sqrt{-(1-t)}} - \sqrt{a_5 t - a_5} \right) \right] \\
 J(y,t) &= 0.5 e^{d_1 t} \left[e^{y\sqrt{d_1 sc}} \operatorname{erfc} \left(\frac{y\sqrt{sc}}{2\sqrt{t}} + \sqrt{d_1 t} \right) + e^{-y\sqrt{d_1 sc}} \operatorname{erfc} \left(\frac{y\sqrt{sc}}{2\sqrt{t}} - \sqrt{d_1 t} \right) \right]
 \end{aligned}$$

$$J_1(y,t) = \operatorname{erfc}\left(\frac{\sqrt{sc}}{2\sqrt{t}}y\right), J_2(y,t) = \operatorname{erfc}\left(\frac{y\sqrt{sc}}{2\sqrt{t}}\right)\left(\frac{y^2 sc}{2} + t\right) - ye^{-\left(\frac{y^2 sc}{4t}\right)}\sqrt{\frac{t sc}{\pi}}$$

$$J_3(y,t-1) = \frac{e^{d_1(t-1)}}{2} \left[\operatorname{erfc}\left(\frac{y\sqrt{sc}}{2\sqrt{(t-1)}} + \sqrt{d_1 t - d_1}\right) e^{y\sqrt{d_1 sc}} + \operatorname{erfc}\left(\frac{y\sqrt{sc}}{2\sqrt{(t-1)}} - \sqrt{d_1 t - d_1}\right) e^{-y\sqrt{d_1 sc}} \right]$$

$$J_4(y,t-1) = \operatorname{erfc}\left(\frac{\sqrt{sc}}{\sqrt{4(t-1)}}y\right)$$

4.1 Nusselt number

From the fluid temperature, heat flow rate can be computed from the non-dimensional form as

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} \quad (30)$$

Using Eq. (25), For Ramped wall temperature, we calculated the Nusselt number as follows:

$$Nu = -[N_1(t) + N_1(t-1)H(t-1)] \quad (31)$$

Using Eq. (28), for isothermal temperature, we calculated the Nusselt as follows:

$$Nu = -[N_2(t)] \quad (32)$$

$$\text{where, } N_1(t) = -\frac{\sqrt{t\alpha}}{2\sqrt{\pi}}\left(e^{-a_5 t} + e^{a_5 t}\right) - t\sqrt{a_5\alpha} \operatorname{erf}\left(\sqrt{a_5 t}\right) - \frac{\sqrt{\alpha}}{2\sqrt{a_5}} \operatorname{erf}\left(\sqrt{a_5 t}\right)$$

$$N_1(t-1) = -\left(e^{-a_5(t-1)} + e^{a_5(t-1)}\right)\left(\frac{\sqrt{(t-1)\alpha}}{2\sqrt{\pi}}\right) - (t-1)\sqrt{a_5\alpha} \operatorname{erf}\left(\sqrt{a_5(t-1)}\right) - \frac{\sqrt{\alpha}}{2\sqrt{a_5}} \operatorname{erf}\left(\sqrt{a_5(t-1)}\right)$$

$$N_2(t) = -\frac{\sqrt{\alpha}}{2\sqrt{\pi t}}\left(e^{-a_5 t} + e^{a_5 t}\right) - 2\sqrt{\alpha a_5} \operatorname{erf}\left(\sqrt{a_5 t}\right)$$

4.2 Sherwood Number

From fluid concentration, mass flow rate computed from the non-dimensional form as

$$sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (33)$$

We derived the sherwood number for as follows using Eq. (26):

$$sh = \sqrt{4sc/\pi} \left[\sqrt{t} - H(t-1)\sqrt{(t-1)} \right] \quad (34)$$

$$\text{where, } f(t) = -\frac{\sqrt{\beta t}}{2\sqrt{\pi}}(e^{-a^*t} + e^{a^*t}) - t\sqrt{a^*\beta} \operatorname{erf}(\sqrt{a^*t}) - \frac{\sqrt{\beta}}{2\sqrt{a^*}} \operatorname{erf}(\sqrt{a^*t})$$

$$f_1(t-1) = -\frac{\sqrt{\beta(t-1)}}{2\sqrt{\pi}}(e^{-a^*(t-1)} + e^{a^*(t-1)}) - \left(\sqrt{a^*\beta} \operatorname{erf}(\sqrt{a^*(t-1)})\right)(t-1) - \frac{\sqrt{\beta}}{2\sqrt{a^*}} \operatorname{erf}(\sqrt{a^*(t-1)})$$

$$g(t) = -\frac{\sqrt{\beta}}{\sqrt{\pi t}}(e^{-(a^*-d)t} + e^{(a^*-d)t}) - 2\sqrt{\beta(a^*-d)} \operatorname{erf}(\sqrt{(a^*-d)t})$$

$$g_1(t) = -\frac{\sqrt{\beta}}{\sqrt{\pi t}}(e^{-a^*t} + e^{a^*t}) - 2\sqrt{\beta a^*} \operatorname{erf}(\sqrt{a^*t})$$

$$g_2(t-1) = -\sqrt{\beta(t-1)\pi}(e^{-(a^*-d)(t-1)} + e^{(a^*-d)(t-1)}) - 2\sqrt{\beta(a^*-d)} \operatorname{erf}(\sqrt{(a^*-d)(t-1)})$$

$$g_3(t-1) = -\sqrt{\beta/(t-1)\pi}(e^{-a^*(t-1)} + e^{a^*(t-1)}) - 2\sqrt{a^*\beta} \operatorname{erf}(\sqrt{(t-1)a^*})$$

$$i(t) = -\sqrt{\beta/\pi t}(e^{-(a^*+d_1)t} + e^{(a^*+d_1)t}) - 2\sqrt{\beta(a^*+d_1)} \operatorname{erf}(\sqrt{(a^*+d_1)t})$$

$$i_1(t) = \sqrt{\beta/\pi t}(e^{-a^*t} + e^{a^*t}) - 2\sqrt{\beta a^*} \operatorname{erf}(\sqrt{a^*t})$$

$$i_2(t-1) = -\sqrt{\beta/\pi(t-1)}(e^{-(a^*+d_1)(t-1)} + e^{(a^*+d_1)(t-1)}) - 2\sqrt{\beta(a^*+d_1)} \operatorname{erf}(\sqrt{(a^*+d_1)(t-1)})$$

$$i_3(t-1) = -\sqrt{\beta/\pi(t-1)}(e^{-a^*(t-1)} + e^{a^*(t-1)}) - 2\sqrt{a^*\beta} \operatorname{erf}(\sqrt{(t-1)a^*})$$

$$r(t) = -\sqrt{\alpha/\pi t}(e^{-(a_5-d)t} + e^{(a_5-d)t}) - 2\sqrt{\alpha(a_5-d)} \operatorname{erf}(\sqrt{(a_5-d)t})$$

$$r_1(t) = -\sqrt{\alpha/\pi t}(e^{-a_5t} + e^{a_5t}) - 2\sqrt{\alpha a_5} \operatorname{erf}(\sqrt{a_5t})$$

$$r_2(t) = -\sqrt{\alpha t/4\pi}(e^{-a_5t} + e^{a_5t}) - t\sqrt{a_5\alpha} \operatorname{erf}(\sqrt{a_5t}) - \frac{\sqrt{\alpha}}{2\sqrt{a_5}} \operatorname{erf}(\sqrt{a_5t})$$

$$r_3(t-1) = -\frac{\sqrt{\alpha}}{\sqrt{\pi t - \pi}}(e^{-(a_5-d)(t-1)} + e^{(a_5-d)(t-1)}) - 2\sqrt{\alpha(a_5-d)} \operatorname{erf}(\sqrt{(a_5-d)(t-1)})$$

$$r_4(t-1) = -\frac{\sqrt{\alpha}}{\sqrt{\pi t - \pi}}(e^{-(a_5t-a_5)} + e^{(a_5t-a_5)}) - 2\sqrt{a_5\alpha} \operatorname{erf}(\sqrt{(t-1)a_5})$$

$$r_5(t-1) = -\sqrt{\alpha(t-1)/(4\pi)}(e^{-a_5(t-1)} + e^{a_5(t-1)}) - (t-1)\sqrt{a_5\alpha} \operatorname{erf}(\sqrt{a_5(t-1)}) - \frac{\sqrt{\alpha}}{2\sqrt{a_5}} \operatorname{erf}(\sqrt{a_5(t-1)})$$

$$j(t) = -\sqrt{\frac{sc}{\pi t}}(e^{-d_1t} + e^{d_1t}) - 2\sqrt{d_1sc} \operatorname{erf}(\sqrt{d_1t})$$

$$j_1(t) = -\sqrt{\frac{sc}{\pi t}}$$

$$j_2(t) = -2\sqrt{\frac{tsc}{\pi}}$$

$$j_3(t-1) = -\sqrt{\frac{sc}{(t-1)\pi}}(e^{-d_1(t-1)} + e^{d_1(t-1)}) - 2\sqrt{scd_1} \operatorname{erf}(\sqrt{(t-1)d_1})$$

$$j_4(t) = -\sqrt{\frac{sc}{\pi(t-1)}}, j_5(t-1) = -\sqrt{4(t-1)sc/\pi}$$

$$J_5(y, t-1) = \left(\frac{y^2 sc}{2} + (t-1)\right) \operatorname{erfc}\left(\frac{y\sqrt{sc}}{2\sqrt{(t-1)}}\right) - y\sqrt{\frac{(t-1)sc}{\pi}} e^{-\frac{y^2 sc}{4(t-1)}}$$

5. Parametric Study

To understand the physical mechanism of present problems, a parametric analysis has been accomplished. We have calculated solutions for the said problems and the values are represented in graphs and tables below. For both ramped ($0 < t \leq 1$) and isothermal ($t \geq 1$) wall boundaries, the physical characteristics of related quantities appearing in the solution model are studied and plotted graphs on flow transport, fluid temperature and species mass besides that tables including numerical simulations are used to explain the role of related parameters in determining heat and mass transmission. Nanoparticles and base fluid physical parameters are listed in Table 1. The Ramping case was depicted by solid lines, whereas the Isothermal case was represented by dotted lines.

Table 1
 Thermophoresis properties of water and nanoparticles [58]

Thermo-physical properties	Gasoline	Al ₂ O ₃
$\rho(kg / m)$	751	3970
$C_p(J/kgK)$	2.06	765
$k(w/mK)$	0.1164	46
$\beta \times 10^5 (K^{-1})$	8.591	0.0217
ϕ	0.00	0.2

The influence of the radiation parameter, as shown in Figure 2, demonstrates with increasing N_r , fluid temperature increases with both ramping and isothermal conditions for both boron and aluminium oxide nanoparticles. In both circumstances, thermal radiation improves the fluid temperature throughout the boundary area. It's stable because, this thermal radiation adds to diffuse energy since the thermal radiation parameter. Figure 3 depict the effect of ϕ (Nano-sized particle volume fraction) on fluid temperature, hence noticed that with increasing ϕ of the nanoparticle decreases the temperature profile for both Ramped and isothermal condition as well as for both nanoparticles.

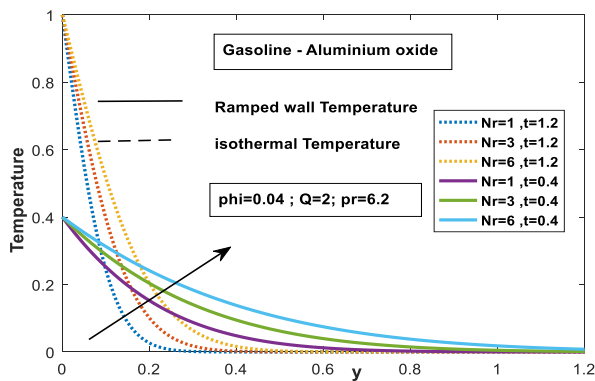


Fig. 2. Various values of N_r for temperature

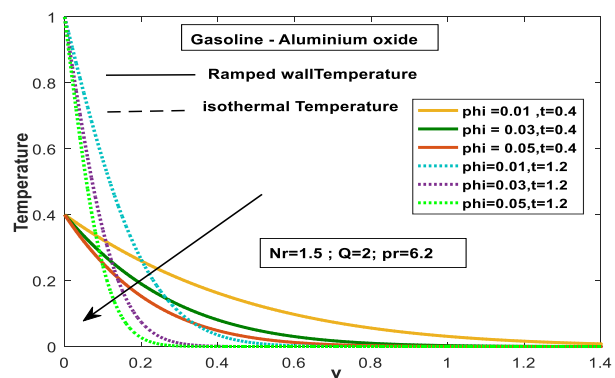


Fig. 3. Various values of volume fraction for temperature

The temperature profiles for various values of Prandtl number are highlighted in Figure 4. The impact of Pr on the distribution of temperature field in a case of ramped temperature as well as isothermal plates. It is noted that the nanofluid temperature diminishes as a result of enlarging Pr values in the case of isothermal. The essence of this decline is that fluids thermal conductivity is weakened by an upturn in Pr values, reducing heat transfer and therefore temperature. And then

Prandtl boundary layer width minimizes. But in ramped temperature, the temperature increased with increases of Prandtl number. The dual character of the flow profile is due to the nonuniform impact of time variables on the thermal boundary layer. The value of $t=0.4$ for ramped conditions and for isothermal condition $t=1.2$.

Figures 5 show the impact of heat source/sink on the temperature of the nanofluids. Notably, temperature decrease with increasing of heat source parameter. As the heat source parameter increases, more heat is absorbed, implying that the hotness of the fluid reduces with the higher magnitude of Q . Heat transmission may be successfully managed by adding a heat sink to the system.

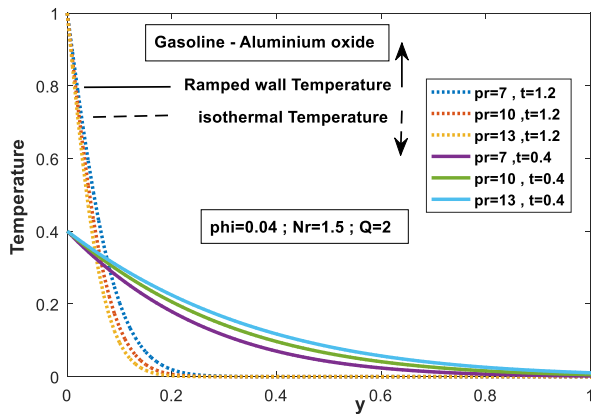


Fig. 4. Various values of Pr for temperature

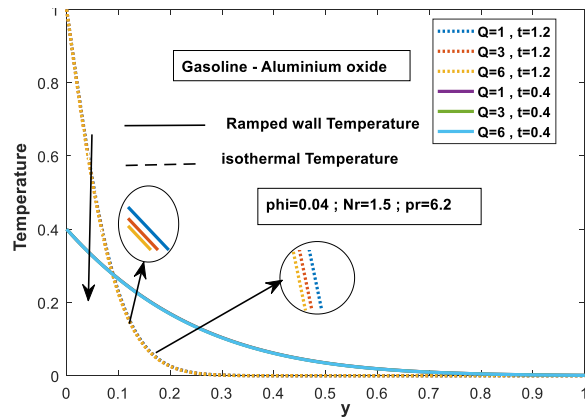


Fig. 5. Various values of heat source for temperature

Figures 6 illustrate how temperature grew as time t increased. Near the plate, the nanofluid's temperature is highest, and it gradually decreases until it reaches a point where it is no longer hotter than the free stream. For different magnitudes sc and t , the curve patterns on the species are represented in Figures 7-8. It has been noticed that in the ramped case also in constant species of mass, the concentration drops as a raise in Schmidt number, but increases as t increases.

The velocity profiles for different values of k are shown in Figure 9 while the other parameters are held constant. Increasing the value of k causes the thickness of the momentum boundary layer to rise, according to the graph. The physical explanation for this is that increasing k lowers the resistance given by a porous medium, which improves the momentum development of the regime and, as a result, increases the fluid's velocity.

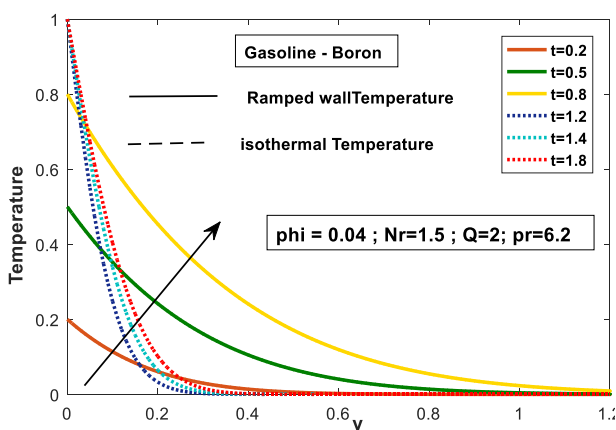


Fig. 6. Various values of t for temperature

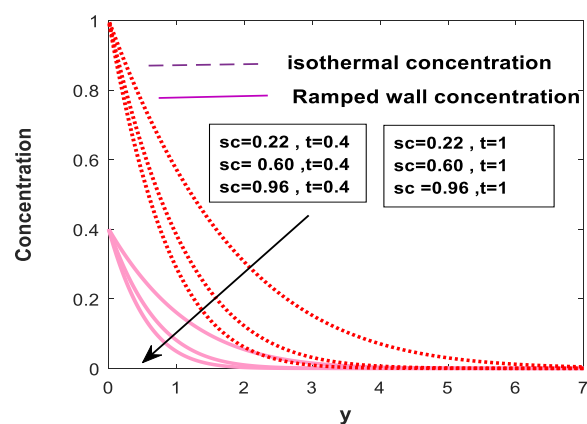


Fig. 7. Various values of sc for concentration

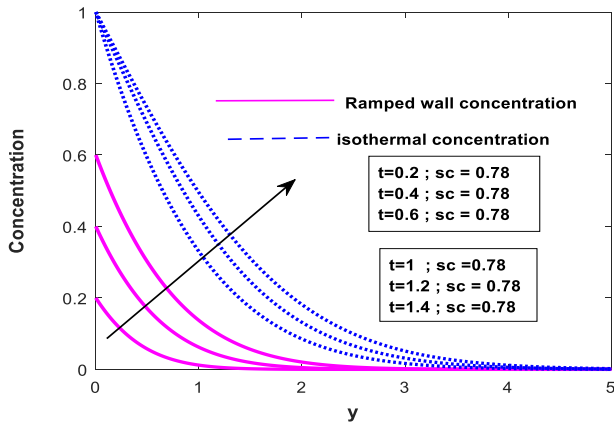


Fig. 8. Various values of t for concentration

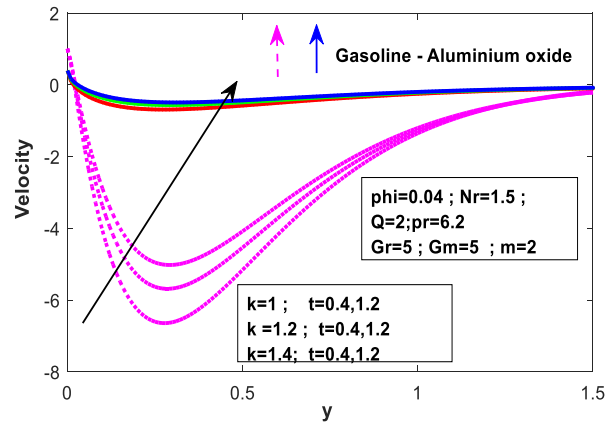


Fig. 9. Various values of porosity for velocity

Magnetic parameter (M) has an impact on velocity distribution in ramping and isothermal conditions, as seen in Figure 10. Nanofluid velocity has been demonstrated to have a negative effect on the function of M . Magnetic lines, also known as the Lorentz force, derived from Ohm's law and one of Maxwell's equations, may have a greater impact on the velocity boundary near the plate and induce slowdowns as a result of this influence. Lorentz force is produced once a magnetic field is applied to an electrified insulated nanofluid, which acts as a dragging force. When M is increased, the Lorentz force becomes more powerful, allowing the nanofluid to slowly come to a stop.

Figure 11 illustrates the influence of the volume fraction of nanoparticle ϕ on dimensionless velocity. In both ramping and isothermal conditions, the velocity of nanofluids is increased. But in general, adding nanoparticles to a fluid increases its density, which reduces both the boundary layer thickness as well as nanofluid velocity.

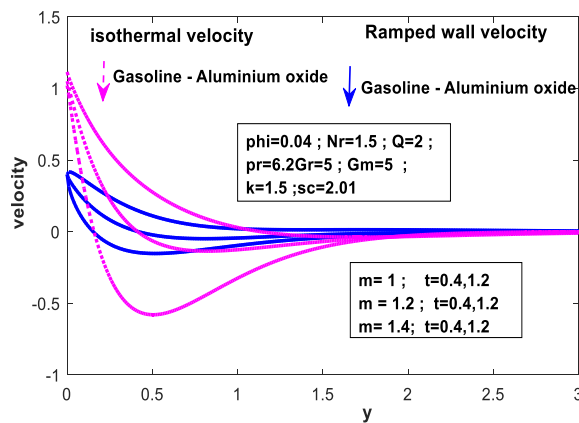


Fig. 10. Various values of magnetic field for velocity

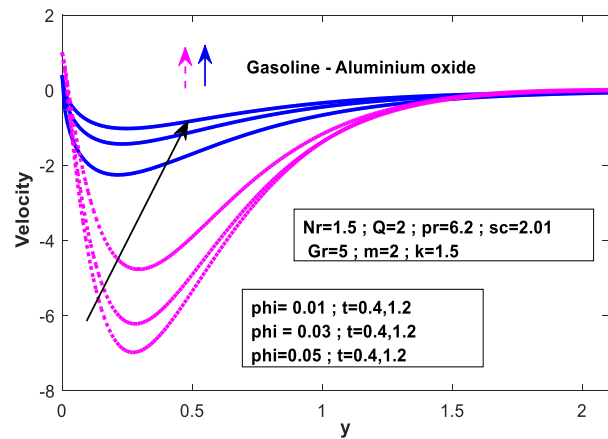


Fig. 11. Various values of porosity for velocity

Isothermal and ramping conditions, as well as the impacts of altering parameters, are depicted in Figures 12-16, some interesting facts noted down for variance parameters, the velocity function exhibits two distinct behaviours. The non-uniform impact of t on the momentum boundary layer thickness accounts for the dual character of the flow profile. For ramping conditions, $t = 0.4$, and for isothermal conditions, $t = 1.2$.

More exactly, Nr and Gr increase the velocity of nanofluid under isothermal conditions and reduce the velocity function under ramping conditions. The major cause of the velocity function's

opposing behaviour is the non-uniform influence of time t on the boundary layer thickness of momentum. The rate of $t=1.2$ under isothermal conditions, which is the reason of momentum boundary layer expands as the rate of Nr and Gr hike, further it decreases under the impact of ramping for increasing variation of Nr and Gr since the time value for this condition is fairly small ($t=0.4$). Figures 14-16 show the same explanation for the inverse effects: when the values of Q , pr , and Gm rise, the velocity for the ramped condition increases, while the velocity for the isothermal state falls due to time influence. The velocity profiles for various Schmidt numbers (Sc) are illustrated in Figure 17, and the velocity rises with higher Schmidt numbers. Schmidt number = kinematic viscosity / momentum diffusivity.

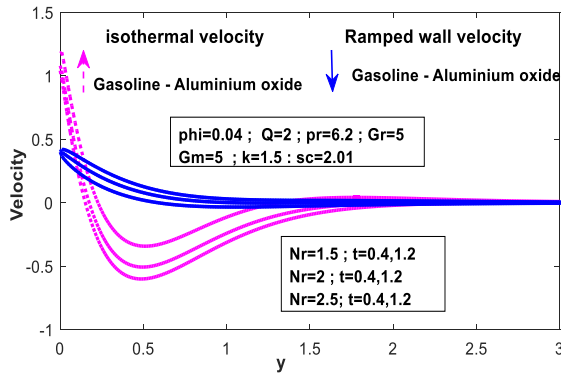


Fig. 12. Various values of Nr for velocity

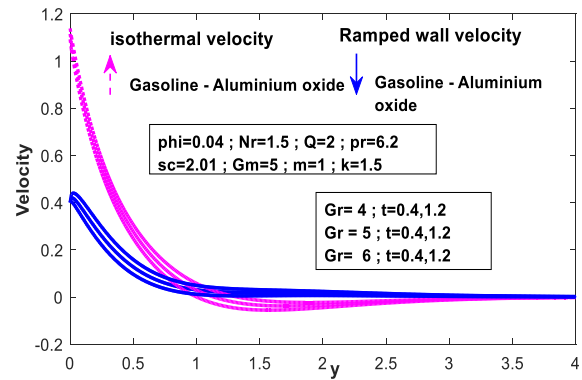


Fig. 13. Various values of Gr for velocity

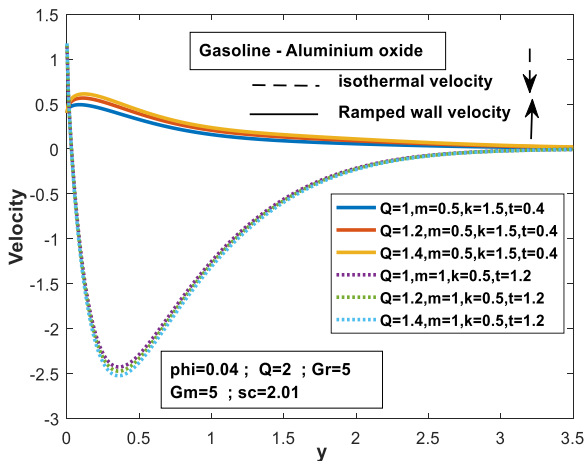


Fig. 14. Various values of heat source for velocity

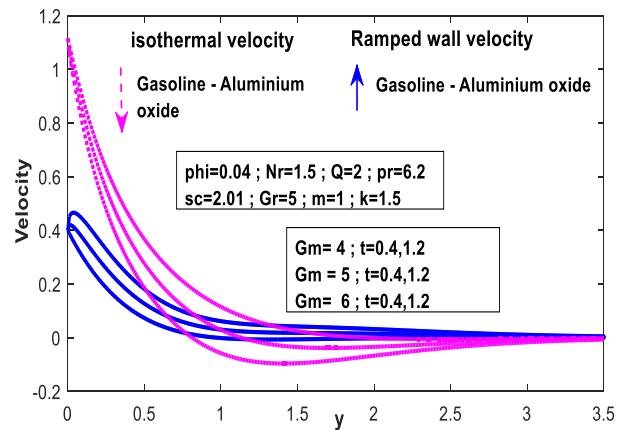


Fig. 15. Various values of Gm for velocity

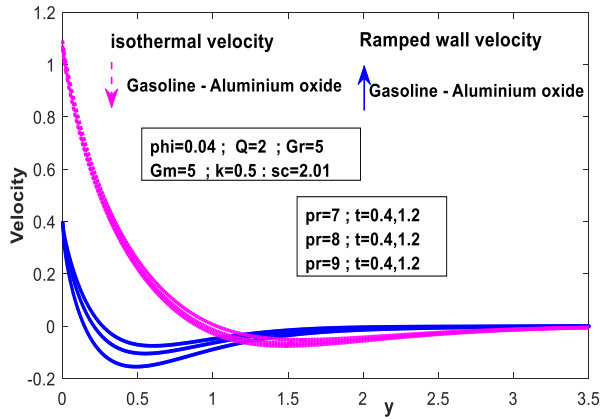


Fig. 16. Various values of Pr for velocity

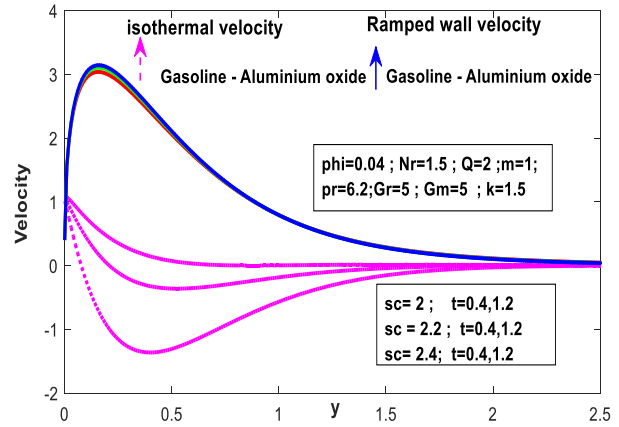


Fig. 17. Various values of sc for velocity

The non-dimensional mass flow rate, heat flow rate and shear stress rate for both (Al₂O₃-H₂O and B-H₂O) nanofluids for different values of several parameters are determined which was mentioned in Table 2 to Table 3. For both ramping and isothermal wall temperatures, the Sherwood number Sh rises in magnitudely with increasing t (Table 2). The Nusselt number Nu drops as pr, Nr, grows for both ramping and isothermal situations, whereas Nu increases when ϕ, Q increases (Table 3).

Table 2
 The Sherwood number variation

Sc	t		Shearwood number for isothermal temperature	Shearwood number for Ramped wall temperature
0.22	1.2	0.2	-0.3430809026	-0.2366908109
0.60	1.2	0.2	-0.5665794634	-0.3908820095
0.78	1.4		-0.5488644301	
0.78	1.6		-0.4886264860	
0.78		0.4		-0.6302783019
0.78		0.6		-0.7719301179

Table 3
 Nusselt number variation

ϕ	Pr	t	Q	Nr	Nusselt number for isothermal temperature		Nusselt number for ramped wall temperature	
					Aluminium Oxide (Al ₂ O ₃)		Aluminium Oxide (Al ₂ O ₃)	
0.01	6.2	1	0.2	2	1.5	0.972278976185802	0.340781511246800	
0.02	6.2	1	0.2	2	1.5	1.187241493920871	0.449726196194241	
0.03	6.2	1	0.2	2	1.5	1.360419306081635	0.533279605411406	
						1.280323188430491	0.501882207693629	
	7.0	1	0.2	2	1.5	1.197632678862229	0.469467817426820	
						1.197632678862229	0.469467817426820	
	8.0	1	0.2	2	1.5	0.874704446391998	0.588126469715958	
						0.874704446391998	0.588126469715958	
		1.2	0.3	2	1.5	0.792526868794893	0.692540897092866	
						0.792526868794893	0.692540897092866	
		1.4	0.4	2	1.5	0.792526868794893	0.788047937296101	
						0.792526868794893	0.788047937296101	
			0.5	2	1.5	0.809946420435267	0.794579969376230	
						0.809946420435267	0.794579969376230	
				2.2	1.5	0.826972053814545	0.800920757393285	
						0.826972053814545	0.800920757393285	
				2.4	1.5	0.796803058413768	0.771702146516740	
						0.796803058413768	0.771702146516740	
					1.7	0.769711938960168	0.745464452254491	
					1.9	0.769711938960168	0.745464452254491	

6. Conclusion

Present investigation PDEs with ramping wall boundary conditions of a nanofluid flow, suspended boron and aluminium oxide nanoparticles in the base fluid, driven by magnetic and gravitational forces in a semi-infinite flow area was solved using the integral transform approach. In the momentum equation, Boussinesq's approximation is used to simplify the pressure gradient and body force, whereas, in the energy equation, temperature-dependent heat absorption and optically thick heat radiating are used. Heaviside, exponential, and complementary error functions were used to address fluid temperature, transport, and species concentration.

Noteworthy results are summarized below:

- i) The velocity profile for both nanofluids are reduced as increasing magnetic field parameter
- ii) For both nanofluids the velocity profiles are increased as an increase of porosity medium
- iii) The momentum boundary layer and concentration boundary layer reduce as an increase in nanoparticles volume fraction, but the energy boundary layer is increased for both the nanofluids
- iv) In both ramped and isothermal conditions, the temperature profiles increase with increasing the values of heat absorption/ generation parameter for both the nanofluids. But in the velocity profile its increases in isothermal, whereas it is noticed that the opposite reaction in ramped condition due to the time nature.
- v) Temperature increases with increasing of time t
- vi) Concentration decreases with an increase in Schmidt number Sc while increases with the progress of time.

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